A Survey Report on "Optimization of different variants of Travelling Salesman Problem"

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Abstract: We have covered a few optimizations of various Traveling Salesman Problem (TSP) variations in this paper, along with Ant Colony Optimization (ACO)-based methods for TSP. Techniques for ant colony optimization were created for static optimization issues, where the input data is predetermined and does not change over time. Among the significant suggestions of this kind are various modifications to the ACO algorithm to improve information reuse and a population-based ACO algorithm created especially for dynamic combinatorial optimization problems. We cover the ACO algorithm for a time-dependent traveling salesman problem, hybrid ACO for a solid multiple traveling salesman problem, and ACO for a multi-conveyance TSP with a cost and time limit among these. An extensive survey of ACO-based solutions for TSP issues is presented in this paper. The experimental section presents computational results using various input data sets.

Keywords: **Travelling Salesman Problem, Multi-objective, Genetic Algorithm, Ant Colony Optimization.**

INTRODUCTION

The traveling salesman problem (GTSP) has been put up by Henry-Labordere, Saksena, and Srivastava about visit sequencing through social organizations and computer record balance since the 1960s. In a sense, the GTSP is a combinatorial optimization problem. It can be defined as the challenge of locating a particular Hamiltonian cycle with the lowest cost in a complete weighted graph.

Review on Optimization of Different variants of TSP: Finding the shortest path between a collection of n vertices so that, save from the starting vertex, each vertex is visited exactly once is the aim of the TSP, a combinatorial optimization problem. This problem is known to be NP-hard, meaning that it cannot be solved exactly in polynomial time, since the tour stops at the initial vertex.

Coello et al. (2005) used an artificial immune system to solve multi-objective optimization problems. In this research, a method based on the clonal selection principle was suggested to tackle multi-objective optimization problems, either confined or unconstrained. They employed two types of mutation: uniform mutation applied to the clones produced and non-uniform mutation applied to the "not so good" antibodies (which are represented by binary strings that encode the decision variables of the problem to be solved).

Bektas (2006) presented an overview of formulations and solution techniques for the multiple traveling salesman issue. The purpose of the overview was to analyze the problem and its practical applications, highlight certain formulations, and explain precise and heuristic solution procedures that have been proposed for this problem.

To address imprecision in vehicle routing difficulties, Goncalves and Djadane Hsu (2007) suggested using fuzzy logic. This article discusses the fleet management of dynamic vehicles that are assigned to the delivery and/or pickup of items on behalf of consumers. Travel times and time windows are subject to change to create a more realistic model. They also present preliminary results from two approaches (heuristic insertion and genetic algorithm) to resolve the vehicle routing problem, as well as a realistic model created by introducing uncertain data (flexible time windows and fuzzy travel times) modeled by fuzzy logic.

Numerous combinatorial optimization issues have been successfully solved using the simulated annealing method. In traditional simulated annealing, simple algorithms with a larger transition probability at the start of the search and a lower probability towards the end of the search dictate temperature and local search repetition. However, these easy techniques may result in a less effective search procedure. To address this flaw, Jeong Kim Lee (2009) introduced an effective search technique for simulated annealing utilizing a fuzzy logic controller. This method offers an adaptive simulated annealing algorithm that makes use of a fuzzy logic controller (FLC). The search process of simulated annealing can be made more effective by FLC by controlling the temperature and the local search repetition.

To tackle the traveling salesman problem, JolaiGhanbari (2010) combined Hopfield neural networks with data transformation techniques. An enhanced artificial neural network (ANN) method for the traveling salesman problem (TSP) is presented in this article. Together, they use data transformation techniques (DTT) and Hopfield neural networks (HNN) to increase the accuracy of the results and find the best tours with shorter overall lengths.

Majumdar Bhunia (2011) presented a genetic solution for an asymmetric traveling salesman issue with inaccurate journey periods. This study provides a version of the asymmetric traveling salesman problem (ATSP) where, instead of a fixed (deterministic) value as in the conventional ATSP, the traveling time between each pair of cities is represented by an interval of values (wherein the actual journey time is predicted to lie).

Deb (2011) provided a succinct overview of an evolutionary optimization method for multi-objective optimization. The principles of evolutionary multi-objective optimization were then covered by him.

Time-dependent fuzzy velocity was employed by Sifa Jiandong Keqiang (2011) to address the problem of urban pickup and delivery. In this research, a realistic model that takes into account the time-dependent fuzzy velocity of vehicles is proposed to handle the urban pickup and delivery problem. First, a simulation of the fuzzy velocity membership function was made using traffic data gathered from typical urban city roadways. Then, using the Alfa-Cut Set Algorithm, the fuzzy arrival time for the consumers was determined. The urban pickup and delivery problem under time-dependent fuzzy velocity was then discussed, and the issue was resolved using a modified version of Tabu Search.

The Fixed Destination Multi-Depot Multiple Travelling Salesmen Problem (MmTSP) is a problem where multiple salesmen leave from multiple starting cities and return to the starting city to form tours such that the tour lengths stay within predetermined bounds and exactly one salesman visits each city. There haven't been many previous studies on this issue because of its complexity. To tackle the problem, Ghafurian Javadian (2011) proposed an ant colony strategy for handling fixed destination multi-depot multiple traveling salesman concerns. The solutions found by using Lingo 8.0, which employs exact methodologies, are compared to the findings obtained by Ghafurian Javadian.

Liao, Yau, and Chen (2012) presented an improved version of the Particle Swarm Optimization (PSO) technique as an evolutionary solution for Travelling Salesman Problems (TSP). This evolutionary process consists of two steps. The first phase includes fuzzy C-Means clustering, a random swap approach, a rule-based route permutation, and a cluster merge technique. By first creating a non-crossing initial route, this technique speeds up the suggested PSO algorithm's ability to solve the TSP. Using sub-clusters reduces complexity and enhances performance when handling scenarios with a high number of cities. The proposed Genetic-based PSO technique is then applied in the second phase to solve the TSP more efficiently.

Robati, Barani, &Anaraki (2012) introduced balanced fuzzy particle swarm optimization for TSP, an adaptation of the realistic particle swarm optimization (PSO) technique, to address combinatorial optimization challenges. The foundation of this algorithm's development is the theory of balanced fuzzy sets. In the new method, both positive and negative membership function information are equally meaningful, whereas, in the classic fuzzy sets theory, they cannot be separated. The balanced fuzzy particle swarm optimization technique is used to solve the traveling salesman problem (TSP), a fundamental optimization problem.

In 2012, Ries Beullens and Salt put forth the parameter setting problem is instance-specific multi-objective parameter tuning based on fuzzy logic to determine optimal parameter values for meta-heuristics. A novel instance-specific way to examine the trade-off between computational time and solution quality is presented: the IPTS strategy for parameter adjustment. A priori statistical analysis to determine the factors influencing heuristic performance in terms of quality and time for a particular kind of problem, as well as the conversion of these insights into a fuzzy inference system rule base to return parameter values on the Pareto-front concerning a decision maker's preference, are two crucial steps in the method.

An optimization method using ant colonies was presented by Changdar and Mahapatra Pal (2013) for the binary knapsack problem under fuzziness. They provide a brand-new ant colony optimization technique for handling binary knapsack problems. By using fuzzy possibility and necessity techniques, the suggested ant colony algorithm arrives at the best possible option. Computational trials using several data sets are provided to bolster the suggested methodology.

Observation from known results from the previous experiments: Based on observations of ants' foraging behavior and their ability to find the shortest path between their colony and food sources, an algorithmic model was developed in 1992 to tackle a combinatorial optimization problem. There has been a rise in interest in developing ant-based algorithms since then, resulting in a wide range of techniques and applications.

Description of considered variants of TSP

1. *The time-dependent traveling salesman problem:* Formulations (Picard and Queyranne, 1978): The time-dependent traveling salesman problem (TDTSP) is a version of the classical traveling salesman problem (TSP) where the transition cost between node i and node j depends on which period node i is visited, knowing that one period is needed to travel from one node to another.

This problem can be addressed more formally as follows. Let G(N; A) be an oriented graph with the node set N = {1, . . . , n}. For each arc (i,j) ∈ A, the cost c_{ij}^{t} of traveling on the arc at period t is known, where t = 1, . . . , n. The time-dependent traveling salesman problem thus consists of finding the least cost Hamiltonian circuit in G,

ξ = (i_1 = 1,i_2, . . . ,i_n,i_n+1 = (i_1 = 1),

where the cost of tour ξ is $c_{\xi}= c_{i_1 i_2}^{1} + c_{i_2 i_3}^{2} + \cdots + c_{i_n i_{n+1}}^{n}$.

Fox (1973, 1980) introduced the time-dependent traveling salesman problem and gave it two very compact (but nonpractical) integer programming formulations. Picard and Queyranne [1978] later proposed two stronger integer programming formulations. The first formulation contains O(n^3) variables and O(n^2) constraints and is very similar to a formulation proposed by Hadley [1964] for the classical traveling salesman problem:

$$Z_{T3}= \min \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{n} c_{ij}^{t}\, x_{ij}^{t}$$

$$\left.\begin{array}{l} \sum_{i=1}^{n} \sum_{t=1}^{n} x_{ij}^{t}=1 \text{ , j=1,.....,n} \\ \sum_{(i,j)\in A} x_{ij}^{1}=1 \text{ ,} \\ \sum_{(i,j)\in A} x_{ij}^{1} - \sum_{(j,l)\in A} x_{ji}^{t+1}=0 \quad \text{t=2,.....n-1,j=1,...n.} \\ \sum_{(i,l)\in A} x_{ii}^{n} =1 \end{array}\right\}$$

$$x_{ij}^{t}\in\{0,1\}\text{,i=1.....n , j=1...n , t=1....n.}$$

This formulation illustrates that the time-dependent traveling salesman problem can be seen as the shortest path problem on a multipartite network with complicating constraints that are used to force the path to visit every node once and only once (see Fig. 1). Let us define $(i,j)^{t}$as the decision of visiting arc (i,j) at position t and (i, t) the state indicating that node i is visited at position t in the tour. In this multipartite network, we simply associate an arc of cost c_{ij}^{t} to every possible decision $(i,j)^{t}$ which consists of going from state (i, t) to state (j, t +1). Considering that states (1, 1) and (1, n+1) are respectively the source and sink nodes in this network, then every path going from the source to the sink that respects constraints (2)–(6) can thus be seen as a feasible solution to the TSTSP since it defines a Hamiltonian circuit and no solution to the TDTSP cannot be transposed as a path in this network. The use of path variables, instead of flow variables x_{ij}^{t}, deduction is possible of the second formulation of Picard and Queyranne for the time-dependent traveling salesman problem:

$$\left.\begin{array}{l} Z_{DW(T3)}= \min\sum_{p\in\Omega}(\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{n} c_{ij}^{t}\, x_{ij}^{t})\theta_p \\ \sum_{p\in\Omega}(\sum_{i=1}^{n} \sum_{t=1}^{n} x_{ij}^{tp})\theta_p \ {\scriptstyle =1,\ j=1,\ldots,n} \\ \Theta_p\in\{0,1\},p \in \Omega \end{array}\right\}$$

where Ω is the set of feasible paths of the multipartite network and x_{ij}^{tp} is a constant taking value 1 if path p visits arc (i,j) at position t and 0 otherwise. This formulation contains only n constraints but an exponential number of columns and can be seen as a Dantzig and Wolfe [1960] reformulation of T3.

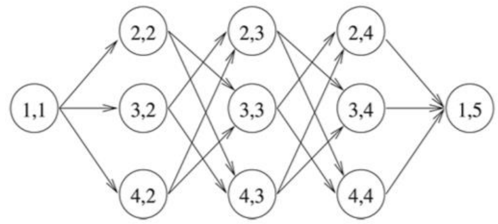

Fig. 1. The multipartite network for a 4 cities problem.

Despite the huge number of columns, it can be easier to solve DW(T3) than T3. Indeed, for a vector of dual variables π associated with constraints (8), one can find the path p associated with the variable θ_p that has the smallest reduced cost by solving the auxiliary problem:

$$Z_{SP(T3)} = \min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{n} (c_{ij}^t - \pi j) x_{ij}^t$$

$$\left.\begin{aligned} &\sum_{(1,j)\in A} x_{ij}^1 = 1, \\ &\sum_{(i,j)\in A} x_{ij}^t - \sum_{(1,j)\in A} x_{ij}^{t+1} = 0, \; t = 2,\ldots, n-1, \; j = 1,\ldots, n, \\ &\sum_{(i,j)\in A} x_{i1}^n = 1, \\ &x_{i1}^n \in \{0,1\}, \quad i = 1,\ldots n, \; j = 1,\ldots,n \end{aligned}\right\}$$

which can be solved as a shortest path problem in the multipartite network described previously. If one uses this subproblem to price the columns within the revised simplex algorithm (instead of the classical procedure), this can alleviate the difficulty of handling a huge number of columns.

Application to single machine scheduling problems:

TDTSP formulations can easily be extended to single-machine problems with sequence-dependent setup times.

Let $N = \{1,\ldots, n\}$ be the set of jobs to process where the assumption is that job 1 is a dummy job used to mark the beginning and the end of the sequence. For each job to be processed, there is a required process time p_j, a released time r_j, and a setup time s_{ij} incurred if job *j* is sequenced right after *i*. By introducing variables C_t that give the completion time of the job in position *t* and by setting :

$C_1 = 0$

$$\left.\begin{aligned} &C_t \geq C_{t-1} + \sum_{i=1}^{n}\sum_{j=1}^{n} (s_{ij} + p_j) x_{ij}^{t-1}, \quad t = 2,\ldots, n, \\ &C_t \geq \sum_{i=1}^{n}\sum_{j=1}^{n} (r_j + p_j) x_{ij}^{t-1}, \quad t = 2,\ldots, n, \end{aligned}\right\}$$

we can enforce constraints on the release date of jobs. If we add these constraints to one of the *TDTSP* formulations presented before and we minimize $\sum_{t=1}^{n} C_t$ instead of $\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{n} c_{ij}^t x_{ij}^t$, we thus obtain a valid integer programming formulation for $1 \mid r_j, s_{ij} \mid \sum C_j$. Note that for instances without release times, $1 \mid s_{ij} \mid \sum C_j$, there is no idle time in any optimal solution and we do not need these inequalities to calculate position completion time. In this case, we just need to minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{n} c_{ij}^t x_{ij}^t$ with costs:

$C_{ij}^t = (n - t + 1)(s_{ij} + p_j)$.

Similarly, problem $1 \mid r_j, s_{ij} \mid \sum Tj$ can be modeled by introducing in *TDTSP* formulations variables Tt which measure the tardiness of the job processed in position t, and by setting:

$T_t \geq C_t - \sum_{i=1}^{n}\sum_{j=1}^{n} d_j x_{ij}^{t-1}, \quad t = 2,\ldots, n,$

$T_t \geq 0$

Since tardiness for job j is defined as $\max\{c_j - d_j, 0\}$, where d_j is the due date of job j. The objective of the problem is now $min_{t=1}^{n} T_t$ (instead of min $\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=1}^{n} c_{ij}^t x_{ij}^t$). Unlike the total flow time problem, extra constraints are needed to evaluate the objective and to our knowledge, there is no way to model $1 \mid s_{ij} \mid \sum T_j$ as a pure *TDTSP*.

In this research, the author demonstrated how integer programming formulations of the time-dependent traveling salesman problem may be applied to single-machine scheduling problems with sequence-dependent setup times. Also, it was discovered how these formulations can be strengthened by applying results from the traveling salesman problem (subtour and 2-matching cuts), the node packing problem (clique cuts), and the vehicle routing problem

(Dantzig-Wolfe reformulation and k-cycle elimination), to finally confirm, with computational experiments, that it often pays to use stronger lower bounds, particularly those obtained with the Dantzig-Wolfe decomposition principle. Several cases were solved with stronger formulations that could not be solved with weaker ones.

2. Solid multiple traveling salesman problem in a fuzzy rough environment.

Problem definition and notation

In this section, at first, the mTSP is described in a crisp environment and then in the fuzz-rough environment.

Single depot solid mTSP: Usually, multiple TSP is formulated by integer programming formulations. However, in the case of integer programming some parts (variables) of the formulation may not be integer. In the proposed problem, the traveling cost is considered as a real number. The mTSP is more capable of modeling real-life applications than TSP since it handles two or more salesmen. In our proposed problem of solid mTSP, assuming that there are different conveyance facilities to travel from any city to any other city and travel cost is different for different conveyance/vehicles.

Consider a complete directed graph $G = (V, A, H)$, where V is the set of n nodes (vertices), A is the set of arcs, $C = c_{ijk}$ is the cost (distance) matrix associated with each arc(i, j, k) ∈A using kth type conveyance, and H is the set of conveyances'/vehicles' type. The cost matrix C can be symmetric, asymmetric, or Euclidean. Let there be m sales-men and all salesman and all salesmen start from depot city 1 and return to the same, depot city. Thus, the single depot solid mTSP consists of finding tours for m salesmen such that all start and end at the depot city, but the tours for each pair of salesmen are likely to be different, along with their conveyances. The maximum and minimum number of nodes visited by a salesman lies within a predetermined interval, and the goal is to minimize the overall cost of visiting all the nodes.

Each salesman starts from the depot city and visits a set of cities exactly once using suitable conveyances available at the cities and returns to the depot city at minimum cost. Let us define x_{ijk} as a binary variable equal to 1, if arc(i, j, k) is in the optimal solution and 0, otherwise. For any salesman u_i is the number of nodes visited on that salesman's path from the origin up to city i(i.e., the visit number of the ith city). Each salesman may visit at most W cities and at least K cities; thus, $1 \le u_i \le W$ for all $i \ge 2$. When $x_{i1} = 1$, then $K \le u_i \le W$. Thus, the problem can be written as follows:

$$
\left.
\begin{aligned}
& Z = \text{Minimize} \sum_{(i,j)\in A \text{ and } k\in H} c_{ijk}x_{ijk} \\
& \text{subject to } \sum_{j=2}^{n} xj1k = m, \\
& \sum_{j=2}^{n} xij1k = m, \\
& \sum_{i=1}^{n} xijk = 1,\ j = 2,3,\ldots, n, \text{ and } k \in H \\
& \sum_{j=1}^{n} xijk = 1,\ i = 2,3,\ldots, n, \text{ and } k \in H \\
& u_i + (W-2)x_{i1k} - x_{i1k} \le W-1;\ i = 2,3,\ldots,n, \text{ and } k \in H \\
& u_i + x_{1ik} + (2-K)x_{i1k} \ge 2,\ i = 2,3,\ldots,n, \text{ and } k \in H
\end{aligned}
\right\}
$$

$x_{ijk} \in \{0,1\}$, such that $(i,j) \in A$.

The above formulation ensures that $2 \le K \le [(n-1)/m]$ and $W \ge K$. For minimum and maximum number of nodes visited by each salesman is defined by constants 5 and 6, and $u_i = 1$ if and only if i is the first node in the tour for any salesman.

The proposed genetic ant colony optimization algorithm:

In this paper, they proposed a new Genetic Ant Colony-based algorithm to solve the proposed problem. At first, the pseudo-code for the proposed genetic ant colony optimization algorithm and then the algorithm is described in detail for the proposed problem.

Genetic ant colony optimization algorithm ( )

(a) Initialization of population:

For one to *popsize* repeat the following steps. [Pop size is the size of the population in GA.]

(i) Randomly generate a set of N random numbers between 1 and N without repetition and each number is again associated with a vehicle type.

(ii) Divide the sets into m subparts/subgroups (for m salesman problem) except the depot city.

(iii) Rearrange each subgroup/subtotal using the ACO algorithm (call subroutine *ALGO_ACO*()).

End For

(b) Evolution

Do While (the termination condition has not been reached)
(i) Selection for mating pool
(ii) Cyclic crossover
(iii) Mutation
(iv) Sequentially extract or sub-divide each chromosome into *m* subparts for each salesman path
(v) Rearrange each sub-tour using ACO algorithm using subroutine *ALGO_ACO*
Calculate fitness for each chromosome of the population
End While
(c) Extract optimal solution for GA
(d) Stop
Subroutine ALGO_ACO( )
Initialize pheromone for each sub-tour based on traveling cost from city to city
Evaluation:
Do While (the termination condition has reached or not converged)
(i) Construct a path/solution for each ant
(ii) Pheromone evaporation
(iii) Pheromone update
(iv) Refinement
End While
Extract the best path and return

Single depot solid mTSP with fuzzy-rough travel cost

If the cost of travel parameter of an objective function described in Eq. (3) is fuzzy-rough, this type of problem is called the single-objective optimization problem under fuzzy-rough environment. Then the above problem may be represented in the following way:

Z= Minimize $\sum_{(i,j)\in A \text{ and } k\in H} \widetilde{cijkxijk}$
subject to $\sum_{j=2}^{n} x1jk$ = m,
$\sum_{j=2}^{n} xj1k$ = m,
$\sum_{i=1}^{n} xijk$ = 1, j=2,3,…,n, and k ∈ H
$\sum_{j=1}^{n} xijk$ = 1, i=2,3,…,n, and k ∈ H
$u_i + (W-2)x_{1ik} - x_{i1k} \leq W - 1$; i= 2,3,…,n, and k ∈ H
$u_i + x_{1ik} + (2-K)x_{i1k} \geq 2$, i= 2,3,…,n, and k ∈ H
$x_{ijk} \in \{0,1\}$, such that (i,j) ∈ A.

Here $\widetilde{cijk}$ ($\tilde{c}$ – L, $\tilde{c}$,$\tilde{c}$+ R) is assumed to be a fuzzy-rough variable, where $\tilde{c}_{ijk}$ = ([c_{ijk1}, c_{ijk2}], [c_{ijk3}, c_{ijk4}]) is a rough variable.

Following existing literature can be re-written as follows:

Z = Minimize $\left[E\left(\sum_{(i,j)\in A,\ k\in H} \widetilde{cijkxijk}\right)\right]$

Minimize $\sum_{(i,j)\in A,\ k\in H} \left(\frac{cijk1+cijk2+cijk3+cijk4}{4} + \left(\frac{(\sigma R-(1-\sigma)L}{2}\right)\right)$

subject to $\sum_{j=2}^{n} x1jk = m$,
$\sum_{j=2}^{n} x1jk = m$,
$\sum_{i=1}^{n} xijk$ = 1, j=2,3,…,n, and $k\in H$
$\sum_{j=1}^{n} xijk$ = 1, i=2,3,…,n, and $k\in H$
$u_i + (W-2)x_{1ik} - x_{i1k} \leq W - 1$; i= 2,3,…,n, and k ∈ H
$u_i + x_{1ik} + (2-K)x_{i1k} \geq 2$; i = 2,3,…,n,
$x_{ijk} \in \{0,1\}$, such that (i,j) ∈ A.

Thus, it describes the formulation of single depot mTSP in a fuzzy-rough environment. In the experimental section, we have depicted the computational results.

The traveling salesman problem (TSP) is a well-known NP-complete problem. Numerous scientific and engineering applications exist for it. A multi-conveyance TSP, wherein many conveyances are present to move from one city to another, was proposed by Mondal and Srivastava (2022). This is a development of traditional TSP. via this TSP, the salesperson travels between cities via various modes of transportation and only makes one visit to each city for the course of the tour. The price of using different forms of transport to go between cities varies. The goal of this research is to satisfy the restrictions of the proposed multi-conveyance TSP to discover the minimal cost tour utilizing an ant colony optimization (ACO) based technique.

In this research, a genetic-ant colony optimization approach for solving a solid multiple Travelling Salesman Problem (mTSP) in a fuzzy rough environment. In solid mTSP, a set of nodes (locations/cities) is provided, and each city must be visited exactly once by the salesman, with each starting and ending at a depot via a separate transportation facility. A solid mTSP is an expansion of mTSP in which travelers move between cities using various modes of transportation. To solve a mTSP, a hybrid algorithm was devised that combines the concepts of two algorithms: the genetic algorithm (GA) and the ant colony optimization (ACO) algorithm. Each salesman chooses his or her route using ACO, while the GA controls the routes of other salesmen (to provide a comprehensive solution). A set of simple ACO features has been further enhanced by integrating a specific feature known as 'refinement'. In this research, the proposed technique uses cyclic crossover and two-point mutation to solve the problem. The journey cost is regarded as inaccurate (fuzzy-rough) and is approximated crisply using fuzzy-rough expectations. Computational findings for various data sets are presented, along with some sensitivity analysis.

3. Solving a Multi-Conveyance Travelling Salesman Problem using an Ant Colony Optimization Method

A multi-conveyance TSP was proposed where different conveyances are present to travel from one city to another city. This is an extension of classical TSP. In this TSP, the salesman visits all the cities only once during his/her tour, using different conveyances to travel from one city to another. The cost of traveling between cities using various modes of conveyance varies. The objective of this research is to find the minimum cost tour using an ant colony optimization (ACO) based approach by satisfying the constraints of the proposed multi-conveyance TSP.

3.1 Problem definition

Classical TSP with cost and time limit constraints

In a classical two-dimensional traveling salesman cities use minimum cost. Let c(i,j) be the cost of traveling from i-th city to j-th city. The total cost limit is $Cost_{MAX}$ and the time limit is Time. Then the problem can be mathematically formulated as:

$$\left.\begin{array}{c} \text{Minimize } Z = \sum_{i}^{N} = 1 \sum_{j}^{N} = 1 \text{yijc}(i,j) \\ \text{Subject to } \sum_{i}^{N} = 1\text{yij} = 1 \text{ for } j = 1,2,\ldots,N \\ \sum_{j}^{N} = 1\text{yij} = 1 \text{ for } i = 1,2,\ldots,N \\ \sum_{i}^{N} = 1 \sum_{j}^{N} = 1\text{yijc}(i,j) \le \text{CostMAX} \\ \sum_{i}^{N} = 1 \sum_{j}^{N} = 1\text{yijt}(i,j) \le \text{TimeMAX} \end{array}\right\} \quad (1)$$

where $y_{ij} = 1$ if the salesman travels from city-i to city-j, otherwise $y_{ij} = 0$ and; $Cost_{MAX}$ and $Time_{MAX}$ are the maximum cost limit and time limit of the tour, respectively. Let $(x_1, x_2, ..., x_N, x_1)$ be a complete tour of a salesman, where xi ∈ {1, 2, ..., N} for I = 1, 2 ,...,N and all x_i are distinct. Then the above problem reduces to:

$$\left.\begin{array}{c} \text{Determine a complete tour}(x1,x2,\ldots,xN,x1) \\ \text{to minimize } Z = \sum_{i=1}^{N-1} co\,(xi, xi+1) + co(xN, x1) \\ \text{subject to } \sum_{i=1}^{N-1} co\,(xi, xi+1) + co(xN, x1) \le \text{CostMAX} \\ \text{and } \sum_{i=1}^{N-1} tm\,(xi, xi+1) + tm(xN, x1) \le \text{TimeMAX} \end{array}\right\} \quad (2)$$

where co(i,j) and tm(i,j) are the travel cost and time, respectively, for traveling from the i-th city to the j-th city.

3.2 Multi-conveyance TSP with time limit constraint

In a multi-conveyance traveling salesman problem, a salesman must travel between N cities for the least amount of cost by taking any of the M available modes of transportation. In his/her trip, the salesman begins in one city, visits all of the cities exactly once using the best mode of transportation available in each city, and then returns to the starting city for the least amount of cost. Traveling from one city to another using various modes of transportation has varied costs and times. Let co(i,j,k) and tm(i, j, k) be the cost and time, respectively, it takes to travel from i-th city to j-th city using the k-th type of conveyance. The salesperson must then determine a complete tour $(x_1, x_2, ..., x_N, x_1)$ in which a specific or different combination of conveyance types (con_1, con_2, ... ,con_N) is to be employed for the tour, where $x_i \in (1, 2, ..., N\}$ for i= 1, 2, ..., N, $con_i \in (1, 2, ..., M\}$ and all x_i 's are distinct. The problem can then be expressed numerically as:

$$\left.\begin{array}{c} \text{Determine a complete tour } (x1, x2, \ldots, xN, x1) \\ \text{and corresponding conveyance types } (con1, con2, \ldots, conN) \\ \text{to minimize } Z = \sum_{i=1}^{N-1} cos\,(xi, xi+1, coni) + co(xN, x1, conN) \\ \text{subject to } \sum_{i=1}^{N-1} co\,(xi, xi+1, coni) + co(xN, x1, conN) \leq CostMAX \\ \text{and } \sum_{i=1}^{N-1} tm\,(xi, xi+1, coni) + tm(xN, x1, conN) \leq TimeMAX \end{array}\right\} \quad (3)$$

where $Cost_{MAX}$ and $Time_{MAX}$ are the maximum travel cost and time limit of a complete tour, respectively, that should be maintained by the salesman.

METHODOLOGY

Proposed ACO algorithm

In 1992, based on observations of ants' foraging behavior and their ability to locate the shortest path between their colony and food sources' created an algorithmic model to solve a combinatorial optimization issue. Since then, there has been a surge in interest in developing ant-based algorithms, resulting in a significant variety of algorithms and applications. A chemical trail called pheromone is left on the ground by ants throughout their travels. The original ACO method is somewhat modified as shown below.

Basic steps of the proposed ACO:
1. Start
2. Initialize parameters
3. Initialize pheromone
4. For i = 1 to MaxIt (Maximum number of iterations)
5. For j = 1 to NOANT
6. Construct a path for each ant
7. End for
8. Perform evaporation
9. Perform a pheromone update for each ant's path.
10. End for
11. Tuning solution
12. End

In this study, the author presents an ACO-based technique for solving a multi-conveyance TSP. This is a new form of TSP in which a different mode of transport is used to move from city to city, and the total travel cost and tour duration are fixed. This is a new type of TSP in which time and expense constraints are set with several transportation options. A unique ACO-based technique was employed to solve the TSP. A new procedure called "tuning solution" has been introduced by ACO to recover better neighbor solutions for each path. This is a unique aspect of the ACO and one of this study's contributions.

CONCLUSION

This review paper looks into a time-sharing program (TSP) where a salesperson has to meet budget and time constraints while traveling between cities. The cost and time required to travel between cities differ depending on the mode of transportation used. The salesman should make the most of both the overall trip duration and total travel expenses.

We have seen in this study how single-machine scheduling problems with sequence-dependent setup durations can be extended to integer programming formulations of the time-dependent traveling salesman problem. Additionally, these formulations can be strengthened by applying results from the vehicle routing problem (k-cycle elimination and Dantzig-Wolfe reformulation), the node packing problem (clique cuts), and the traveling salesman problem (subtour and 2-matching cuts). Ultimately, computational experiments confirm that using stronger lower bounds—especially those derived using the Dantzig-Wolfe decomposition principle—pays off. Stronger formulations were necessary in certain cases, and weaker formulations could not solve the problem.

ACO-based approaches to solving multi-conveyance TSPs—a unique kind of TSP in which the total travel cost and tour duration are fixed—have not been widely used in methodology. This is a new version of the TSP where different conveyance facilities are used along with time and cost constraints. A novel ACO-based method was used to solve the TSP. ACO has implemented a novel process known as a "tuning solution" to obtain superior neighbor solutions for every path. This is a distinctive feature of the ACO and a contribution to the research.

Conflict of Interest: The author has no conflict of interest to declare.

Future Research Scope: The implementation of a probabilistic and multi-objective TSP involves many more steps. Additionally, a bee colony optimization algorithm can solve a random fuzzy type-2 TSP. Additionally unknown are the outcomes of multi-depot multi-conveyance TSP using a bat algorithm. In a fuzzy random environment, precedence constraints and multiple-conveyance TSP can also be used to solve the TSP problems.

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