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Intuitionistic Fuzzy normal subgroups and its Properties

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Abstract- In this paper we have studied the Intuitionistic fuzzy sets as defined by K.T.Atanassov [10] and intuitionistic fuzzy normal subgroups as de- fined by Li Xiaoping. We established some properties of Intuitionistic fuzzy normal subgroups under homomorphism and prove a few independent equivalent propositions.

Keywords : Intuitionistic fuzzy set, Intuitionistic fuzzy group, Intuitionistic fuzzy normal subgroups.

1 INTRODUCTION

The concept of fuzzy set[4] was first introduced by L.A. Zadeh(1965) have laid the foundation of new branch of mathematics. In 1976 A. Rosenfeld [1] have introduced the concept of fuzzy subgroups. K.T. Atanassov[10] was introduced the concept of intuitionistic fuzzy set. R.Biswas [8] Li Xi- aoping and many more have introduced the concept of intuitionistic fuzzy subgroups in short $[IFS(G)]$ and intuitionistic fuzzy normal subgroups in short $[IFNS(G)]$. On the basis of intuitionistic fuzzy subgroups and intuitionistic fuzzy normal subgroups, we established some properties of intuitionistic fuzzy normal subgroup $[IFNS(G)]$ under homomorphism and prove a few independent propositions.

2 PRELIMINARIES

In this section, we recall the basic definitions of intuitionistic fuzzy set, intuitionistic fuzzy subgroups, intuitionistic fuzzy normal subgroups and other definition which play an important rule in proving some independent propositions.

2.1 Intuitionistic fuzzy set

Definition 2.1 *[10] Intuitionistic fuzzy set Let X be a non empty set.* A set $A = \{< x, \phi A(x), \psi A(x) >: x \in X\}$ *is called an intuitionistic fuzzy set on* X *, where* $\phi A : X \to [0, 1]$ *and* $\psi A : X \to [0, 1]$ *denote the degree of membership and degree of non-membership of each element* $x \in X$ *to the set* A *respectively. Also* $0 \le \phi A(x) + \psi A(x) \le 1$ *for each* $x \in X$.

In short intuitionistic fuzzy sets on X is written as $IFS(X)$.

Definition 2.2 *Let X be any non empty set, $A, B \in IFS(X)$, and* $A = \{< x, \phi A(x), \psi A(x) >: x \in X\}$ *and* $B = \{< x, \phi B(x), \psi B(x) >: x \in X\}$

then their operations are defined as follows.

$A \subseteq B \; iff \phi A(x) \le \phi B(x), \& \psi A(x) \ge \psi B(x), \forall x \in X$

$A = B \; iff \phi A(x) = \phi B(x), \& \psi A(x) = \psi B(x)$

$A \cap B = \{< x, min\{\phi A(x), \phi B(x)\}, max\{\psi A(x), \psi B(x)\} >: x \in X\}$

$A \cup B = \{< x, max\{\phi A(x), \phi B(x)\}, min\{\psi A(x), \psi B(x)\} >: x \in X\}$

$A = \{< x, \psi A(x), \overline{\phi} A(x) >: x \in X\}$

$\Box A = \{< x, \phi A(x), 1 - \phi A(x) >: x \in X\}$

$\diamond A = \{< x, 1 - \psi A(x), \psi A(x) >: x \in X\}$

Definition 2.3 *Let X be a non- empty crisp set and* $\{A_i : i \in I\} \subset$

$IFS(X)$. If $A_i = \{< x, \phi A_i(x), \psi A_i(x) >: x \in X\}$, we define.

$\cap_{i \in I} A_i = \{< x, inf(\phi A_i(x)), sup(\psi A_i(x)) >: x \in X\}$

Definition 2.4 *Let G be a group the intuitionistic fuzzy set*

$A = \{< x, \phi A(x), \psi A(x) >: x \in X\}$

is called an intuitionistic fuzzy subgroup $IFS(G)$ of a group G if the fol- lowing conditions are satisfied.

$\phi A(xy) \geq min\{\phi A(x), \phi A(y)\}$

$\phi A(x^{-1}) \geq \phi A(x)$

$\psi A(xy) \leq max\{\psi A(x), \psi A(y)\}$

$\psi A(x^{-1}) \leq \psi A(x)$

2.2 The Intuitionistic Fuzzy normal subgroups[7]

Definition 2.5 *Let G be a group, $A = \{< x, \phi A(x), \psi A(x) >: x \in X\}$ be an intuitionistic fuzzy set on G then A is called an intuitionistic fuzzy normal subgroup on G if*

$\phi A(xyx^{-1}) \geq \phi A(y)$, $\psi A(xyx^{-1}) \leq \psi A(y)$

All the intuitionistic fuzzy normal subgroups on G are denoted as $IFNS(G)$

Proposition 2.1*: Let G be a classical group. If $A, B \in IFNS(G)$ then show that $A \cap B \in IFNS(G)$.*

Proof: Let $A = \{< x, \phi A(x), \psi A(x) >: x \in X\}$ and $B = \{< x, \phi B(x), \psi B(x) >: x \in X\}$ be any two intuitionistic fuzzy normal subgroup of a group G,then

we have

$A \cap B = \{< x, min\{\phi A(x), \phi B(x)\}, max\{\psi A(x), \psi B(x)\} > x \in X\}$

let $\delta A{\cap}B(x) = min\{\phi A(x), \phi B(x)\}$

and $\gamma A{\cap}B(x) = max\{\psi A(x), \psi B(x)\}$ Therefore, we have

$A \cap B = \{< x, \delta A{\cap}B(x), \gamma A{\cap}B(x) >: x \in G\}$

Suppose that $x, y \in G$ and $A, B \in IFNS(G)$, then by the definition of intuitionistic fuzzy normal subgroup $IFNS(G)$ of a group G we have

$\delta A{\cap}B(xyx^{-1}) = min\{\delta A(xyx^{-1}), \delta B(xyx^{-1})\}$

$\geq min\{\delta A(y), \delta B(y)\}$

$= \delta A{\cap}B(y)$

On the other hand we have

$\gamma A{\cap}B(xyx^{-1}) = max\{\gamma A(xyx^{-1}), \gamma B(xyx^{-1})\}$

$\leq max\{\gamma A(y), \gamma B(y)\}$

$= \gamma A{\cap}B(y)$ therefore, $A \cap B \in IFNS(G)$

Proposition 2.2*: Let G be a classical group. Let $\{A_i : i \in I\} \subset IFNS(G)$. Then show that $\cap_i A_i \in IFNS(G)$*

Proof: Let $A_i = \{< x, \phi A_i(x), \psi A_i(x) >: x \in X\} \subset IFNS(G)$ of a group G then

Let and

$\cap_i A_i = \{< x, inf\ \phi A_i(x), sup\ \psi A_i(x), >: x \in X\}$ $\delta{\cap}A_i(x) = inf\{\phi A_i(x)\}$

$\gamma{\cap}A_i(x) = sup\{\psi A_i(x)\}$

putting these value in above equation we have

$\cap A_i(x) = \{< x, \delta{\cap}A_i(x), \gamma{\cap}A_i(x) > x \in X\}$

suppose that $x, y \in G$ and $A, B \in IFNS(G)$,then by the definition of intuitionistic fuzzy normal subgroup $IFNS(G)$ of a group G we have

$\delta\cap Ai\ (xyx^{-1}) = inf\{\phi Ai\ (xyx^{-1}) : i \in I\}$
$\geq inf\{\phi Ai\ (y) : i \in I\}$
$= \delta\cap Ai\ (y), \quad \forall y \in Y$
Similartly,
$\gamma\cap A\ (xyx^{-1}) = sup\{\psi A\ (xyx^{-1}) : i \in I\}$
$\leq sup\{\psi Ai\ (y) : i \in I\}$
$= \gamma\cap Ai\ (y), \ \forall y \in Y$
This implies that $IFNS(G) \in G$

Proposition 2.3: *Let f: $G1 \rightarrow G2$, be a homomorphism of a group $G1$ to group $G2$. Let $\square B \in IFNS(G)$ of a group $G2$, then show that $f^{-1}(\square B) \in IFNS(G)$ of a group $G1$.*
Proof: Let $\square B \in IFNS(G)$ of a group $G2$. Let $y1, y2 \in G2$, then there exists $x1, x2 \in G1$ such that $f(x1) = y1$ and $f(x2) = y2$, we have

$\square B = \{< y, \phi B\,(y), 1-\phi B\,(y) >: y \in G2\}$ .... (I)
Let $\delta B\,(y) = 1-\phi B\,(y) \qquad \forall y \in G2$
Since $\square B \in IFNS(G)$ of a group $G2$ implies that

1 $\phi B\,(y1.\ y2.y^{-1}) \geq \phi B\,(y2)$... (II)
Also
$\delta B\,(y1.y2.y^{-1}) \qquad = 1-\phi B\,(y1.y2.y^{-1})$
1 1
.... $\leq 1-\phi B\,(y2) \qquad (III)$
$= \delta B\,(y2)$
Now by extension principle we have

$$f^{-1}(\square B)(x_1.x_2.x^{-1}) = (\square B)\{f(x_1.x_2.x_1^{-1})\}$$
$$= (\square B)\{f(x_1).f(x_2).f(x_1^{-1})\} \quad \because f \text{ is homomorphism} \quad 1$$
$$= (\square B)(y_1.y_2.y_1^{-1}) \quad \because f(x_1^{-1}) = \{f(x_1)\}^{-1}$$
$$= \{\phi_B(y_1.y_2.y_1^{-1}), 1-\phi_B(y_1.y_2.y_1^{-1})\}$$
$$= \{\phi_B(y_2), \delta_B(y_2)\}$$

Now using equation (II) and equation (III), we have

$$f^{-1}(\square B)(x_1.x_2.x^{-1}) = \{\phi_B(f(x_2), \delta_B(f(x_2))\}$$
$$= \{\phi_{f^{-1}(B)}(x_2), \delta_{f^{-1}(B)}(x_2)$$
$$= f^{-1}(\square B)(x_2), \quad \forall x_1, x_2 \in G_1$$

Hence $f^{-1}(\square B) \in IFNS(G)$ of a group $G1$.

Proposition 2.4 *Let G be a classical group. If $A \in IFNS(G)$, then we have to show that $\diamond A \in IFNS(G)$.*
Proof: We have $A = \{< x, \phi A(x), \psi A(x) >: x \in G\}$, then by the definition we know that
$\diamond A = \{< x, 1-\psi A(x), \psi A(x) >: x \in G\}$
Let
$\beta A(x) = 1-\psi A(x), \qquad \forall x \in G$
For arbitrary $x, y \in G$, and $A \in IFNS(G)$ we have

$\psi A(x.y.x^{-1}) \leq \psi A(y)$

Thus i.e.

$\beta A(x.y.x^{-1}) = 1 - \psi A(x.y.x^{-1}) \geq 1 - \psi A(y) = \beta A(y)$ $\beta A(x.y.x^{-1}) \geq \beta A(y)$ $x, y \in G$

$\Rightarrow \diamond A \in IFNS(G)$

Proposition 2.5 *Let f: $G_1 \rightarrow G_2$ be a homomorphism of group G_1 to group G_2, and $\diamond B \in IFNS(G)$ of a group G_2. Then show that $f^{-1}(\diamond B) \in IFNS(G)$ of a group G_1.*

Proof : Let $\diamond B \in IFNS(G)$ of a group G_2, and $y_1, y_2 \in G_2$, then there exists $x_1, x_2 \in G_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$ We have

$\diamond B = \{< y, 1 - \psi B(y), \psi B(y) >: y \in G_2\}$

Let

$\delta B(y) = 1 - \psi B(y), \quad \forall y \in G_2$

Also $\diamond B \in IFNS(G)$ of a group G_2, implies that

$\psi B(y_1.y_2.y_1^{-1}) \leq \psi B(y_2), \quad \forall y_1, y_2 \in G_2$ $\delta B(y_1 y_2 y_1^{-1}) = 1 - \psi B(y_1 y_2 y_1^{-1})$

$\geq 1 - \psi B(y_2)$

$= \delta B(y_2)$

Now applying extension principle, we have

$f^{-1}(\diamond B)(x_1.x_2.x_1^{-1}) = (\diamond B)\{f(x_1.x_2.x_1^{-1})\}$

$= (\diamond B)\{f(x_1).f(x_2).f(x_1^{-1})\}$ $\quad \because f$ *is homomorphism*

$= (\diamond B)(y_1.y_2.y_1^{-1})$

$= \{1 - \psi B(y_1.y_2.y_1^{-1}), \psi B(y_1.y_2.y_1^{-1})\}$

where and

$= \{\delta B(y_2), \psi B(y_2)\}$.......................(1)

$\delta B(y_1.y_2.y_1^{-1}) \geq \delta B(y_2), \quad \forall y_1, y_2 \in G_2$ $\psi B(y_1.y_2.y_1^{-1}) \leq \psi B(y_2), \quad \forall y_1, y_2 \in G_2$

Now from equation (1) we have

$f^{-1}(\diamond B)(x_1.x_2.x_1^{-1}) = \{\delta B(f(x_2)), \psi B(f(x_2))$

$= \{\delta_{f^{-1}(B)}(x_2), \psi_{f^{-1}(B)}(x_2)\}$

$= f^{-1}(\diamond B)(x_2), \quad \forall x_2 \in G_2$

Hence $f^{-1}(\diamond B) \in IFNS(G)$ of a group G_2.

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