Performance analysis of controller design methods for unstructured uncertainty based modeled uncertain system

Dr. Supanna S. Shirguppe

Professor & Head, Electrical and Electronics Engineering Department, S. G Balekundri Institute of Technology, Belagavi, Karnataka, India

***Abstract*—This paper presents the performance of a robust PID controller designed using a simple and efficient technique for a system with uncertain parameters. The uncertain system considered is a warm air-drying chamber represented by unstructured uncertainty based model. The design of PID controller is based on a technique that uses Small Gain theorem which guarantees closed loop stability for variations in plant input across its entire working range. The designed controller is compared with those designed using D curves and D partition methods. The control input plots exhibit the superiority of the designed controller over the other methods for the uncertain plant considered. Also, the integral absolute error (IAE), integral squared error (ISE), integral time weighted absolute error (ITAE) and integral time weighted squared error (ITSE) performance indices are determined using MATLAB®. Comparison of the results shows that the designed controller gives a better performance over those obtained from other design methods.**

***Index Terms*—Additive uncertainty, Performance index, PID controller, Robust control, Small gain theorem.**

I. INTRODUCTION

The mathematical model used to represent any physical system is mostly inaccurate. This is because of the introduction of modelling errors due to factors like time delay, parameter variations, non-linearity etc. which are ignored during simplification of the model leading to unpredictable plant behaviour. Hence, an uncertain model of the plant is necessary to avoid the impact of uncertainty on the system stability and performance.

An unstructured uncertainty is one in which a perturbation block is used to represent dynamic perturbations due to neglected plant dynamics or unmodelled high frequency dynamics. This type of uncertainty can be represented either as an additive perturbation model represented by the equation, $G_p(s) = G_0(s) + \Delta(s)$ or the multiplicative perturbation model represented by the equation, $G_p(s) = G_0(s)[1 + \Delta(s)]$. $G_0(s)$, $G_p(s)$ and $\Delta(s)$ are the nominal plant model, actual perturbed plant dynamics and the uncertain block respectively. $\Delta(s)$ is normally bounded i.e. $\overline{\sigma}[\Delta(j\omega)] \leq \delta(j\omega)\ \forall\ \omega \in [0, \infty]$. Here, $\overline{\sigma}$ and $\delta(s)$ are the largest singular value of the matrix and a known scalar transfer function respectively[1]. During the design of a controller for a physical process we need to consider one or more parameters to obtain the best performance. These parameters which are a measure of the performance are called as the performance indices. The integral absolute error (IAE), integral squared error (ISE), integral time weighted absolute error (ITAE) and integral time weighted squared error (ITSE) performance indices are commonly used as a tool to compare between different controller design techniques.

Due to ease of implementation, PID controllers have become popular in industries. However their drawback is that they lack in generality since they use heuristic tuning methods such as Zeigler-Nichols [2]. An easy technique of designing a PI or PID controller for uncertain systems using small gain theorem has been implemented by Dubravska and Harsanyi [3]. Harsanyi and Dubravska have also demonstrated the design of a robust controller for linear systems with parametric and dynamic uncertainties [4]. Huang and Wang have implemented a robust PID controller for an uncertain system with bounded parametric variations using Kharitonov theorem [5]. The limitations of model predictive control such as unstable open loop system, unknown plant orders etc. have been overcome by generalized predictive control (GPC) introduced by Clarke, Mohtadi and Tuffs [6]. Bordons and Camacho have shown that a significant reduction in computational time is possible through their proposed GPC technique for First Order Plus Dead Time (FOPDT) model as in [7], [8]. Several researchers have formulated a control problem as a mathematical H∞ optimization problem [9]-[11]. Anushree Das and Veena Sharma have modelled an uncertain system and designed a controller using H∞ technique [12]. A linear uncertain system has been represented using unstructured uncertainty model and robust stability analysis has been carried out as in [13], [14].

D curves method and D partition method have been used to design robust controllers for a linear interval model and for an affine model of an uncertain warm air-drying chamber respectively as in [15]. Since these approaches involve graphical methods based on Kharitonov theorem they are time consuming. A simple, frequency domain based approach for design of a robust PID controller for a warm air-drying chamber has been proposed in [16]. Reference [17] shows the comparison of the error performance indices in an optimized PID controller and PID controllers tuned by other methods for a nuclear reactor. In this paper, comparison of the performance indices for different controllers has been done to show that the proposed PID controller is superior to the other controllers designed using the D curves and D partition methods.

II. PID CONTROLLER DESIGN METHODOLOGY

Small gain theorem states that, if the magnitude of the open loop gain is < 1, then the closed loop system will be robustly stable. i.e, if the open loop gain magnitude $|\mathrm{L}(s)| < 1\ \forall\omega \in [0, \infty]$ then, the closed loop system will be stable. If the uncertain system is described by a family of stable transfer functions $\mathrm{G_x}(s)$, then the system can be represented by an additive unstructured uncertainty model given by,

$$G(s) = G_0(s) + W_a(s)\Delta(s) \qquad (1)$$

where $\mathrm{G_0}(s)$ is the nominal transfer function, $\mathrm{W_a}(s)$ is the additive weighting transfer function and $\Delta(s)$ is a set of transfer functions having peak magnitudes less than or equal to 1 for all frequencies. If $\Delta(s) \leq 1$ then, $|\mathrm{W_a}(s).\Delta(s)| \leq |\mathrm{W_a}(s)|$ which implies that,

$$|W_a(s)| \geq Max|G_x(s) - G_0(s)|\ \forall\omega \in [0, \infty] \qquad (2)$$

where x = 1, 2, 3... n and $G_x(s)$ is the family of stable transfer functions representing the uncertain system. A suitable choice of $W_a(s)$ is made based on the condition given in (2).

The characteristic equation of the closed loop control system is given by,

$$1 + C(s)G(s) = 0$$

Using (1) in the above equation, we have,

$$1 + C(s)[G_0(s) + W_a(s).\Delta(s)] = 0$$

$$[1 + C(s)G_0(s)]\left[1 + \frac{C(s)G_0(s)W_a(s)\Delta(s)}{[1 + C(s)G_0(s)]G_0(s)}\right] = 0$$

We have,

$$N_0(s) = C(s)G_0(s)/[1 + C(s)G_0(s)] \qquad (3)$$

where $N_0(s)$ represents a stable nominal closed loop transfer function. Therefore we have,

$$[1 + C(s)G_0(s)][1 + N_0(s)W_a(s)\Delta(s)/G_0(s)] = 0 \qquad (4)$$

Since the nominal closed loop system is stable, it implies that the nominal closed loop characteristic equation $[1 + C(s)G_0(s)] = 0$ is stable. Now for (4) to be stable, $[1 + N_0(s).W_a(s)\Delta(s)/G_0(s)] = 0$ should satisfy small gain theorem condition. Hence the following condition must be satisfied.

$$\left|N_0(s).\frac{W_a(s)\Delta(s)}{G_0(s)}\right| < 1\ \forall\omega \in [0, \infty]$$

In the above equation, using the worst case value of $\Delta(s)$ which is $|\Delta(s)| = 1\ \forall\omega \in [0, \infty]$, we have,

$$|N_0(s)| < |G_0(s)/W_a(s)| \qquad (5)$$

$N_0(s)$ is selected to satisfy (5) as follows. Rearranging (3) we get,

$$C(s) = N_0(s)/\{G_0(s)[1 - N_0(s)]\} \qquad (6)$$

Expressing $C(s)$, $N_0(s)$ and $G_0(s)$ as ratios of pertinent polynomials and rearranging, the controller transfer function is given by,

$$\frac{C_1(s)}{C_2(s)} = \frac{G_{02}(s)N_{01}(s)}{G_{01}(s)[N_{02}(s) - N_{01}(s)]} \qquad (7)$$

Expressing the transfer function in (3) also in terms of ratios of pertinent polynomials we get,

$$\frac{N_{01}(s)}{N_{02}(s)} = \frac{G_{01}(s)}{C_2(s)[G_{02}(s)/C_1(s)] + G_{01}(s)}$$

$$\frac{N_{01}(s)}{N_{02}(s)} = \frac{G_{01}(s)}{C_2(s)[P_0(s)] + G_{01}(s)} \qquad (8)$$

Equating the numerators and denominators on both sides of (8),

$$N_{01}(s) = G_{01}(s) \qquad (9)$$

$$N_{02}(s) = C_2(s)P_0(s) + G_{01}(s) \qquad (10)$$

where

$$P_0(s) = G_{02}(s)/C_1(s) \qquad (11)$$

Using (9) in (7) we have,

$$\frac{C_1(s)}{C_2(s)} = \frac{G_{02}(s)}{[N_{02}(s) - G_{01}(s)]} \qquad (12)$$

A PID controller is described by a general transfer function,

$$\frac{C_1(s)}{C_2(s)} = \frac{1}{K}\left(\frac{X(s)}{s}\right) = \frac{1}{K}\left(\frac{x_d s^2 + x_p s + x_i}{s}\right)$$

From (10) we have,

$$N_{02}(s) - G_{01}(s) = P_0(s)C_2(s) \qquad (13)$$

From (11) we have,

$$G_{02}(s) = P_0(s)C_1(s) \qquad (14)$$

Using (13) and (14), the right hand side of (12) becomes,

$$\frac{G_{02}(s)}{[N_{02}(s) - G_{01}(s)]} = \frac{P_0(s)C_1(s)}{P_0(s)C_2(s)}$$

Using the general transfer function of a PID controller in the above equation,

$$\frac{G_{02}(s)}{[N_{02}(s) - G_{01}(s)]} = \frac{P_0(s)}{P_0(s)}\frac{1}{K}\left(\frac{X(s)}{s}\right)$$

Thus we have,

$$G_{02}(s) = P_0(s)\left(x_d s^2 + x_p s + x_i\right) \qquad (15)$$

$$N_{02}(s) - G_{01}(s) = P_0(s).K.s \qquad (16)$$

Consider (15). We choose the degree of $P_0(s)$ such that it is equal to the degree of $G_{02}(s)$ minus the degree of $X(s)$. Thus $P_0(s)$ and $X(s)$ can be determined. Using (9) and (16), the nominal closed loop transfer function can be expressed as a function of K.

$$N_0(s) = \frac{N_{01}(s)}{N_{02}(s)} = \frac{G_{01}(s)}{P_0(s).K.s + G_{01}(s)} \qquad (17)$$

Now, a suitable choice of K is made such that the condition given by (5) is satisfied. From the selected value of K, the PID parameters are determined using (18).

$$K_p = \frac{x_p}{K}, \quad K_i = \frac{x_i}{K}, \quad K_d = \frac{x_d}{K} \qquad (18)$$

III. PID CONTROLLER DESIGN OF THE WARM AIR DRYING CHAMBER

PID controller for the warm air-drying chamber uncertain system used in [15] has been designed using the small gain theorem based methodology as in [16]. The transfer functions for the uncertain system for step change in input power in steps of 20 % are given by,

$$G_1 = \frac{10.6}{12893s^3 + 8058s^2 + 226.7s + 1}(0 - 20\%)$$

$$G_2 = \frac{10.5}{2078s^3 + 10346s^2 + 231s + 1}(20 - 40\%)$$

$$G_3 = \frac{11.4}{40184s^3 + 10506s^2 + 256s + 1}(40 - 60\%)$$

$$G_4 = \frac{12.8}{13681s^3 + 11606s^2 + 279s + 1}(60 - 80\%)$$

$$G_5 = \frac{8}{32324s^3 + 10370s^2 + 277s + 1}(80 - 100\%)$$

The values of K_p, K_d and K_i determined for $K = 60$ are,

$$K_p = 4.2; \quad K_i = 0.0167; \quad K_d = 160.9$$

IV. RESULTS

A. Comparison with PID controller by D curves method

In the practical system considered, the control input is the output of a triac controller in watts which controls the power consumption of the electric heater [15]. Fig.1 and Fig.2 show control signal plots with zoomed in Y-axis for the proposed PID controller in [16] and the PID controller by D curves method in [15] for K=60.

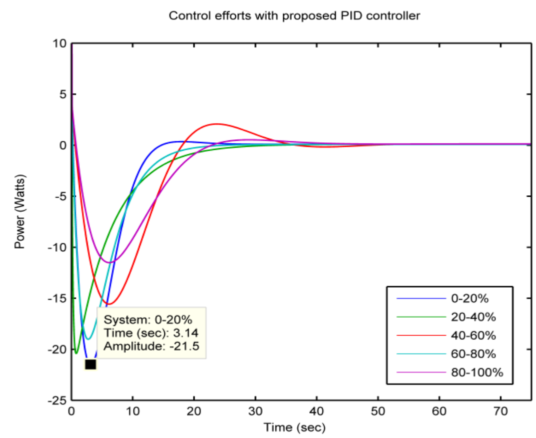

Fig.1 Control signals with the proposed PID controller for K=60 with zoomed in Y-axis.

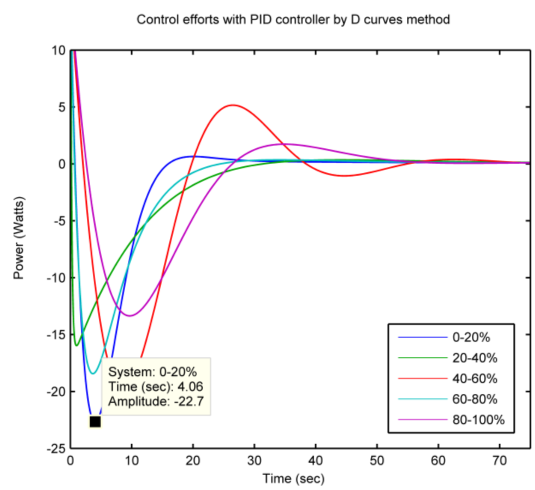

Fig.2 Control signals with the PID controller by D curves method with zoomed in Y-axis.

Fig.1 and Fig.2 indicate that the maximum value of the control effort and average value of the area under the control effort curves for the proposed PID controller in [16] are lesser compared to the PID controller by D curves method in [15].

Fig.3 shows the IAE and ISE with the proposed PID controller designed using Small gain theorem for K=60. Fig.4 shows the IAE and ISE with the PID controller by D curves method. Fig.5 shows the ITAE and ITSE with the proposed PID controller designed using Small gain theorem for K=60. Fig.6 shows the ITAE and ITSE with the PID controller by D curves method.

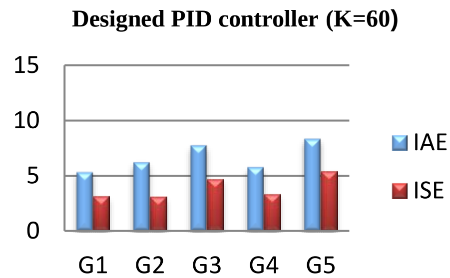

Fig.3 IAE and ISE with the proposed PID controller for K=60

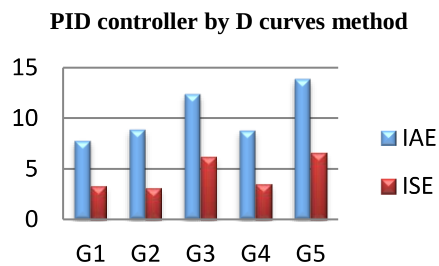

Fig. 4 IAE and ISE with the PID controller by D curves method

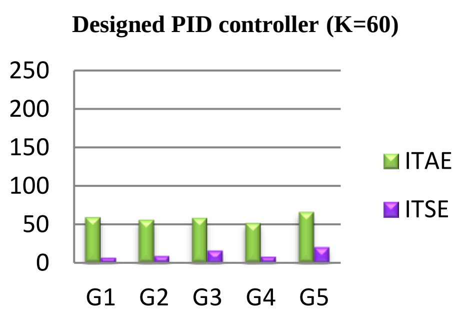

Fig. 5 ITAE and ITSE with the proposed PID controller for K=60

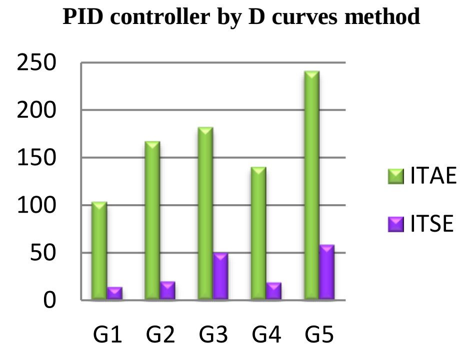

Fig. 6 ITAE and ITSE with the PID controller by D curves method

It can been observed from Fig.3, Fig.4, Fig.5 and Fig.6 that the PID controller designed using small gain theorem gives better performance indices than the PID controller designed using D curves method.

B. Comparison with PID controller by D-partition approach

PID controller designed to give a minimum phase margin of 45.8 degrees [15] in addition to a reduced peak overshoot requires K=120. The PID controller parameters determined for K=120 are given by [16],

$$K_p = 2.0984; \quad K_d = 80.4349; \quad K_i = 0.0083$$

Fig.7 shows the IAE and ISE with the proposed PID controller designed using Small gain theorem for K=120. Fig.8 shows the IAE and ISE with the PID controller by D partition method. Fig.9 shows the ITAE and ITSE with the proposed PID controller designed using Small gain theorem for K=120. Fig.10 shows the ITAE and ITSE with the PID controller by D partition method.

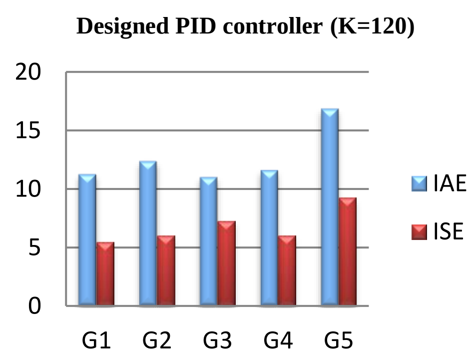

Fig. 7 IAE and ISE with the proposed PID controller for K=120

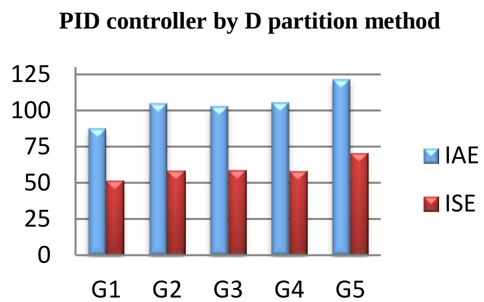

Fig. 8 IAE and ISE with the PID controller by D partition method

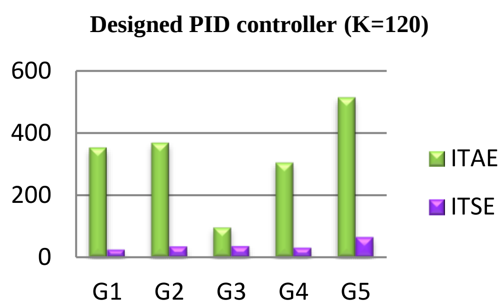

Fig. 9 ITAE and ITSE with the proposed PID controller for K=120

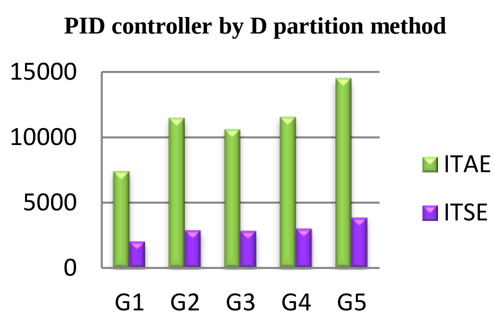

Fig. 10 ITAE and ITSE with the PID controller by D partition method

It can been observed from Fig.7, Fig.8, Fig.9 and Fig.10 that the PID controller designed using small gain theorem gives better performance indices compared to the PID controller designed using D partition method.

V. CONCLUSION

The robust PID controller design methodology using small gain theorem [16] which is simple and easy to use can be successfully applied to a warm air-drying chamber which is an uncertain system. The results show that the control efforts for the proposed controllers lie within the maximum load capacity of about 16.5 kW for a 3-phase triac controller which proves the feasibility of implementation of the designed controllers. The average energy consumption of the electric heater in case of PID controller designed by D curves method is greater than that of the corresponding proposed small gain theorem based PID controller. .Also, the IAE, ISE, ITAE and ITSE performance error indices are improved with controllers designed using small gain theorem.

REFERENCES

[1] D. -W. Gu, P. H. Petkov, and M. M. Konstantinov, *Robust Control Design with MATLAB®*, London: Springer, 2005.

[2] J. G. Zeigler and N. B. Nichols, -Optimum settings for automatic controllers, ‖ *Transactions ASME*, vol. 64, pp. 759-768, 1942.

[3] M. Dubravska and L. Harsanyi, -Control of Uncertain systems, ‖ *Journal of Electrical Engineering*, vol. 58, no. 4, pp. 228-231, 2007.

[4] L. Harsanyi and M. Dubravska, -Robust Controller Design for Linear Systems with Parametric and Dynamic Uncertainties, ‖ *Journal of Electrical Engineering*, vol. 52, no. 9-10, pp. 307-310, 2001.

[5] Y. J. Huang and Y. -J. Wang, -Robust PID tuning strategy for uncertain plants based on the Kharitonov theorem, ‖ *ISA Transactions*, vol. 39, pp. 419-431, 2000.

[6] D. W. Clarke, C. Mohtadi, and P. S. Tuffs, -Generalized Predictive Control-Part I. The Basic Algorithm, ‖ *Automatica*, vol.23, no.2, pp. 137-148, 1987.

[7] E. F. Camacho and C. Bordons, -Implementation of self-tuning generalized predictive controllers for the process industry, ‖ *International Journal of Adaptive Control and Signal Processing*, vol. 7, no. 1, pp. 63-73, 1993.

[8] C. Bordons and E. F. Camacho, -A generalized predictive controller for a wide class of industrial processes, ‖ *IEEE Transactions on Control Systems Technology*, vol. 6, no. 3, pp. 372-387, 1998.

[9] S. P. Bhattacharya, H. Chapellat, and L. H. Keel, *Robust Control: The parametric approach*, Prentice Hall, 1995.

[10] J.C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis, -State space solutions to standard H2 and H∞ control problems, ‖ *IEEE Transactions on Automatic Control*, vol. 34, no. 8, pp. 831-847, 1989.

[11] K. B. Datta and V. V. Patel, -H∞ based synthesisfor a Robust controller of Interval plants, ‖ *Automatica*, vol.32, no.11, pp. 1575-1579, 1996.

[12] A. Das and V. Sharma, -Modelling of uncertainty in control systems and design of a robust controller using H∞ method, ‖ in *2017 International Conference on Intelligent Computing and Control Systems (ICICCS)*, Madurai, India, 2017, pp. 1008-1013, doi: 10.1109/ICCONS.2017.8250617, 2017.

[13] Matušů R, Şenol B and Yeroğlu C, -Linear systems with unstructured multiplicative uncertainty: Modeling and robust stability analysis, ‖ *PLoS ONE*, vol. 12, no. 7, e0181078, https://doi.org/10.1371/journal.pone.0181078, 2017.

[14] Keshtkar, Najmeh, and Klaus Röbenack, -Unstructured Uncertainty Based Modeling and Robust Stability Analysis of Textile-Reinforced Composites with Embedded Shape Memory Alloys, ‖ *Algorithms*, vol. 13, no. 1: 24, https://doi.org/10.3390/a13010024, 2020.

[15] J. Danko, M. Ondrovicova, and V. Vesely, -Robust Controller Design to control a warm air-drying chamber, ‖ *Journal of Electrical Engineering*,vol.55, no. 7-8, pp. 207-211, 2004.

[16] S. Chokkadi and S. S. Kumar, -Design of Robust Controller for an Uncertain System described by Unstructured Uncertainty Model using Small Gain Theorem, ‖ in *2019 International Conference on Automation, Computational and Technology Management (ICACTM)*, London, UK, 2019, pp. 459-463, doi: 10.1109/ICACTM.2019.8776718.

[17] Seyed Mohammad Hossein Mousakazemi, -Comparison of the error-integral performance indexes in a GA-tuned PID controlling system of a PWR-type nuclear reactor point-kinetics model, ‖ *Progress in Nuclear Energy*, vol. 132, 2021, 103604, ISSN 0149-1970, https://doi.org/10.1016/j.pnucene.2020.103604.