Properties Of Petersen Graph

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Abstract— In this paper I present the Petersen graph in a mathematical way some interesting properties of this graph have been presented in this paper. For example we seen it will be proved that this graph has exactly five maximum independent set and any two maximum independent sets intersect in a single vertex. Any vertex of this graph is contain in exactly two maximum independent sets. The independent edge numbers of this graph is exactly three and graph is vertex transitive graph.

Index Terms- Petersen graph, independent numbers, maximum independent set, independent edge set, edge independent number, vertex transitive graph.

I. INTRODUCTION

In graph theory there are graph properties which have investigated for different types of graph. Some of the properties which have been investigated generally are one independent number, two domination number, three independent domination number and others. In this paper I will present the Petersen graph in a mathematical way. In addition to above mention properties. I will also present some other interesting facts above the Petersen graph.

II. PRELIMINARIES AND RESULTS

Let G be a graph. Then V (G) denotes the vertex set of G and E (G) denotes the edge set of G. we will always consider only undirected finite simple graph.

A. Definition: (independent set) [2]

A subset S of V (G) is said to be an independent set if no two distinct vertices are adjacent in S.

B. Definition: (maximum independent set)[4]

An independent set have been maximum cardinality is called maximum independent set. It is also called βß$_0$ set.

C. Definition: (independence number) [4]

The cardinality of maximum independent set is called independence number of G. it is denoted βß$_0$ (G).

D. Definition: (dominating set, minimum dominatingset, domination number) [4]

Let G be a graph and S$\subset$V (G). Then S is said to be a dominating set if for every vertex v in V (G)-S. There is a vertex u in S such that u and v are adjacent.

A dominating set with minimum cardinality of a minimum dominating set. It is also called γ-set.
A cardinality of a minimum dominating set is called domination number of G and it is denoted γ (G).

E. Definition: (independent dominating set, minimum independent dominating set, independent domination number) [4]

Let G be a graph and S$\subset$V (G) if S is an any independent set as well as a dominating then it is called an independent dominating set.

An independent dominating set with minimum cardinality is called a minimum independent dominating set. It is also called i-set.

The cardinality of minimum dominating set is also called independent domination number and it is denoted i (G).

The cardinality of minimum dominating set is also called independent domination number and it is denoted i (G).

F. Definition: (edge independent set, maximum edge independent set, edge independence number) [4]

Let F be a subset of E (G) then F is said to be an edge independent set if no two edges in F are adjacent.

An edge independent set with maximum cardinality is called edge maximum independent set. It is also called $\beta\beta_1$ set.

The cardinality of maximum edge independent set is called the edge independent number of G. it is denoted by $\beta_1(G)$.

G. Definition:(vertex transitive graph) [4]

Let G be a graph then G is said to be a vertex transitive graph if for any two vertex u, v $\in \in V(G)$, there is an automorphism f from $G \to G$ such that $f(u)=v$.

Every vertex transitive graph is a regular graph.

III. PETERSEN GRAPH

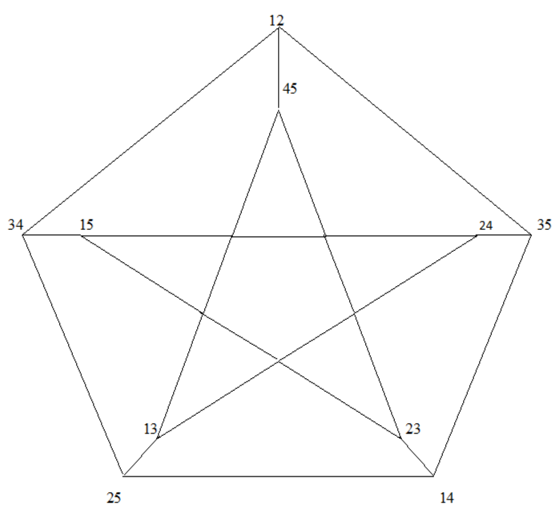
Now I present the Petersen graph somewhat different way. Instead of, drawing the graph directly. I will mention the verities of the graph & I will also prove criteria which will determine when two verities are adjacent in this graph.

Let X= {1, 2, 3, 4, 5} and

Let

V(G)={{1,2},{1,3},{1,4},{1,5},{2,3},{2,4},{2,5}, {3,4},{3,5},{4,5}}

Consider the graph G whose vertex set is V (G) & two vertices {i, j} and {a, b} are adjacent if and only if $\{i, j\} \cap \{a, b\} = \Phi\emptyset$.



This graph is in fact isomorphic to the Petersen graph and therefore it is called Petersen graph.

IV. PROPERTIES OF PETERSEN GRAPH

- Result 1: the independent number of Petersen graph is equal to four.

Proof: consider the set S={{1,2},{1,3},{1,4},{1,5}}this is a maximum independent set of G. thus $\beta_0(G)=4$.

- Remarks: there are other maximum independent set also. They are as follows:

S_1= {{2, 1}, {2, 3}, {2, 4}, {2, 5}}
S_2= {{3, 1}, {3, 2}, {3, 4}, {3, 5}}
S_3= {{4, 1}, {4, 2}, {4, 3}, {4, 5}}
S_4= {{5, 1}, {5, 2}, {5, 3}, {5, 4}}

Thus, there are exactly five maximum independent set. Intact, there are the only maximum independent set of G.

Also may be note that any two maximum independent sets intersect in a single vertex.

For e.g.: S_2& S_3 also it is true that every vertex of G lies in exactly two maximum independent set.

- Result 2: the independent domination number is equal to three.

Proof: consider the set S= {{1, 2}, {2, 3}, {1, 3}} then obviously S is an independent dominating set in G. since independent domination number of G is equal to three.

- Remarks: there are other independent dominating set also. They are as follows:

S_1= {{1, 3}, {3, 4}, {1, 4}}
S_2= {{1, 4}, {4, 5}, {1, 5}}
S_3= {{1, 2}, {2, 5}, {1, 5}}
S_4= {{2, 3}, {3, 4}, {2, 4}}
S_5= {{2, 5}, {2, 4}, {4, 5}}
S_6= {{1, 3}, {3, 5}, {1, 5}}
S_7= {{1, 2}, {2, 4}, {1, 4}}
S_8= {{2, 3}, {2, 5}, {3, 5}}
S_9= {{3, 5}, {3, 4}, {4, 5}}

Thus, there are exactly ten independent dominating set.

- Result 3: the edge independence number is equal to three.

Proof: consider the edges

e_1= {1, 2} {3, 4}
e_2= {1, 3} {2, 4}
e_3= {1, 4} {2, 3}

Let F= { e_1, e_2, e_3}

Then it can be proved that F is a maximum edge independent set of G and

Therefore, $\beta\beta_1(G) =3$.

V. PETERSEN GRAPH AN VERTEX TRANSITIVITY

Any one-one & onto function from f: X→X indecies a one-one & onto function from f*: V (G)→ V(G) as follows
f*({i, j})={f(i),f(j)}
Image of i, j under f*

In fact f* is an automorphism of the Petersen graph of G because if {i, j}, {a, b} are adjacent then {i, j} $\bigcap$ {a, b} =Φ.∅

Therefore, {f (i),f(j)} $\bigcap$ {f (a),f(b)} = Φ ∅and

Conversely, also

These means that {i, j} is adjacent to {a, b} if and only if f*({i , j}) is adjacent to f*({a, b}) this proves that f*is an automorphism of G.

To prove that G is a vertex transitive. Let {i, j}and{a, b} be two vertices of G. now there is a function from f:X→X such that f(i)=a & f(j)=b.

Now consider corresponding function f* define on V (G).

Then f*({i, j}) = {a, b}.

This proves that G is a vertex transitive.

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