© May 2023| IJIRT | Volume 9 Issue 12 | ISSN: 2349-6002

Buckling of Re-Inforced Anisotropic Cylindrical Shell with Rings and Stringers Without Shear Load

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Abstract - Object of this paper is to formulate and find the solution of the differential equation for the buckling of re-inforced anisotropic cylindrical shells of uniform thickness re-inforced by closely spaced rings and stringers or both without shear load. Solution for shells without shear load in case of two-way compression is obtained. The corresponding equation for gridwork cylindrical shells are deduced as a special case which are found to be identical with the previous result. Numerical results vide Timoshenko and Woinowsky Kreiger are obtained and the buckling diagram of re-inforced anisotropic cylindrical shells subject to two-way compression is drawn and stable domain is shown in the diagram.

Key Words - Re-inforced, Anisotropic, Rings or Stringers, Shear load, Grid work, Critical load, twisting moments, Shearing stresses, Twisting rigidity, Torsional rigidity.

1.INTRODUCTION

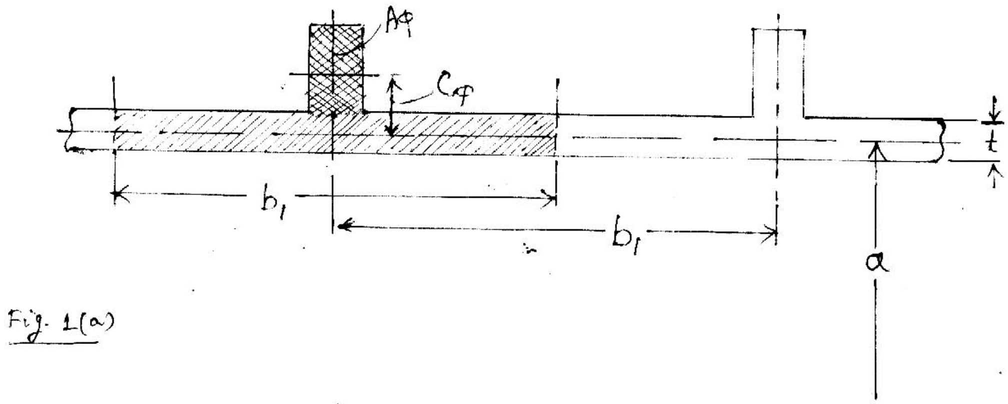
Solution of buckling of cylindrical shells in case of isotropic material is known from the literature on shells e.g. Flugge[1] (1973). An isotropic shell consists of composite materials such as Boron epoxy, re-inforced plastics and whiskers. They are used in many advanced structural applications and quite often in cylindrical shells.

Buckling problem of an isotropic cylindrical shells has occupied the interest of many researchers such as Tasi[2] , Cheng & Kuenzi[3] , Hess T.E[4], Thielemann, Schnell and Fisher[5], Tasi, Fellmann and Strang[6], Cheng & Ho[7,8], Lie and Cheng[9], De,A[10-14].

The object of this paper is to formulate the differential equation for re-inforced anisotropic cylindrical shell of uniform thickness re-inforced by closely spaced rings or stringers or both. The corresponding solution of the differential equation so formulated is found out for shells without shear-load in the case of two-way compression.

The re-inforced anisotropic shell is most important one. The shell of uniform thickness re-inforced by closely spaced rings or stringers or both. We may handle this case in two ways, either we superpose the stress resultants of an isotropic shell and those of a grid work, or we use (5.109)[vide Flugge[1], page-300] and (5.110) with the understanding that the slab integrals are now to be extended over one slab only. This second way is to be recommended for concrete shells and similar structures, for which it will represent the facts.

For the thin shell of air plane fuse lages(5.109) [vide Flugge[1], page-300] have a serious drawback, which excludes their use. In a double-walled shell the twisting moments are carried by shearing stresses $\tau_{x\varphi}$ or $\tau_{\varphi x}$ having opposite directions in the two slabs. The contribution of the ribs is practically nil and has been neglected in (5.109 g,h). It is quite different when the shell consists of only one very thin wall and a set of sturdy stiffeners, particularly when this have tubular cross sections. Then the twisting rigidity of the wall is next to nothing, and almost all the twisting stiffness of the shell comes from the torsional rigidity of the ribs.



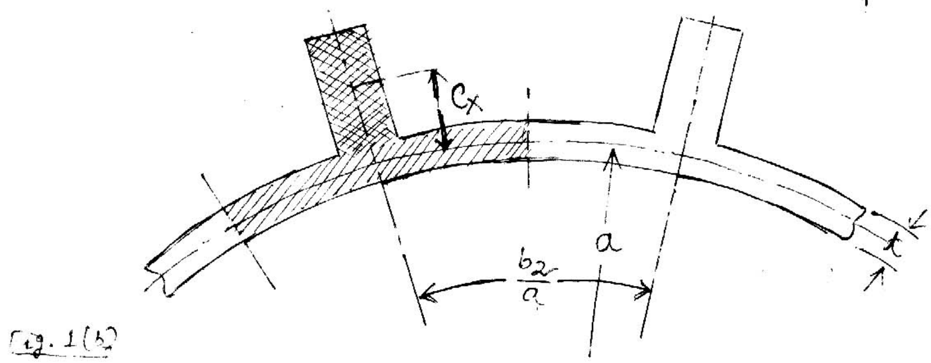


FIG. (1-a) - SECTIONS φ = CONSTANT THROUGH A SHELL WITH RINGS
FIG. (1-b)- SECTIONS X = CONSTANT THROUGH A SHELL WITH STRINGERS

2.BASIC EQUATION

Fig. 1(a) shows the section φ = Constant through a reinforced shell i,e shell with rings and stringers and fig. 1 (b) shows the sections x= Constant through the reinforced cylindrical shell .
We use the middle surface of the wall as the middle surface of the entire shell. We arrive at the following elastic law; given by the Flugge[1],W. (Page-309)

$$N_{\varphi} = \frac{D_{\varphi}}{a}(\dot{v}+w) + \frac{D_{v}}{a}u' - \frac{S_{\varphi}}{a^{2}}\ddot{w} \qquad \text{------------------------------(1-a)}$$

$$N_{x} = \frac{D_{x}}{a}u' + \frac{D_{v}}{a}(\dot{v}+w) - \frac{S_{x}}{a^{2}}w'' \qquad \text{-----------------------------(1-b)}$$

$$N_{\varphi x} = \frac{D_{x\varphi}}{a}(\dot{u}+v') + \frac{K_{\varphi x}}{a^{3}}\dot{w}' \qquad \text{-----------------------------(1-c)}$$

$$N_{x\varphi} = \frac{D_{x\varphi}}{a}(\dot{u}+v') \qquad \text{------------------------------(1-d)}$$

$$M_{\varphi} = \frac{K_{\varphi}}{a^{2}}\ddot{w} + \frac{K_{v}}{a^{2}}w'' - \frac{S_{\varphi}}{a}(\dot{v}+w) \qquad \text{-----------------------------(1-e)}$$

$$M_x = \frac{K_x}{a^2} w^{//} + \frac{K_\nu}{a^2} \ddot{w} - \frac{S_x}{a} u^{/} \quad \text{------------------------------(1-f)}$$

$$M_{\varphi x} = \frac{K_{\varphi x}}{a^2} w^{/\bullet} \quad \text{-------------------------------(1-g)}$$

$$M_{x\varphi} = \frac{K_{x\varphi}}{a^2} w^{/\bullet} \quad \text{------------------------------(1-h)}$$

where rigidity are defined by,

$$D_\varphi = \frac{Et}{1-\nu^2} + \frac{EA_\varphi}{b_1} \quad \text{-------------------------------(2-a)}$$

$$D_x = \frac{Et}{1-\nu^2} + \frac{EA_x}{b_2} \quad \text{--------------------------------(2-b)}$$

$$D_\nu = \frac{E\nu}{1-\nu^2} \quad \text{--------------------------------(2-c)}$$

$$D_{x\varphi} = \frac{Et}{2(1+\nu)} \quad \text{-------------------------------(2-d)}$$

$$S_\varphi = \frac{EA_\varphi C_\varphi}{b_1} \quad \text{--------------------------------(2-e)}$$

$$S_x = \frac{EA_x C_x}{b_2} \quad \text{------------------------------------(2-f)}$$

$$K_\varphi = \frac{Et^3}{12(1-\nu^2)} + \frac{E(I_\varphi + A_\varphi C_\varphi{}^2)}{b_1} \quad \text{------------------------------(2-g)}$$

$$K_x = \frac{Et^3}{12(1-\nu^2)} + \frac{E(I_x + A_x C_x{}^2)}{b_2} \quad \text{------------------------------(2-h)}$$

$$K_\nu = \frac{Et^3\nu}{12(1-\nu^2)} \quad \text{------------------------------(2-i)}$$

$$K_{\varphi x} = \frac{Et_3}{12(1+\nu)} + \frac{GJ_\varphi}{b_1} \quad \text{---(2-j)}$$

$$K_{x\varphi} = \frac{Et_3}{12(1+\nu)} + \frac{GJ_x}{b_2} \quad \text{-----------------------------------(2-k)}$$

The equation of equilibrium in case of buckling of cylindrical shell vide Flugge(1973)(p-448) are given by,

$$a N_x{}^{/} + a \dot{N}_{\varphi x} - pa(\ddot{u} - w^{/}) - Pu^{//} - 2Tu^{/\bullet} = 0 \quad \text{-------------------------------(3-a)}$$

$$a \dot{N}_\varphi + a N_{x\varphi}{}^{/} - \dot{M}_\varphi - M_{x\varphi}{}^{/} - pa(\ddot{v} + \dot{w}) - P v^{//} - 2T(v^{/\bullet} + w^{/}) = 0 \quad \text{------------------------(3-b)}$$

$$\ddot{M}_{\varphi} + \dot{M}_{x\varphi}^{\,/} + M_{\varphi x}^{\,/\bullet} + M_{x}^{\,//} + a\,N_{\varphi} + pa(u^{/} - \dot{v} + \ddot{w}) + P\,w^{//} - 2T(v^{/} - w^{/\bullet}) = 0 \text{ ----------(3-c)}$$

where ( )$^/$ and ($^\bullet$) denotes a $\frac{\partial}{\partial x}(\ldots)$ and $\frac{\partial}{\partial \phi}(\ldots)$ respectively.

The shell being subject to three simple loads -

(i) a uniform normal pressure on its wall, $p_r = -p$ (Fig.2)

(ii) an axial compression applied at the edges, the force per unit of circumference being P

(iii) a shear load applied at the same edges so as to produce a torque in the cylinder. The shearing force (shear flow) is T.

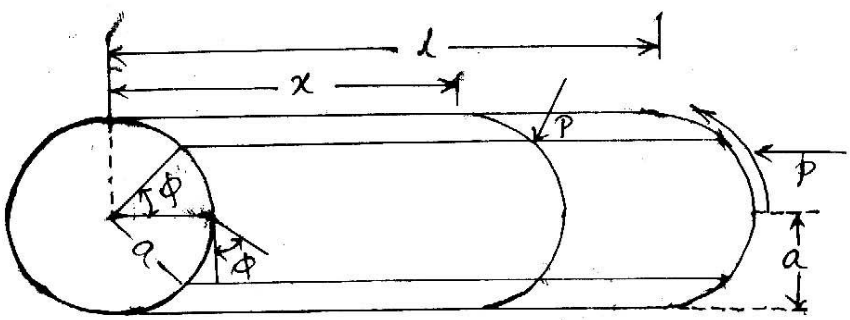


Fig. 2(a)

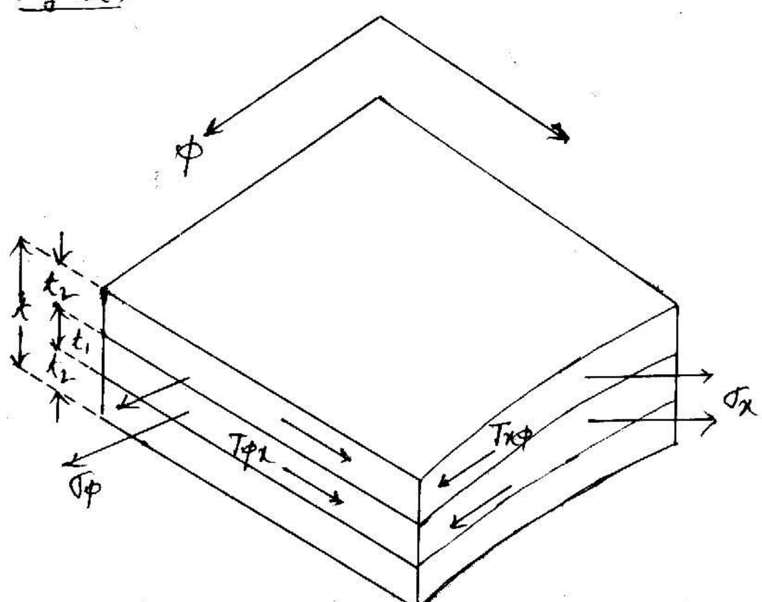


Fig. 2(b)

FIG. (2-a) - CYLINDRICAL SHELL: CO-ORDINATES AND BASIC LOADS.
FIG. (2-b) - ELEMENTS OF A PLYWOOD SHELL.

Substituting (1) in (3), the differential equation for the re-inforced anisotropic cylindrical shell after proper simplification reduces to,

$$u^{//} + A_1\ddot{u} + A_2 v^{/\bullet} + A_3 w^{/} - k_1(A_4 w^{///} - A_5 w^{/\bullet\bullet}) - q_1(\ddot{u} - w^{/}) - q_2 u^{//} - 2q_3 u^{/\bullet} = 0 \text{ ....(4-a)}$$

$$A_6\, u^{/\bullet} + \ddot{v} + A_7 v^{///} + \dot{w} + k_1 [A_8(\ddot{v} + \dot{w} - \dddot{w}) - A_9 w^{//\bullet} - A_{10} \dddot{w}\,] - A_{11}[\, q_1 (\ddot{v} + \dot{w}) + q_2\, v^{//} + 2\, q_3 (\, v^{/\bullet} + w^{/})] = 0 \quad \text{.................(4-b)}$$

$$A_{12}\, u^{/} + \dot{v} + w - k_1 [A_8(\dddot{v} + 2\ddot{w}) + A_{13} u^{///} - A_{10} \ddddot{w} - A_{14} w^{//\bullet\bullet} - A_{11} w^{////}] + A_{11}[\, q_1 (u^{/} - \dot{v} + \ddot{w}) + q_2\, w^{//} - q_3 (\, v^{/} - w^{/\bullet})] = 0 \quad \text{....................(4-c)}$$

where $A_1 = \dfrac{D_{x\varphi}}{D_x}$, $A_2 = \dfrac{D_\nu + D_{x\varphi}}{D_x}$, $A_3 = \dfrac{D_\nu}{D_x}$, $A_4 = \dfrac{aS_x}{K_x}$,

$A_5 = \dfrac{K_{\varphi x}}{K_x}$, $A_6 = \dfrac{D_\nu + D_{x\varphi}}{D_\varphi}$, $A_7 = \dfrac{D_{x\varphi}}{D_\varphi}$, $A_8 = \dfrac{aS_\varphi D_x}{K_x D_\varphi}$, ...(5)

$A_9 = \dfrac{D_x (K_\nu + K_{x\varphi})}{K_x D_\varphi}$, $A_{10} = \dfrac{K_\varphi D_x}{K_x D_\varphi}$, $A_{11} = \dfrac{D_x}{D_\varphi}$, $A_{12} = \dfrac{D_\nu}{D_\varphi}$,

$A_{13} = \dfrac{aS_x D_x}{K_x D_\varphi}$, $A_{14} = \dfrac{2K_\nu + K_{\varphi x} + K_{x\varphi}}{K_x D_\varphi}$

And $k_1 = \dfrac{K_x}{a^2 D_x}$, $q_1 = \dfrac{pa}{D_x}$, $q_2 = \dfrac{P}{D_x}$, $q_3 = \dfrac{T}{D_x}$ ....(6)

The equations (4) describes the differential equations for the buckling of re-inforced anisotropic cylindrical shells with rings and stringers. In these equations the parameters defined by (6) are small quantities. For k_1, it is obvious since we are concerned with the shells where t<<a. The three load parameters q_1, q_2, q_3 are approximately the elastic strains caused by the corresponding basic loads, and since all our theory is based on the assumption that such strains are small compared with unity, we shall neglect q_1, q_2, q_3 compared with 1 whenever the opportunity comes.

3.SOLUTION OF THE PROBLEM

When there are no shear load (ie, T = 0 and hence $q_3 = 0$) the differential equations (4) admit a solution of the form,

$$u = A \cos(m\varphi) . \cos\frac{\lambda x}{a}$$

$$v = B \sin(m\varphi) . \sin\frac{\lambda x}{a} \quad \text{...................................(7)}$$

$$w = C \cos(m\varphi) . \sin\frac{\lambda x}{a}$$

where $\lambda = \dfrac{n\pi.a}{l}$ (n being an integer) ....(8)

Solution (7) describes a buckling mode with n half waves along the length of the cylinder and with 2m half waves around its circumference. Although this is far from being the most general solution, it is one which fulfils reasonable boundary conditions. We assume the edges of the cylinder to be at x =0 and x = l, and we see at a glance that there v = w = 0. From the elastic law (1) we see that $N_x = 0$, $M_x = 0$.

Thus we see that the solution (7) represents the buckling of a shell whose edges are supported in the tangential and radial direction but are neither restricted in the axial direction nor clamped.

When we introduce the solution (7) in the differential equation (4) with $q_3 = 0$, the trigonometrical function drop out entirely and we are left with the following three equations,

$$A[\lambda^2 + A_1 m^2 - q_1 m^2 - q_2 \lambda^2] + B[- A_2\lambda m] + C[- A_3\lambda + k_1(- A_4\lambda^3 + A_5\lambda m^2) - q_1 \lambda] = 0 \quad \text{(9-a)}$$

$$A[-A_6\lambda m] + B[m^2 + A_7\lambda^2 + k_1 A_8 m^2 - A_{11}(q_1 m^2 + q_2 \lambda^2)] + C[m + k_1\{A_8(m + m^3) + A_9\lambda^2 m + A_{10} m^3\} - A_{11} q_1 m] = 0 \quad \text{(9-b)}$$

$$A[-A_{12}\lambda - k_1 A_{13}\lambda^3 - A_{11} q_1 \lambda] + B[m + k_1 A_8 m^3 - A_{11} m q_1] + C[1 + k_1(2A_8 m^2 + A_{10} m^4 + A_{14}\lambda^2 m^2 + A_{11}\lambda^4) - A_{11}(q_1 m^2 + q_2 \lambda^2)] = 0 \quad \text{(9-c)}$$

The equations (9) are three linear equations with the buckling amplitudes A, B, C as unknowns and with the brackets as coefficients. Since these equations are homogeneous they admit, in general, only the solution A = B = C = 0 , indicating that the shell is not in neutral equilibrium.

A non-vanishing solution A, B, C is possible only if,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} == 0 \quad \text{(10)}$$

where a_{11}, a_{12}, a_{13} ……….. etc. are the coefficients of A, B, C in the equations (9-a), (9-b), (9-c) respectively.

Thus vanishing of the det. (10) is the buckling condition of the shell whenever the buckling condition is fulfilled, any two of the three equations (9) determine the ratios A/C and B/C and thus the buckling mode according to equation (7). As in the case of neutral equilibrium, the magnitude of the possible deformation remains arbitrary.

The buckling condition contains four unknowns: the dimensionless loads q_1, q_2 and the modal parameters m and λ. Of m we know that it must be an integer (0,1,2,3,4……..) and of λ, that it must be an integer multiple n of $\frac{\pi . a}{l}$.

We may write the buckling condition separately for every pair (m, λ) fulfilling these requirements and consider it as a relation between q_1 and q_2 which describes this combinations of the load for which the shell is in neutral equilibrium as a curve in the q_1 - q_2 plane, we obtain a diagram like fig.3 which can be interpreted as follows,

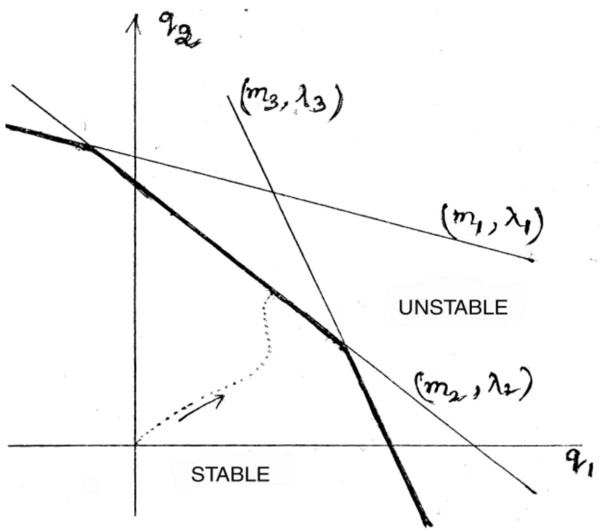


FIG. BUCKLING DIAGRAM

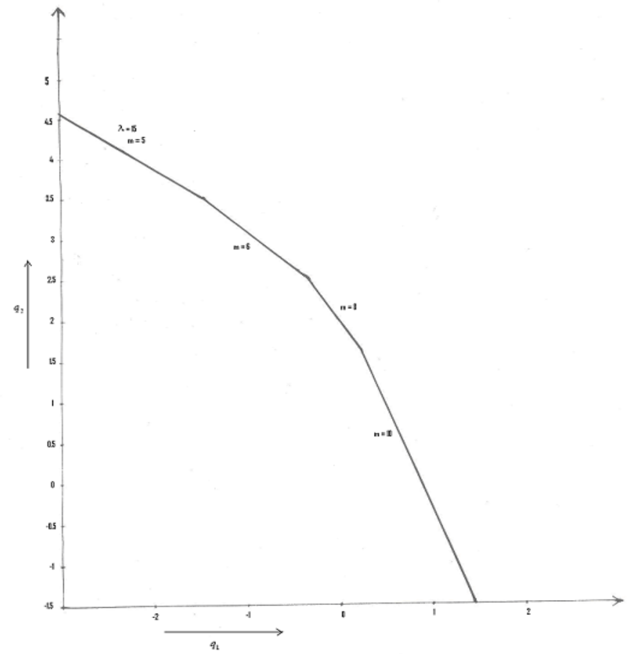


Fig. 3 : Stable and unstable regions in the q_1 - q_2 plane

The origin $q_1 = q_2 = 0$ represents the unloaded shell. When a load is gradually applied, the corresponding diagram point moves along a certain path, as shown by the dotted line. As long as it does not meet any of the curves, the shell is in stable equilibrium, but as soon as one of the curves is reached, equilibrium becomes neutral with the buckling mode defined by the parameters m, λ of this curve. The stable domain in the q_1 - q_2 plane is therefore, bounded by the envelope of all the curves, which is shown in fig.3 by a heavy line.

The coefficients of (9) are linear functions of k, q_1, q_2. The expanded determinant (10) is therefore a polynomial of third degree in these parameters. Since they are very small quantities, it is sufficient to keep only the linear terms and we can write the buckling conditions as,

$$C_1 + C_2 \mathrm{k}_1 = C_3 q_1 + C_4 q_2 \quad \text{..........................(11)}$$

The equation (11) describes a straight line in the q_1 - q_2 plane and the limit of the stable domain as shown in fig.3, is a polygon consisting of sections of straight lines for different pairs (m, λ).

The coefficients C_1, C_2, C_3, C_4 of the equation (11) may easily be found out by directly expanding the det. (10) and putting it equal to zero. Since C_1, turns out to be proportional to λ^4, we may drop the terms with λ^4 in all other coefficients and we obtain,

$$C_1 = A_7(1 - A_{12})\, \lambda^4 \quad \text{...(12-a)}$$

$$C_2 = (\lambda^2 + A_1 m^2)(m^2 + A_7\lambda^2)(A_{10}m^4 + A_{14}\lambda^2 m^2 + A_{11}\lambda^4) - [A_{15}\lambda^6 - A_{16}\lambda^4 m^2 - A_{17}\lambda^2 m^4$$
$$- A_{18}m^6 + A_{19}\lambda^2 m^6 + A_{20}\lambda^4 m^4 + A_{21}\lambda^6 m^2 - A_{22}\lambda^2 m^2 - A_{23}\lambda^3 m^3 - A_{24}\lambda^5 m] \quad \text{............(12-b)}$$

$$C_3 = (\lambda^2 + A_1 m^2)(m^2 + A_7\lambda^2)A_{11}m^2 + \lambda^2 m^2(A_{25}\lambda^2 - A_{21}m^2) + m^2(A_{26}\lambda^2 - A_{27}m^2) \quad \text{..........(12-c)}$$

$$C_4 = (\lambda^2 + A_1 m^2)(m^2 + A_7\lambda^2)A_{11}\lambda^2 - A_{21}\lambda^4 m^2 + A_{27}\lambda^2 m^2 \quad \text{...................................(12-d)}$$

where $A_{15} = A_7(A_3 A_{13} + A_4 A_{12})$

$A_{16} = 2\,A_7 A_8 + A_9 + A_2 A_{13} + A_2 A_9 A_{12} - A_3 A_{13} + A_4 A_6 - A_4 A_{12}$

$A_{16} = A_{10} + 2\,A_1 A_7 A_8 + A_1 A_9 - 2\,A_2 A_6 A_8 + A_2 A_8 A_{12} + A_2 A_{10} A_{12} + A_3 A_6 A_8$

$A_{18} = A_1 A_{10}$

$A_{19} = A_2 A_6 A_{10}$

$A_{20} = A_2 A_6 A_{14}$

$A_{21} = A_2 A_6 A_{11}$ ………………………………………………………………… …..(13)

$A_{22} = A_8 A_{12} (A_2 - A_3)$

$A_{23} = A_5 (A_{12} - A_6)$

$A_{24} = A_5 A_7 A_{12}$

$A_{25} = A_2 A_{11} A_{12}$

$A_{26} = A_7 - A_{11} - A_2 A_{11} + A_3 A_6 A_{11} + A_3 A_{11} - A_3 A_{11} A_{12} - A_6 + A_{12}$

$A_{27} = A_1 A_{11}$

From the formulae (11) & (12) the stability curve may easily be constructed when l and k_1 are given.

Particular Case :

Putting $D_\nu = 0$, $K_\nu = 0$ in the equations (1), (4), (9), (12) reduces to the corresponding equations for Gridwork Cylindrical shells which are found to be identical with De, A[13](1987) .

4. NUMERICAL CALCULATIONS:

From the formulae (11) & (12) the stability curve may easily be drawn when l and k_1 are given.

Taking $t_1 = 3$ cm., $t_2 = 2$ cm., $t = t_1 + 2\,t_2 = 7$ cm. & $k_1 = 10^{-5} = k$ and considering the shell to be made up of same matter as that of Gaboon (Okoumme)-3 ply so that,

$E = 1.28 \times 10^6$ psi, $G = 0.085 \times 10^6$ psi

$\nu = 0.378$, $b_1 = b_2 = 1$ cm.

$C_x = C_\varphi = 0.5$ cm.,

$A_x = A_\varphi = 1$ sq. cm,

$I_x = I_\varphi = 10$ units,

$J_x = J_\varphi = 1$ unit,

(see Timoshenko and Woinowsky Kreiger[10](1959)) The buckling diagram of re-inforced anisotropic cylindrical shell subject to two-way compression is sketched(fig.) and the following conclusion may be drawn.

Although the load and the basic stress system have axial symmetry, the buckling mode does not($m \neq 0$) but develop nodal generators, their number varies as q_1 increases.

The diagram shows that as m increases q_1 gradually decreases and q_2 increases.

The stable domain in the q_1 - q_2 plane is bounded by the envelope of all the curves which is shown in the figure by a heavy line.

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