Crack width analysis for circular reinforced concrete columns subjected to axial load and uniaxial bending

B. Meena Sai Lakshmi[1], P K Priyesh[2], and Suresh R[3]
[1]*Asst. Engineering Manager (Design), L&T construction.*
[2]*Asst. Engineering Manager (Design), L&T construction*
[3]*Sr.Engineering Manager (Design), L&T construction*

Abstract **- This paper presents analytical approach to calculate crack width for reinforced concrete (RC) circular columns under axial load with uniaxial bending as per Euro code 2. Compression members are most commonly encountered in reinforced concrete buildings as columns. Circular columns are efficient in the use of formwork and are elegant in appearance. Crack width analysis of RC structures is not much easy due to complexity of the parameters which affect the crack width and there is no specific simplified guidance for crack width calculations of circular RC columns. This Analytical method will be helpful to enhance the efficiency of the design work related to the crack width calculations of circular RC columns. The present study was carried out to evaluate the behavior of cracks in circular RC columns as per the design standard, EN 1992-1-1. An analytical approach is introduced to calculate the crack width of RC Circular columns and Oasys adsec software was used to perform crack width calculations for validation process.**

Keywords – columns, Compression, uniaxial, crack width, analytical approach.

I. INTRODUCTION

Columns are the primary vertical carrying members of a typical multistoried buildings. The loads coming on the floors and beams are transmitted to the foundation through columns. In addition to direct vertical loads, bending moments and shear forces in one or two directions are often carried by columns. If columns have axial loads and bending moment about either the x- or y-axes only, they are classified as uniaxial eccentrically loaded columns.

This section deals with the behaviour and design of short compression members subject to axial compression combined with uniaxial bending. The loading condition is statically equivalent to a condition of uniaxial eccentric compression wherein the factored axial load Pu is applied at an eccentricity e = Mu/Pu with respect to the centroid axis, Mu being the factored bending moment. The elastic behaviour will be dealt with on the basis of strip concept.

Columns may be rectangular and non-rectangular cross sections such as square, circular, hexagonal, octagonal cross sections etc. Circular columns are efficient in the use of formwork and are elegant in appearance. Circular columns are mostly use in buildings and bridges due to its efficiency.

Crack width analysis of reinforced concrete(RC) structures under serviceability conditions has gained considerable importance due to buildings tend to be taller and structural spans tend to be greater. The appearance of cracks raises concern about the durability and safety of the structure.

Crack width of reinforced concrete (RC) structures should be controlled for satisfying the serviceability and durability requirements of the structures. However, the crack width analysis of RC structures is not much easy due to complexity of the parameters which affect the crack width and there is no specific simplified guidance in terms charts and tables for crack width calculations of circular RC columns. This Analytical method will be helpful to enhance the efficiency of the design work related to the crack width calculations of circular RC columns.

The present study was carried out to evaluate the behaviour of cracks in circular RC columns as per the design standard, EN 1992-1-1. An analytical approach is introduced to calculate crack width of RC Circular columns and Oasys adsec software was used to perform crack width calculations for validation process.

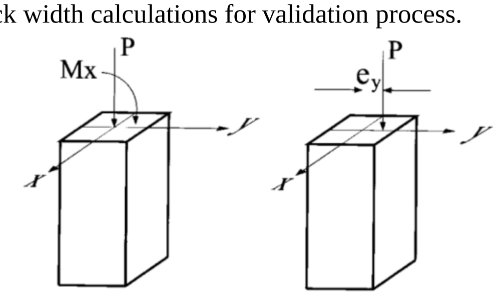


Fig.1 Uniaxial eccentrically loaded columns.

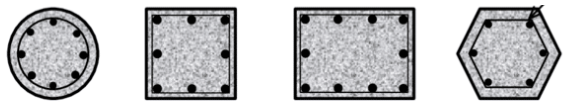


Fig.2 Types of cross sections for RC columns

In the present study, the following cases are considered to evaluate crack width of RC circular columns by Analytical method.

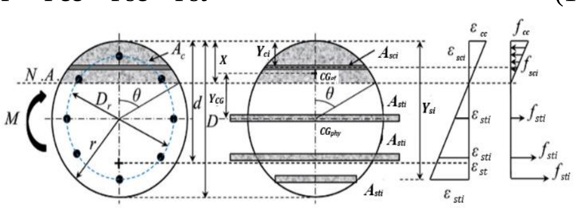
Table.1 Different types of column

S.No.	Column Type	Diameter(mm)	Axial Load (kN)	Bending moment (kNm)
1	C1	1000	300	675
2	C2	800	-150	500
3	C3	600	500	175
4	C4	500	-100	75

II. ANALYTICAL METHOD FORMULATION

The stress and strain distribution of a circular column section (uniaxial column) for the calculation of Crack width is given in Figure 4. The resultant force (P) is equal to the summation of all internal forces,

$$P = Fcc + Fsc – Fst \qquad (1)$$



RC Column cross section Transformed cracked section Strains Stresses

Fig.4 stress and strain distribution for RC columns

The following steps revealed the calculation of the required internal forces for a circular uniaxial RC column.

A. Plain Concrete section

The internal concrete compression force (F_{cc}) is computed as $F_{cc} = \sum f_{ci} A_{ci}$ (2)

B. Compression Steel Section.

The internal steel compressive force (F_{sc}) is computed as

$$F_{sc} = \sum f_{yki} A_{sci} \qquad (3)$$

C. Tension Steel section.

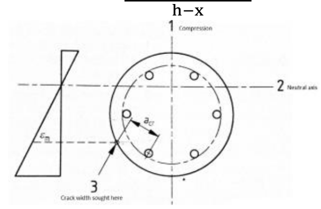
The internal tensile force (F_{st}) is computed as

$$F_{st} = \sum f_{yki} A_{sti} \qquad (4)$$

D. Crack width Calculation.

Crack width calculations for non-rectangular tension zones as per PD 6678-1:2010 sec 2.22,

$$w_k = \frac{3\, a_{cr}\, \varepsilon_m}{1+\frac{2(a_{cr}-c_{min})}{h-x}} \qquad (5)$$



Where,

a_{cr} = The distance from the point considered to the surface of the nearest longitudinal bar, C, h and X as defined in BS EN 1992-1-1:2004, cl. 7.3.

ε_m = The average strain at the level at which cracking is considered, assuming the concrete stress varies linearly from zero at the neutral axis to 0.7 N/mm² at the extreme fibre in tension.

III. ANALYTICAL METHOD PROCEDURE

In analytical method, the following steps are used to calculate stress, strain and crack width of the circular RC column section.

1) Circular section properties:

Area of concrete under compression (A_c),

Area of rebar in compression (Transformed) (A_{sc}),

Area of rebar in tension (Transformed) (A_{st}),

Transformed area of cracked section,

$$A_e = (A_c+(m-1)A_{sc}+ mA_{st}) \qquad (6)$$

2) Distance (Y_{CG}) between physical centroid (CG_{phy}) to the centroid of effective section (CG_{ef}):

Centroid of compression steel,

$$Y_{sc} = (m-1) \sum A_{sci}\, y_i / (m-1) \sum A_{sci} \qquad (7)$$

Centroid of tensile steel,

$$Y_{st} = m \sum A_{sti}\, y_i / m\sum A_{sti} \qquad (8)$$

Distance of effective centroid from physical centroid,

$$Y_{CG} = A_c.C_y + (m-1) \sum A_{sci}\, y_{sci} + m \sum A_{sti}\, y_{sti}]/ [Ac + (m-1) \sum A_{sci} + m \sum A_{sti}] \qquad (9)$$

3) Second moment of area of cracked section about centroid,

$$I_e = I_G + (m-1) \sum [A_{sci} (y_{sci} - Y_{CG})^2] + m \sum [A_{sti} (y_{sti} - Y_{CG})^2] \qquad (10)$$

4) Stress and strain of circular RC section,

If Circular section is under axial force combined with uniaxial bending, the following stresses & strains will be acting on the section.

Axial stress in rebar, $f_a = m \times P / A_e$ (11)

Axial strain, $e_a = f_a / E_s$ (12)

Distance from CG of section to top rebar, Y_t

Flexural Stress in top rebar,

$f_{sc} = m \times (M - P.Y_{CG}) / (I_e \times Y_t)$ (13)

Flexural Strain in top rebar, $e_{sc} = f_{sc} / E_s$ (14)

Distance from CG of section to bottom rebar, Y_b

Flexural Stress in bottom rebar,

$f_{st} = m \times (M - P.Y_{CG}) / I_e \times Y_b)$ (15)

Flexural Strain in bottom rebar, $e_{st} = f_{st} / E_s$ (16)

Rate of change of strain, $n = (e_{sc} - e_{st}) / (y_t - y_b)$ (17)

Strain in extreme fibre of concrete at top,

$e_{ct} = (n . x) + e_a$ (18)

Strain in extreme fibre of concrete at bottom

$e_{cb} = (n . (x - D)) + e_a$ (19)

5) Internal forces of circular section due to applied forces & moments (axial force, P & Moment, M) are as follows,

The internal compression force in concrete,

$F_{cc} = \sum f_{ci} . A_{ci}$ (20)

The internal compression force in steel,

$F_{sc} = \sum f_{sci} . A_{sci}$ (21)

Total compressive force, $F_c = F_{cc} + F_{sc} - P$

The internal tensile force, $F_{st} = \sum f_{sti} . A_{sti}$ (22)

Check for equilibrium, $F_{cc} + F_{sc} - P - F_{st} = 0$ (23)

6) Check for Crack width

Strain after accounting for tension stiffening of concrete,

$e_m = e_1 - e_2$ (24)

Distance from surface point to nearest rebar (a_{cr}),

Cover considered for design (C_{min}),

Depth of structure (h)

Crack width,

$$w_k = \frac{3\, a_{cr}\, \varepsilon_m}{1+2\frac{(a_{cr}-c_{min})}{h-x}} \quad (25)$$

IV. NUMERICAL EXAMPLE FOR UNIAXIAL COLUMNS

Detailed numerical example for circular column (C1) is shown below

Input Data: Figure

Axial Load, P = 300 kN

Bending Moment, M = 675 kNm

Strength Class C35/45

Grade of rebar Fe 500

Diameter of Column = 1000 mm

Cover to reinforcement = 50 mm

Diameter of link = 12 mm

Main reinforcement, 20 No's of 25mm dia.

1) Modulus of Elasticity of concrete, E_{cm} = 34077 N/mm² (As per Table 3.1 of BS EN 1992-1-1),

Creep coefficient = 1.45 (Cl. 3.1.4 of BS EN 1992-1-1),

Effective modulus of Elasticity, E_{ceff} = 13909 N/mm²,

Modular ratio (m) = E_s/E_{ceff} = 200000/14081 = 14.38

2) Assume depth of neutral axis (x) = 301 mm (Trail & error)

3) Properties for transformed section

Area of concrete under compression, A_c = 199399 mm²,

Area of rebar in compression, A_{sc} = 45972 mm²,

Area of rebar in tension, A_{st} = 91758 mm²,

Centroid of Transformed section Y_{CG} = 187 mm from pile centre,

Transformed area of cracked section, A_e = 337130 mm²,

Second moment of area of cracked section about centroid, I_e = 2.256E+10 mm^4.

4) Stress & strain due to axial load,

Axial stress in rebar, f_a = 12.8 N/mm²,

Axial strain, e_a = 0.00006

5) Stress & strain due to applied moment,

(a) Flexural Stress in top rebar, f_{st} = 93.9 N/mm²,

Flexural Strain in top rebar, e_{st} = 0.00047.

(b) Flexural Stress in bottom rebar, f_{sb} = 241.8 N/mm²,

Flexural Strain in bottom rebar, e_{sb} = 0.001209.

6) Internal forces of circular section due to applied forces,

Compressive force, F_{cc} = 900.42 kN, F_{sc} = 259.07 kN,

Tensile force, F_{st} = 859.49 kN.

7) Check for equilibrium,

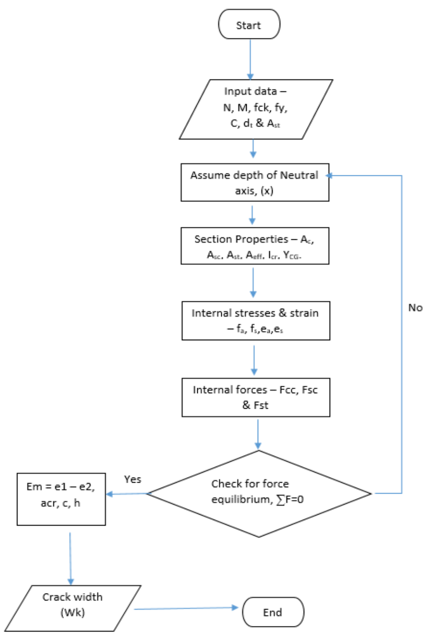
$F_{cc} + F_{sc} - P - F_{st}$ = 900.42 + 259.07 - 300 – 859.49
= 1159.49 -1159.49 = 0,

Hence assumed depth of neutral axis is satisfying to balance external forces & internal forces. If equilibrium equation is not equal to zero means internal and external forces need to be balanced by changing depth of neutral axis. Try again with another depth of neutral axis (x) until to satisfy equilibrium of forces eq (23).

8) Crack width calculation,

Strain due tension stiffening of concrete, e_2 = 0.00014,
Strain at the point under consideration, e_1 = 0.00129,
Average strain at the point under consideration,
$e_m = e_1 - e_2$ = 0.00129 - 0.00014 = 0.00115.
Distance from surface point to nearest rebar,
a_{cr} = 87.404 mm,
Cover considered for design, C = 62 mm,
Depth of structure, h = 1000 mm,
Crack Width, w_k= 0.281 mm.

V. FLOW CHART



VI. VALIDATION OF ANALYTICAL METHOD

In order to validate the proposed analytical method, the model is validated with computer software adsec. Four reinforced circular columns (C1 to C4) having different column sizes, different forces are analysed by using analytical method and also validated with adsec software for the same data.

The design input data for these columns are illustrated in Table 1. The four columns C1 to C4 are analysed by using analytical method to find stress, strain of bar and crack width of section and results are shown in Table 2. These values are also compared and validated with the computer software adsec. The obtained results are depicted in Table 3. The results obtained from computer software adsec are quite close to the analytical method results, showing satisfactory computational results.

Table 1: Input data for columns (C1 to C4)

Col. type	Dia. (mm)	Pu (kN)	Mu (kNm)	Ast (mm^2)	fck (Mpa)	fy (Mpa)
C1	1000	300	675	9817	45	500
C2	800	-150	500	12868	45	500
C3	600	500	175	2413	45	500
C4	500	-100	75	6434	45	500

Table 2: Analytical method results (C1 to C4)

Col. type	Max. stress in bar (MPa)	Avg. Strain in bar	crack width (mm)
C1	229	0.0011	0.281
C2	217	0.0012	0.284
C3	227	0.0012	0.272
C4	196	0.0012	0.265

Table 3: Computer software Adsec results (C1 to C4)

Col. type	Max. stress in bar (MPa)	Avg. Strain in bar	crack width (mm)
C1	230	0.0011	0.278
C2	217	0.0012	0.286
C3	238	0.0012	0.280
C4	196	0.0011	0.250

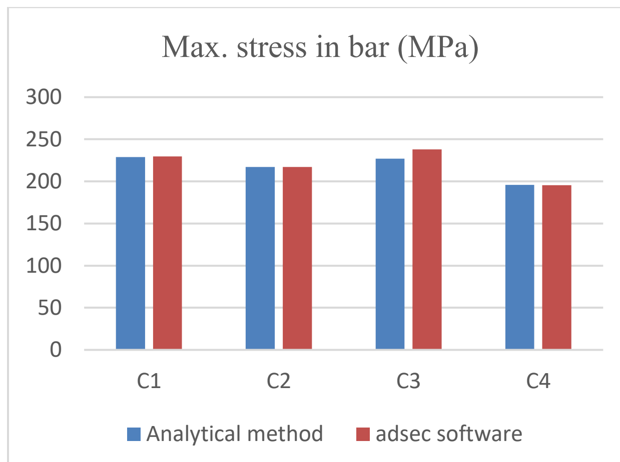


Figure 1: Max. Stress in bar (MPa) for columns C1 to C4

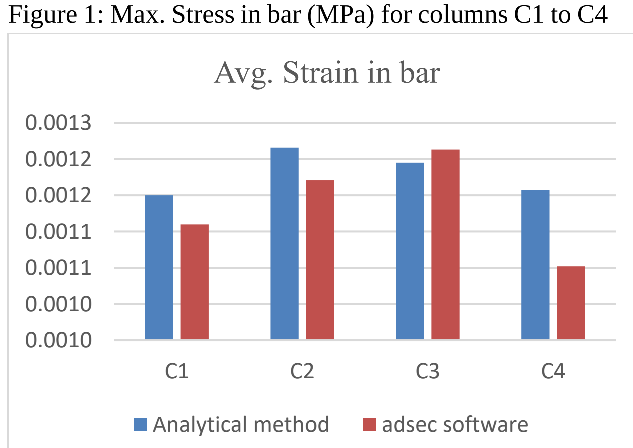


Figure 2: Avg. Strain in bar for columns C1 to C4

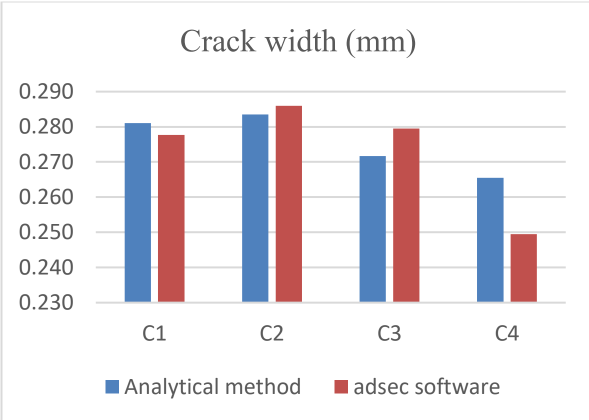


Figure 3: Crack width (mm) for columns C1 to C4

VII. RESULTS AND DISCUSSIONS

The four columns C1 to C4 are analysed by using analytical method to find stress, strain of bar and crack width of section and results are shown in Table 2. The results obtained from computer software adsec are quite close to the analytical method results, showing satisfactory computational results.

The results obtained from analytical method for circular columns showing a safe and conservative design method.

VIII. CONCLUSION

In this study, the analytical method is presented to analyse circular section for crack width as there is no direct method. Four reinforced circular columns (C1 to C4) having different column sizes, different forces are analysed by using analytical method and also validated with adsec software for the same data.

The results of stress and strain in a bar obtained from computer software adsec are quite matching to the analytical method results, showing conservative design method.

This newly analytical proposed mathematical method is a conservative and direct approach to analyse circular section for crack width. This method can also help the students and the academic researchers to analyse crack width of circular sections with less time.

IX. FUTURE SCOPE OF WORK

Direct approach is required to analyse stress and strain behaviour for non-rectangular sections such as hexagon, hollow sections. Need for development of charts to calculate stresses for non-rectangular sections which can be reduced time.

REFERENCE

[1] Basic reinforced concrete design by CHAS. E. REYNOLDS.

[2] Concrete Bridge Practice: Analysis, Design & Economics by V. K. Raina.

[3] Crack Width Calculation for Columns Subject to Biaxial Bending by R. Gong, S. Cao.

[4] Cracks in circular reinforced concrete columns occurring during the construction process by Marta Lutomirska, Szczepan Lutomirski

[5] Eurocode 2 – Design of concrete structures, Part 1: General Rules and Rules for Buildings. BSI British Standard, BS EN 1992-1-1:2004+A1:2014.

[6] A. K. H. Kwan and F. J. Ma, "Crack width analysis of reinforced concrete under direct tension by finite element method and crack queuing algorithm," Engineering Structures, vol. 126, pp. 618-627, 2 016, doi: 10.1016/j.engstruct.2016.08.027.

[7] Clark L.A. (1983). Concrete bridge design to BS5400.

[8] BS8110-1: 1997. Structural use of concrete – Part 1: Code of practice for design and construction.

[9] BS8110-2: 1985. Structural use of concrete – Part 2: Code of practice for special circumstances.

[10] J. Zięba, L. B. Ożóg, and I. Skrzypczak, "Probabilistic method and F EM analysis in the design and analysis of cracks widths," Engineering Structures, vol. 209, 2020, doi: 10.1016/j.engstruct.2019. 110022.

[11] F. A. Mohammad and B. Merrony, "Design charts for reinforced concrete circular columns in accordance with Eurocode 2,", Proc. Of the Institution of Civil Engineers - Structures and Buildings, vol. 110, no. 4, pp. 410-416, 1995.

[12] M. Lutomirska and S. Lutomirski, "Cracks in circular reinforced concrete columns occurring during the construction process," Procedia Engineering, vol. 153, pp. 419-426, 2016.

[13] I. A. Tegos, N. C. Giannakas, and T. A. Chrysanidis, "Serviceability cracking check of circular section piers," Bridge Structures, vol. 7, no.1, pp. 43-52, 2011.

[14] Steel, concrete and composite bridges, Part 4, Code of practice for the d esign of concrete bridges, BS 5400 - 4: 1990.