Common Fixed point theorems for fuzzy mappings in intuitionistic fuzzy metric spaces and application to ordinary differential equations

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Abstract. This paper present some common fixed point results for a pair of fuzzy mappings satisfying an almost generalized contractive condition in partially ordered complete intuitionistic fuzzy metric spaces by employing the idea of combining the ideas in the contraction principle with those in the monotone iterative technique. Motivated by this, the result generalizes the recent result of Nashine et al. [9] results and other existing results in intuitionistic fuzzy metric space. Also some examples and an application are given to illustrate the result.

1.INTRODUCTION

In many branches of mathematical analysis the Banach contraction principle [3] is a very popular tool to solve existence problems. Zadeh [19] investigation of the notion of fuzzy set has led to rich growth of fuzzy Mathematics. Many authors as Singh and chouhan [13], Jain et al. [4], Verma and Chandel [18] have studied the concept of fuzzy metric space. Heilpern [6] introduced the concept of fuzzy mappings and proved a fixed point theorem for fuzzy contraction mapping. George and Veeramani [5] modified the concept of fuzzy metric space. Atanassov [2] introduced and studies the concept of intuitionistic fuzzy sets. Further, using the idea of intuitionistic fuzzy metric set, Alaca et al. [1] defined the notion of intuitionistic fuzzy metric space. Park [10] introduced a notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t- conorms, as a generalization of fuzzy metric space due to Kramosil and Michalek [8]. In this paper a common fixed point theorem proved for a pair of fuzzy mappings without taking into account any commutativity condition in complete ordered intuitionistic fuzzy metric spaces. The main result is based on an almost generalized contractive condition and generalizes the result of Nashine et al. [9].

Definition 1.2 [7] $\varphi: [0, +\infty) \to [0, +\infty)$ is called an altering distance function if the following properties are satisfied:

1. φ is continuous and nondecreasing,
2. $\varphi(t) = 0 \Leftrightarrow t = 0$.

Theorem 1.1 [3] Let (X, d) be a complete metric space and T be a mapping of X into itself satisfying $d(Tx, T\,y) \leq k(x, y)$ for all $, y \in X$,
where k is a constant in (0, 1). Then T has a unique fixed point$x \in X$. There is a great number of generalizations of the Banach contraction principle. Taskovic [16] presented a comprehensive survey of such results in metric spaces. A new category of contractive fixed point problems was addressed by Khan et al. [7] that introduced the concept of altering distance function which is a control function that alters distance between two points in a metric space.

2. BASIC DEFINITIONS AND PRELIMINARIES

Definition 2.1[9] An intuitionistic fuzzy set A in a universe X is an object $A = \{x, \mu_A(x), \gamma_A(x): x \in X\}$, where for all $x \in X, \mu_A(x)(\in [0,1])$ is called the degree of membership of x in A, $\gamma_A(x)(\in [0,1])$ is called the degree of non - membership of x in A, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for every $x \in X$.

Definition 2.2[9] A fuzzy set A in a metric linear space is said to be an approximate quantity if and only if A_α is compact and convex in X for each $\propto \in (0, 1]$ and $sup_{x\epsilon X} Ax = 1$. Let I = [0, 1] and $W(X) \subset I^X$ be the collection of all approximate quantities in X. For $\propto \in [0, 1]$, the family $W_\alpha(X)$ is given by $\{A \in I^X: A_\alpha$ is nonempty and compact$\}$.

Definition 2.3[9] Let $A, B \in V(X)$ where V(X) denotes metric space,$\propto \in [0, 1]$, then

$$p_\alpha(A,B) = \inf_{x\in A_\alpha, y\in B_\alpha} d(x,y), \qquad D_\alpha(A,B) = H(A_\alpha, B_\alpha),$$
$$p'_\alpha(A,B) = \sup_{x\in A_\alpha, y\in B_\alpha} d(x,y), \qquad D'_\alpha(A,B) = H'(A_\alpha, B_\alpha),$$

where H and H' are the Hausdorff distance.

Definition 2.4[9] Let $A, B \in V(X)$. Then A is said to be more accurate than B denoted by $A \subset B$, iff $Ax \leq Bx$ for each $x \in X$. For $\alpha \in (0,1]$ the fuzzy point x_α of X is the fuzzy set of X given by $x_\alpha(x) = \alpha$ and $x_\alpha(z) = 0$ if $z \neq x$.

Definition 2.5[14] Let x_α be a fuzzy point of X. Then x_α is a fixed fuzzy point of the fuzzy mapping F over X if $x_\alpha \subset F_x$(i.e. the fixed degree of x for F, say $(Fx)(x)$, is at least α.

In particular and according to [6], if $\{x\} \subset Fx$, we can say that x is a fixed point of F.

Definition 2.6[12]. A binary operation *:[0,1]×[0,1]→[0,1] is called a *t-norm* * satisfies the following conditions:

i. * is commutative and associative,
ii. * is continuous,
iii. a * 1 =a for all a ∈ [0, 1],
iv. a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0,1].

Examples of *t-norm:* a * b = ab and a * b=min{a, b}.

Definition 2.7[1]. A binary operation ◊:[0,1]×[0,1]→[0,1] is said to be continuous t-co norm if it satisfied the following conditions:

i. ◊ is associative and commutative,
ii. ◊ is continuous,
iii. a ◊ 0 = a for all a ∈ [0,1],
iv. a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for each a, b, c, d ∈ [0,1]

Examples of t-conorm : a ◊ b = min(a+b , 1) and a ◊ b = max(a, b)

Remark 2.1.[12] The concept of triangular norms (*t-norm) and* triangular conorms (*t-conorm)* are knows as axiomatic skeletons that we use for characterizing fuzzy intersections and union respectively.

Definition 2. 8[1]. A 5- tuple (X, M, N, *, ◊) is called intuitionistic fuzzy metric space if X is an arbitrary non empty set, * is a continuous *t-norm,* ◊ continuous *t-conorm* and M, N are fuzzy sets on $X^2 \times [0,\infty]$ satisfying the following conditions:

For each x, y, z, ∈ X and t, s > 0

(IFM-1) $M(x,y,t) + N(x,y,t) \leq 1$,
(IFM-2) $M(x,y,0) = 0$, for all $x, y \in X$
(IFM-3) $M(x,y,t) = 1$ for all $x,y \in X and\ t > 0$ if and only if $x = y$
(IFM-4) $M(x,y,t) = M(y,x,t)$, for all $x,y \in X and\ t > 0$
(IFM-5) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,
(IFM-6) $M(x,y,.): [0,\infty] \to [0,1]$ is left continuous,
(IFM-7) $\lim_{t\to\infty} M(x,y,t) = 1$,
(IFM-8) $N(x,y,0) = 1$, for all x, y in X,
(IFM-9) $N(x,y,t) = 0$, for all x, y in X and t > 0 if and only if x = y,
(IFM-10) $N(x,y,t) = N(y,x,t)$, for all x, y in X and t>0,
(IFM-11) $N(x,y,t) \lozenge N(y,z,s) \geq N(x,z,t+s)$,
(IFM-12) $N(x,y,.): [0,\infty) \to [0,1]$ is right continuous,
(IFM-13) $\lim_{t\to\infty} N(x,y,t) = 0$, for all x, y in X and $t > 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The function $M(x,y,t)$and $N(x,y,t)$ denote the degree of nearness and degree of non- nearness between x and y with respect to t, respectively.

Remark 2.2.[17]. An Intuitionistic Fuzzy Metric space with continuous t-norm * and continuous *t-conorm* ◊ defined by a * a ≥ a, and (1-a) ◊ (1-a) ≤ (1-a)for all a ∈ [0,1]. Then for all $x,y \in$ X, $M(x,y,*)$is non decreasing and $N(x,y,\lozenge)$ is non increasing.

By using the concept of Nashine et al. [9] now define some definitions as follows:

Definition 2.9 Let (X, M, N) be a intuitionistic fuzzy metric space,$x, y \in X$ and $A, B \in W(X)$:

(i) if $p_\alpha(x, A, t) = 0, then\ x_\alpha \subset A,$

(ii) $p_\alpha(x, A, t) \leq M(x, y, t) * p_\alpha(y, A, t)$

(iii) $p'_\alpha(x, A, t) \geq N(x, y, t) \lozenge p'_\alpha(y, A, t)$

(iv) if $x_\alpha \subset A, then\ p_\alpha(x, B, t) \leq D_\alpha(A, B, t)$ and $p'_\alpha(x, B, t) \geq D'_\alpha(A, B, t)$

Definition 2.10. Let X be a nonempty set. Then $(X, M, N, \preccurlyeq)$ is called an ordered intuitionistic fuzzy metric space if and only if :

1. (X, M, N) is a metric space.
2. $(X, \preccurlyeq)$ is partially ordered.

Definition 2.11. Let $(X, \preccurlyeq)$ is partially ordered set. Then $x, y \in X$ are called comparable if $x \preccurlyeq y$ or $y \preccurlyeq x$ holds.

3.MAIN RESULT

Denote with φ, ψ are non decreasing and non increasing functions $\varphi, \psi : [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=1}^{n} \varphi^n < \infty$ and $\sum_{n=1}^{n} \psi^n > 0$ for all $x, y \in X\ and\ t > 0$. The next lemma is obvious.

Lemma 3.1[15] If φ, ψ are non decreasing and non increasing functions $\varphi, \psi : [0, +\infty) \to [0, +\infty)$, then $\varphi(0) = 0\ and\ \psi(0) = 1$ $\varphi(t) < t, \psi(t) > t$ for all $x, y \in X$ and for each $t > 0$.

Theorem 3.1. Let $(X, M, N, \preccurlyeq)$ be a complete ordered intuitionistic fuzzy metric space and $T_1, T_2 : X \to W_\alpha(X)$ be two fuzzy mappings satisfying

$$D_\alpha(T_1x, T_2y, t) \geq \psi\big(m(x, y, t)\big) + \min\{ap_\alpha(x, T_1x, t), bp_\alpha(y, T_2y, t), cp_\alpha(x, T_2y, t), dp_\alpha(y, T_1x, t)\}$$

and

$$D'_\alpha(T_1x, T_2y, t) \leq \varphi\big(n(x, y, t)\big) + \max\{ap'_\alpha(x, T_1x, t), bp'_\alpha(y, T_2y, t), cp'_\alpha(x, T_2y, t), dp'_\alpha(y, T_1x, t)\} \quad \ldots\ldots(1)$$

for all comparable elements $x, y \in X$, where $a, b, c, d \geq 0$ and

$$m(x, y, t) = \max\{M(x, y, t), p_\alpha(x, T_1x, t), p_\alpha(y, T_2y, t), \tfrac{1}{2}[p_\alpha(x, T_2y, t) * p_\alpha(y, T_1x, t)]\}$$

and

$$n(x, y, t) = \min\{N(x, y, t), p'_\alpha(x, T_1x, t), p'_\alpha(y, T_2y, t), \tfrac{1}{2}[p'_\alpha(x, T_2y, t) \lozenge p'_\alpha(y, T_1x, t)]\}$$

Also suppose that

(i) if $y \in (T_1x_0)_\alpha\ then\ y, x_0 \in X$ are comparable,

(ii) if $x, y \in X$ are comparable then every if $u \in (T_1x)_\alpha$ and every if $v \in (T_2y)_\alpha$ are comparable,

(iii) if a sequence if $\{x_n\}$ in X converge to $x \in X$ and its consecutive terms are comparable then x_n and x are comparable for all n.

Then there exists a point $x \in X$ such that $x_\alpha \subset T_1x$ and $x_\alpha \subset T_2x$.

Proof. Let $x_0 \in X$. Since $(T_1x_0)_\alpha \neq \emptyset$, then there exists $x_1 \in X$ such that $x_1 \in (T_1x_0)_\alpha$. By assumption (i) x_0 and x_1 are comparable. Since $(T_2x_1)_\alpha$ is a nonempty compact subset of X, there exists $x_2 \in (T_2x_1)_\alpha$ such that

$$M(x_1, x_2, t) = p_\alpha(x_1, T_2x_1, t) \geq D_\alpha(T_1x_0, T_2x_1, t)$$

and $N(x_1, x_2, t) = p'_\alpha(x_1, T_2x_1, t) \leq D'_\alpha(T_1x_0, T_2x_1, t).$

Moreover x_1 and x_2 are comparable. Continuing this process, one obtains a sequence $\{x_n\}$ in X such that $x_{2n+1} \in (T_1x_{2n})_\alpha$ and $x_{2n+2} \in (T_2x_{2n+1})_\alpha$ for all $n \geq 0$, x_{2n} and x_{2n+1} are comparable and

$$M(x_{2n+1}, x_{2n+2}, t) \geq D_\alpha(T_1x_{2n}, T_1x_{2n+1}, t)$$

and $N(x_{2n+1}, x_{2n+2}, t) \leq D'_\alpha(T_1x_{2n}, T_1x_{2n+1}, t)$

Since x_{2n} and x_{2n+1} are comparable, by taking x_{2n} for x and $x_{2n+1}\ for\ y$ in the inequality (1), it follows that

$M(x_{2n+1}, x_{2n+2}, t) \geq D_\alpha(T_1x_{2n}, T_1x_{2n+1}, t)$

$$\geq \psi\big(m(x_{2n}, x_{2n+1}, t)\big) + \min\{ap_\alpha(x_{2n}, T_1x_{2n}, t), bp_\alpha(x_{2n+1}, T_2x_{2n+1}, t), cp_\alpha(x_{2n}, T_2x_{2n+1}, t), dp_\alpha(x_{2n+1}, T_1x_{2n}, t)\}$$

and

$$N(x_{2n+1}, x_{2n+2}, t) \leq D'_\alpha(T_1x_{2n}, T_1x_{2n+1}, t)$$

$$\leq \varphi\big(n(x_{2n}, x_{2n+1}, t)\big) + max\{ap'_\alpha(x_{2n}, T_1x_{2n}, t), bp'_\alpha(x_{2n+1}, T_2x_{2n+1}, t), cp'_\alpha(x_{2n}, T_2x_{2n+1}, t), dp'_\alpha(x_{2n+1}, T_1x_{2n}, t)\} \quad \ldots\ldots(2)$$

where

$$m(x_{2n}, x_{2n+1}, t) = max\{M(x_{2n}, x_{2n+1}, t), p_\alpha(x_{2n}, T_1x_{2n}, t), p_\alpha(x_{2n+1}, T_2x_{2n+1}, t), \frac{1}{2}[p_\alpha(x_{2n}, T_2x_{2n+1}, t) * p_\alpha(x_{2n+1}, T_1x_{2n}, t)]\}$$

$$= max\{M(x_{2n}, x_{2n+1}, t), p_\alpha(x_{2n}, T_1x_{2n}, t), p_\alpha(x_{2n+1}, T_2x_{2n+1}, t), \frac{1}{2}p_\alpha(x_{2n}, T_2x_{2n+1}, t)\}$$

$$\geq max\{M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t), \tfrac{1}{2}M(x_{2n}, x_{2n+2}, t)\}$$

$$= max\{M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t)\}$$

and

$$n(x_{2n}, x_{2n+1}, t) = min\{N(x_{2n}, x_{2n+1}, t), p'_\alpha(x_{2n}, T_1x_{2n}, t), p'_\alpha(x_{2n+1}, T_2x_{2n+1}, t), \frac{1}{2}[p'_\alpha(x_{2n}, T_2x_{2n+1}, t) \lozenge p'_\alpha(x_{2n+1}, T_1x_{2n}, t)]\}$$

$$= min\{N(x_{2n}, x_{2n+1}, t), p'_\alpha(x_{2n}, T_1x_{2n}, t), p'_\alpha(x_{2n+1}, T_2x_{2n+1}, t), \frac{1}{2}p'_\alpha(x_{2n}, T_2x_{2n+1}, t)\}$$

$$\leq min\{N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t), \tfrac{1}{2}N(x_{2n}, x_{2n+2}, t)\}$$

$$= min\{N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t)\}$$

Therefore from (2), $M(x_{2n}, x_{2n+1}, t) \geq \psi(\min\{M(x_{2n}, x_{2n+1}, t), M(x_{2n+1}, x_{2n+2}, t)\})$
and $N(x_{2n}, x_{2n+1}, t) \leq \varphi(\max\{N(x_{2n}, x_{2n+1}, t), N(x_{2n+1}, x_{2n+2}, t)\})$.
If $M(x_{2n}, x_{2n+1}, t) = 1$ and $N(x_{2n}, x_{2n+1}, t) = 0$,
it follows that $M(x_{2n+1}, x_{2n+2}, t) = 1$ and $N(x_{2n+1}, x_{2n+2}, t) = 0$.
Now $x_{2n} = x_{2n+1} = x_{2n+2}$ implies $x_{2n+1} \in (T_1x_{2n})_\alpha = (T_1x_{2n+1})_\alpha$ and
$x_{2n+1} = x_{2n+2} \in (T_2x_{2n+1})_\alpha$, then the proof is finished. Therefore, we assume $M(x_{2n}, x_{2n+1}, t) < 1$ and $N(x_{2n}, x_{2n+1}, t) > 0$ By lemma, we get $\psi(t) > t$ *and* $\varphi(t) < t$ for each t > 0.
consequently,
if $M(x_{2n+1}, x_{2n+2}, t) < M(x_{2n}, x_{2n+1}, t)$ and $N(x_{2n+1}, x_{2n+2}, t) > N(x_{2n}, x_{2n+1}, t)$,
for some n, then we have

$$M(x_{2n+1}, x_{2n+2}, t) \geq \psi\big(M(x_{2n+1}, x_{2n+2}, t)\big) > M(x_{2n+1}, x_{2n+2}, t)$$

and

$$N(x_{2n+1}, x_{2n+2}, t) \leq \varphi\big(N(x_{2n+1}, x_{2n+2}, t)\big) < N(x_{2n+1}, x_{2n+2}, t)$$

which is a contradiction. Therefore

$$M(x_{2n+1}, x_{2n+2}, t) \geq \psi\big(M(x_{2n}, x_{2n+1}, t)\big) > M(x_{2n}, x_{2n+1}, t)$$

and

$$N(x_{2n+1}, x_{2n+2}, t) \leq \varphi\big(N(x_{2n}, x_{2n+1}, t)\big) < N(x_{2n}, x_{2n+1}, t)$$

that is

$$M(x_{2n+1}, x_{2n+2}, t) > M(x_{2n}, x_{2n+1}, t)$$

and

$$N(x_{2n+1}, x_{2n+2}, t) < N(x_{2n}, x_{2n+1}, t)$$

Similarly it can be shown that

$$M(x_{2n+3}, x_{2n+2}, t) \geq \psi\big(M(x_{2n+2}, x_{2n+1}, t)\big) > M(x_{2n+2}, x_{2n+1}, t)$$

and

$$N(x_{2n+3}, x_{2n+2}, t) \leq \varphi\big(N(x_{2n+2}, x_{2n+1}, t)\big) < N(x_{2n+2}, x_{2n+1}, t)$$

that is

$$M(x_{2n+3}, x_{2n+2}, t) > M(x_{2n+2}, x_{2n+1}, t)$$

and

$$N(x_{2n+3}, x_{2n+2}, t) < N(x_{2n+2}, x_{2n+1}, t)$$

Therefore, for all n, we get

$$M(x_n, x_{n+1}, t) \geq \psi\big(M(x_{n-1}, x_n, t)\big)$$
$$\ldots\ldots\ldots\ldots..$$
$$\geq \psi^n\big(M(x_0, x_1, t)\big).$$

and

$$N(x_n, x_{n+1}, t) \leq \varphi\big(N(x_{n-1}, x_n, t)\big)$$
$$\ldots\ldots\ldots\ldots..$$
$$\leq \varphi^n\big(N(x_0, x_1, t)\big).$$

Hence

$$\begin{aligned} M(x_n, x_{n+m}, t) &\geq M(x_n, x_{n+1}, t) * M(x_n, x_{n+2}, t) * M(x_n, x_{n+3}, t) * \\ &\quad \ldots\ldots..* M(x_{n+m-1}, x_{n+m}, t) \\ &\geq \psi^n\big(M(x_0, x_1, t)\big) * \ldots\ldots\ldots\ldots * \psi^{n+m-1}(M(x_0, x_1, t)) \\ &= \textstyle\sum_{k=n}^{n+m-1} \psi^k\big(M(x_0, x_1, t)\big). \end{aligned}$$

and

$$\begin{aligned} N(x_n, x_{n+m}, t) &\leq N(x_n, x_{n+1}, t) \lozenge N(x_n, x_{n+2}, t) \lozenge N(x_n, x_{n+3}, t) \lozenge \\ &\quad \ldots\ldots..\lozenge N(x_{n+m-1}, x_{n+m}, t) \\ &\quad \leq \varphi^n\big(N(x_0, x_1, t)\big) \lozenge \ldots\ldots\ldots\ldots \lozenge \varphi^{n+m-1}(N(x_0, x_1, t)) \\ &= \textstyle\sum_{k=n}^{n+m-1} \varphi^k\big(N(x_0, x_1, t)\big). \end{aligned}$$

Since $\sum_{n=1}^{\infty} \psi^n\big(M(x_0, x_1, t)\big) > 0$ and $\sum_{n=1}^{\infty} \varphi^n\big(N(x_0, x_1, t)\big) < \infty$, then $\{x_n\}$ is a Cauchy sequence in X.

Now from the completeness of X, there exists $x \in X$ such that $x_n \to x$ as $n \to \infty$ and since consecutive terms of $\{x_n\}$ are comparable, by hypothesis also x_n and x are comparable for all n. Now we claim that $p_\alpha(x, T_2x, t) = 1$ and $p'_\alpha(x, T_2x, t) = 0$ for each $\alpha \in [0,1]$. If not, then for some $\alpha \in [0,1]$, we have $p_\alpha(x, T_2x, t) < 1$ and $p'_\alpha(x, T_2x, t) > 0$ Consider

$$\begin{aligned} p_\alpha(x, T_2x, t) &\geq M(x, x_{2n+1}, t) + p_\alpha(x_{2n+1}, T_2x, t) \\ &\geq M(x, x_{2n+1}, t) + D_\alpha(T_1x_{2n}, T_2x, t) \\ &\geq M(x, x_{2n+1}, t) + \psi\left(\min\begin{Bmatrix} M(x_{2n}, x, t), +p_\alpha(x_{2n}, T_1x_{2n}, t), p_\alpha(x, T_2x, t), \\ \frac{1}{2}[p_\alpha(x_{2n}, T_2x, t) + p_\alpha(x, T_1x_{2n}, t)] \end{Bmatrix}\right) \\ &\quad + max\,\{ap_\alpha(x_{2n}, T_1x_{2n}, t), bp_\alpha(x, T_2x, t), \\ &\quad cp_\alpha(x_{2n}, T_2x, t), dp_\alpha(x, T_1x_{2n}, t)\} \\ &\geq M(x, x_{2n+1}, t) + \psi\left(\min\begin{Bmatrix} M(x_{2n}, x, t), +M(x_{2n}, x_{2n+1}, t), p_\alpha(x, T_2x, t), \\ \frac{1}{2}[p_\alpha(x_{2n}, T_2x, t) + p_\alpha(x, T_1x_{2n}, t)] \end{Bmatrix}\right) \\ &\quad + max\,\{aM(x_{2n}, x_{2n+1}, t), bp_\alpha(x, T_2x, t), \\ &\quad cp_\alpha(x_{2n}, T_2x, t), dp_\alpha(x, T_1x_{2n}, t)\} \end{aligned}$$

and

$$\begin{aligned} p'_\alpha(x, T_2x, t) &\leq N(x, x_{2n+1}, t) + p'_\alpha(x_{2n+1}, T_2x, t) \\ &\leq N(x, x_{2n+1}, t) + D'_\alpha(T_1x_{2n}, T_2x, t) \\ &\leq N(x, x_{2n+1}, t) + \varphi\left(\max\begin{Bmatrix} N(x_{2n}, x, t), +p'_\alpha(x_{2n}, T_1x_{2n}, t), p'_\alpha(x, T_2x, t), \\ \frac{1}{2}[p'_\alpha(x_{2n}, T_2x, t) + p'_\alpha(x, T_1x_{2n}, t)] \end{Bmatrix}\right) \\ &\quad + min\,\{ap'_\alpha(x_{2n}, T_1x_{2n}, t), bp'_\alpha(x, T_2x, t), \\ &\quad cp'_\alpha(x_{2n}, T_2x, t), dp'_\alpha(x, T_1x_{2n}, t)\} \\ &\leq N(x, x_{2n+1}, t) + \varphi\left(\max\begin{Bmatrix} N(x_{2n}, x, t), +N(x_{2n}, x_{2n+1}, t), p'_\alpha(x, T_2x, t), \\ \frac{1}{2}[p'_\alpha(x_{2n}, T_2x, t) + p'_\alpha(x, T_1x_{2n}, t)] \end{Bmatrix}\right) \\ &\quad + max\,\{aN(x_{2n}, x_{2n+1}, t), bp'_\alpha(x, T_2x, t), \\ &\quad cp'_\alpha(x_{2n}, T_2x, t), dp'_\alpha(x, T_1x_{2n}, t)\}\,. \end{aligned}$$

We note that $M(x_{2n}, x, t) \to 1, M(x_{2n}, x_{2n+1}, t) \to 1$
and $N(x_{2n}, x, t) \to 0, N(x_{2n}, x_{2n+1}, t) \to 0$,
$p_\alpha(x_{2n}, T_2x, t) \to p_\alpha(x, T_2x, t)$ and $p'_\alpha(x_{2n}, T_2x, t) \to p'_\alpha(x, T_2x, t)$ as $n \to \infty$.
This implies that there exists $n_0 \in \mathbb{N}$ such that

$$\min\left\{M(x_{2n}, x, t), M(x_{2n}, x_{2n+1}, t), \frac{1}{2}[p_\alpha(x_{2n}, T_2x, t) + M(x, x_{2n+1}, t)]\right\} \geq p_\alpha(x, T_2x, t)$$

$$\max\left\{N(x_{2n}, x, t), N(x_{2n}, x_{2n+1}, t), \frac{1}{2}[p'_\alpha(x_{2n}, T_2x, t) + N(x, x_{2n+1}, t)]\right\} \leq p'_\alpha(x, T_2x, t)$$

for all $n \geq n_0$. Consequently, we have

$$p_\alpha(x, T_2x, t) \geq \psi\big(p_\alpha(x, T_2x, t)\big) + \min\{aM(x_{2n}, x_{2n+1}, t), bp_\alpha(x, T_2x, t), cp_\alpha(x_{2n}, T_2x, t), dp_\alpha(x, T_1x_{2n}, t)$$

and

$$p'_\alpha(x, T_2x, t) \leq \varphi\big(p_\alpha(x, T_2x, t)\big) + \max\{aN(x_{2n}, x_{2n+1}, t), bp'_\alpha(x, T_2x, t), cp'_\alpha(x_{2n}, T_2x, t), dp'_\alpha(x, T_1x_{2n}, t)$$

for all $n \geq n_0$, which on taking the limit as $n \to +\infty$ gives

$$p_\alpha(x, T_2x, t) \geq \psi\big(p_\alpha(x, T_2x, t)\big) > p_\alpha(x, T_2x, t)$$

and

$$p'_\alpha(x, T_2x, t) \leq \varphi\big(p_\alpha(x, T_2x, t)\big) < p'_\alpha(x, T_2x, t),$$

which is a contradiction. Hence $p_\alpha(x, T_2x, t) = 1$ and $p'_\alpha(x, T_2x, t) = 0$ and so $x_\alpha \subset T_2x$. Similarly we deduce that $x_\alpha \subset T_1x$.
From Theorem 3.1, assuming $\psi(t) = \varphi(t) = qt$ with 0 < q < 1 and a, b, c, d = 0, then following result deduced
Corollary 3.1 Let $(X, M, N, \preccurlyeq)$ be a complete ordered intuitionistic fuzzy metric space and $T_1, T_2: X \to W_\alpha(X)$ be two fuzzy mappings satisfying

$$D_\alpha(T_1x, T_2y, t) \geq \max\{M(x, y, t), p_\alpha(x, T_1x, t), p_\alpha(y, T_2y, t), \frac{1}{2}[p_\alpha(x, T_2y, t) * p_\alpha(y, T_1x, t)]\}$$

and

$$D'_\alpha(T_1x, T_2y, t) \leq \min\{N(x, y, t), p'_\alpha(x, T_1x, t), p'_\alpha(y, T_2y, t), \frac{1}{2}[p'_\alpha(x, T_2y, t) \lozenge p'_\alpha(y, T_1x, t)]\} \quad \ldots\ldots(1)$$

for all comparable elements $x, y \in X$
Also suppose that

(i) if $y \in (T_1x_0)_\alpha$ *then* $y, x_0 \in X$ are comparable,
(ii) if $x, y \in X$ are comparable then every if $u \in (T_1x)_\alpha$ and every if $v \in (T_2y)_\alpha$ are comparable,
(iii) if a sequence if $\{x_n\}$ in X converge to $x \in X$ and its consecutive terms are comparable then x_n and x are comparable for all n.

Then there exists a point $x \in X$ such that $x_\alpha \subset T_1x$ and $x_\alpha \subset T_2x$.

Example 3.1 Let X = [0,1] endowed with the usual order of real numbers and the Euclidean metric $d(x, y) = |x - y|$ for all $x, y \in X$. Clearly (X, d) is a complete (ordered) metric space. Let $\alpha \in (0, 1/2)$ and M, N are two fuzzy sets defined as let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$M(x, y, t) = \frac{t}{t + d(x,y)}$ and $N(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$,for all t > 0

now define

$$\psi(t) = \varphi(t) = \begin{cases} \frac{t^3}{1+t} & if\ x \in [0,1] \\ \frac{1}{2} & if\ x \in (1, +\infty) \end{cases},$$

$$(T_1 0)(x) = (T_2 1)(x) = \begin{cases} 1 & if\ x = 0 \\ \alpha & if\ x \in (0, 1/2] \\ \alpha/2 & if\ x \in (1/2, 1], \end{cases}$$

$$(T_1 1)(x) = (T_2 0)(x) = \begin{cases} 1 & if\ x = 0 \\ 2\alpha & if\ x \in (0, 1/2] \\ \alpha/2 & if\ x \in (1/2, 1], \end{cases}$$

$$(T_1z)(x) = (T_2z)(x) = \begin{cases} 1 & if\ x = 0 \\ \alpha & if\ x \in (0, 1/2] \\ 0 & if\ x \in (1/2, 1], \end{cases} \quad \text{where } z \in (0,1).$$

Now discuss the existence of fixed fuzzy points of mappings T_1 and T_2. To this aim, it is note that $(T_i0)_\alpha = (T_iz)_\alpha = (T_i1)_\alpha = [0, 1/2]$, $(T_i0)_{\alpha/2} = (T_i1)_{\alpha/2} = [0,1]$, and $(T_iz)_{\alpha/2} = [0, 1/2]$, where i = 1, 2. Consequently, it is easy to show that all the hypotheses of theorem 3.1 are satisfied. In particular, condition (1) holds trivially since $D_\alpha(T_1x, T_2y, t) = 1$ and $D'_\alpha(T_1x, T_2y, t) = 0$ for all $x, y \in X$. It can be conclude that each $x \in [0, 1/2]$ is such that $x_\alpha \subset T_1x$ and $x_\alpha \subset T_2x$.

On the other hand, in view of Definition 2.4, we can apply our theorem 3.1 establish the existence of a common fixed point of T_1 and T_2. In this case, we note that $(T_i0)_1 = (T_iz)_1 = (T_i1)_1 = \{0\}$, hence $x = 0$ is common fixed point of T_1 and T_2.

4.APPLICATION TO ORDINARY FUZZY DIFFERENTIAL EQUATION[9]

In this section, we present a solution where our obtained results can be applied. Precisely, we study the existence of solution for the second order nonlinear boundary value problem.

$$\begin{cases} x^n(t) = k(t, x(t), x'(t), & t \in [0, \Lambda], \Lambda > 0 \\ x(t_1) = x_1, & \\ x(t_2) = x_2, & t_1, t_2 \in [0,1] \end{cases}$$

where $k: [0,1] \times W(X) \times W(X) \to W(X)$ is a continuous function. This problem is equivalent to the integral equation

$$x(t) = \int_{t_1}^{t_2} G(t,s)k\big(s, x(s), x'(s)\big)ds + \beta(t), \qquad t \in [0, \Lambda],$$

where the Green's function G is given by

$$G(t,s) = \begin{cases} \dfrac{(t_2 - t)(s - t_1)}{t_2 - t_1} & if\ t_1 \le s \le t \le t_2 \\ \dfrac{(t_2 - s)(t - t_1)}{t_2 - t_1} & if\ t_1 \le t \le s \le t_2, \end{cases}$$

and $\beta(t)$ satisfies $\beta''(t) = 0, \beta(t_1) = x_1, \beta(t_2) = x_2$. Let us recall some properties of $G(t,s)$, precisely then

$$\int_{t_1}^{t_2} |G(t,s)|\, ds \le \frac{(t_2 - t_1)^2}{8}$$

and

$$\int_{t_1}^{t_2} |G(t,s)|\, ds \le \frac{(t_2 - t_1)}{2}$$

Now prove the result, by establishing the existence of a common fixed point for a pair of integral operators defined as

$$T_i(x)(t) = \int_{t_1}^{t_2} G(t,s)k_i\big(s, x(s), x'(s)\big)ds + \beta(t), \qquad t \in [0, \Lambda], i \in [1,2],$$

where $k_1, k_2 \in C([0, \Lambda] \times \mathrm{W(X)} \times \mathrm{W(X)}, \mathrm{W(X)}, \mathrm{x} \in \mathrm{C}^1\big([0, \Lambda], \mathrm{W(X)}\big)$, and

$$\beta \in C\big([0, \Lambda], \mathrm{W(X)}\big).$$

5 CONCLUSIONS

Our Theorem 3.1 gives a contribution to the ‘fixed point arena’ in the sense of generalization by using fuzziness under ordered intuitionistic fuzzy metric spaces and by assuming the validity of the contractive condition only on elements that are comparable in respect to partial ordering.

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