Homotopy Perturbation Method for the Analysis of Radiative Fin with Temperature Dependent Thermal Properties

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Abstract- In this research works, Homotopy Perturbation Method (HPM) has been employed to analyze the conductive-radiative fin with temperature depended themo-physical parameter. The results obtained from HPM has been validated and compared with the other methods available in the literature. It has been found that the present methods perform well with the other methods available in the literature. The effects of various thermo-physical parameter are presented and discussed.

Nomenclature

N_r **Radiation-conduction parameter**
C **Constant which represents the temperature**
k **Temperature dependent thermal conductivity,** $W/(mK)$
k_a **Thermal conductivity corresponding to ambient condition,** $W/(mK)$
ε_s **The surface emissivity corresponding to radiation sinks temperature,** T_s
T **Temperature,** K
P **Fin perimeter,** m
T_b **Fin's base temperature,** K
T_a **Sink temperature corresponding to** k_a, K
T_s **Sink temperature for radiation,** K
b **Length of the fin,** m
x **Axial co-ordinate of the entire fin,** m
A_c **Cross-sectional area of the entire fin,** m^2
X **Dimensionless axial co-ordinate**
A **Thermal conductivity parameters**
B **The surface emissivity parameters**

Greek symbols

α **Slope of the thermal conductivity-temperature curve,** K^{-1}
β **Slope of the surface emissivity-temperature curve,** K^{-1}
θ **Dimensionless temperature of the fin,**
θ_a **Dimensionless sink temperature of the fin corresponding to** k_a**,**
θ_s **Dimensionless radiation sinks temperature,**
σ **Stefan-Boltzmann constant**
ε **Emissivity**

1. INTRODUCTION

The fins are the extended surface which is used in thermal engineering applications in which energy transfer takes place from hot surface to the atmosphere [1]. In some of the thermal engineering applications such as heat pipe, space radiator, where heat transfer takes place solely by radiation only. The governing equation in conduction radiation heat transfer problem can be expressed in ordinary differential equations with relevant boundary conditions. The many analysis of radiation-conduction problem can be made based constant values of thermophysical parameter for reducing the mathematical complexity [2]. But in actual situation thermophysical parameter of the heat conducting material may vary with temperature and the governing equation became highy non-linear. In solving the non-linear problem, many analytical methods developed such as Variation Iteration Method (VIM), Homotopy Analysis Method (HAM), Lest Square Method (LSM), Adomian Decomposition Method (ADM) etc. The mathematical modeling of circular convective-radiative porous extended surface with various geometries is analyzed using Lest Square Method (LSM) and fourth-order Runge-Kutta Method [3]. Similarly same methods are extended for longitudinal porous fin [4]. Kumar et al [5] investigated the annular fin with multiboiling heat transfer characteristics with two analytical methods. Internal heat generation of convective–conductive–radiative annular porous fin with variable thermal parameters is presented by Venkitesh and Mallick [6]. M.Hatami et.al [7] applied, three analytical methods such as Differential Transformation Method (DTM), Collocation Method (CM) and Least Square Method (LS) for the temperature distribution in porous fin materials such as Si_3N_4, Al with temperature dependent internal heat generation. Patel and Meher [8] investigated longitudinal porous fin using uniform magnetic field using ADM. Singla and Das [9] predicted the heat generation number and fin tip temperature using Adomian decomposition method and Genetic Algorithm. Roy et.al [10] presented the effect of sink temperature and internal heat generation number on the temperature distribution of a convective-radiative rectangular using decomposition method. Therefore the presents works aims at finding the effects of environmental temperature and surface emissivity parameters on the temperature distribution of a conductive radiative fin with variable thermal conductivity.

2. MATHEMATICAL FORMULATIONS

The construction of radiating space radiator is shown in Figure 1. The heat pipes are connected in series and for the analysis one heat transfer module is taken for the present analysis. The base temperature T_b, is constant and the radiation mixing between the heat pipe fin is neglected. The fin has length b, thickness w and the both surfaces of the fin can radiate heat to the surrounding temperature. The one dimensional energy equation is given by

$$2w\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-2\varepsilon(T)b\sigma\left(T^4-{T_s}^4\right)=0 \tag{1}$$

The thermal conductivity as well as the surface emissitivity of the fin material is linear function of temperature.

$$k(T)=k_a\left[1+\alpha(T-T_a)\right] \tag{2}$$

$$\varepsilon(T)=\varepsilon_s\left[1+\beta(T-T_s)\right] \tag{3}$$

Employing the following dimensionless parameter

$$\theta=\frac{T}{T_b}\ ,\ \theta_a=\frac{T_a}{T_b}\ ,\ \theta_s=\frac{T_s}{T_b}\ ,X=\frac{x}{b}\ ,\quad A=\alpha T_b\ ,\quad B=\beta T_b\ ,\quad N_r=\frac{\varepsilon_s\sigma {T_b}^3 b^2}{k_a w} \tag{4}$$

The formulation of the fin problem reduces to the following equations:

$$\frac{d^2\theta}{dX^2}+A\theta\frac{d^2\theta}{dX^2}+A\left(\frac{d\theta}{dX}\right)^2-A\theta_a\frac{d^2\theta}{dX^2}-N_r\left[1+B(\theta-\theta_s)\right]\left[\theta^4-{\theta_s}^4\right]=0 \tag{5}$$

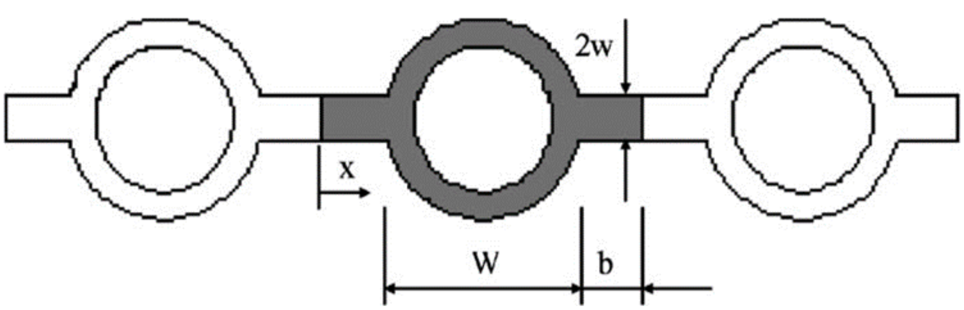

Figure 1. The geometry of heat pipe/fin space radiator.

With the following boundary conditions:

$$\frac{d\theta}{dX}=0 \ \ at \ \ X=0 \tag{6}$$

$$\theta=1 \ \ at \ \ X=1$$

3. HOMOTOPY PERTURBATION METHOD (HPM)

Applying the homotopy perturbation method [21-25] to the fin equation (5) can be express as

$$H(\theta,p)=(1-p)L(\theta-\theta_0)+p\begin{bmatrix} A\theta\dfrac{d^2\theta}{dX^2}+A\left(\dfrac{d\theta}{dX}\right)^2-A\theta_a\dfrac{d^2\theta}{dX^2} \\ -N_r\left(\theta^4-\theta_s{}^4+B\theta^5-B\theta_s{}^4\theta-B\theta_s\theta^4+B\theta_s{}^4\right)\end{bmatrix}=0 \tag{7}$$

Where $L=\dfrac{d^2}{dX^2}$ is called the linear operator and embedding parameter, $p\in[0,1]$ is known as homotopy parameter. The θ_0 is initial approximation that satisfies the boundary condition (6). When the value of p changes from 0 to 1 the homotopy equation (7) also changes, this is called the deformation. The perturbation solution for θ in the form of power series in p as under:

$$\theta(X)=\theta_0(X)+p\theta_1(X)+p^2\theta_2(X)+\cdots\cdots=\sum_{i=0}^{\alpha}p^i\theta_i(X) \tag{8}$$

Substituting equation (8) into equation (7) and rearranging based on the power of p

p^0 :

$$\frac{d^2\theta_0}{dX^2}=0 \tag{9a}$$

$$\frac{d\theta}{dX}=0 \ \ at \ \ X=0, \quad \theta_0=C \ \ at \ \ X=0 \tag{9b}$$

and

p^1 :

$$\frac{d^2\theta_1}{dX^2}+\begin{bmatrix} A\theta_0\dfrac{d^2\theta_0}{dX^2}+A\left(\dfrac{d\theta_0}{dX}\right)^2-A\theta_a\dfrac{d^2\theta_0}{dX^2} \\ -N_r\left(\theta_0{}^4-\theta_s{}^4+B\theta_0{}^5-B\theta_s{}^4\theta_0-B\theta_s\theta^4+B\theta_s{}^5\right)\end{bmatrix}=0 \tag{10a}$$

$$\frac{d\theta_1}{dX}=0 \ \ at \ \ X=0, \quad \theta_1=0 \ \ at \ \ X=0 \tag{10b}$$

and

p^2 :

$$\frac{d^2\theta_2}{dX^2}+\begin{bmatrix} A\left(\theta_0\dfrac{d^2\theta_1}{dX^2}+\theta_1\dfrac{d^2\theta_0}{dX^2}\right)+A\left(2\dfrac{d\theta_0}{dX}\dfrac{d\theta_1}{dX}\right)-A\theta_a\dfrac{d^2\theta_1}{dX^2} \\ -N_r\left(4\theta_0{}^3\theta_1+5B\theta_0{}^4\theta_1-B\theta_s{}^4\theta_1-B\theta_s4\theta_0{}^3\theta_1\right)\end{bmatrix}=0 \tag{11a}$$

$$\frac{d\theta_2}{dX}=0 \ \ at \ \ X=0, \quad \theta_2=0 \ \ at \ \ X=0 \tag{11b}$$

$p^3:$

$$\frac{d^2\theta_3}{dX^2}+\left[\begin{array}{l} A\left(\theta_0\frac{d^2\theta_2}{dX^2}+\theta_1\frac{d^2\theta_1}{dX^2}+\theta_2\frac{d^2\theta_0}{dX^2}\right)+A\left(2\frac{d\theta_0}{dX}\frac{d\theta_2}{dX}+\left(\frac{d\theta_1}{dX}\right)^2\right) \\ -A\theta_a\frac{d^2\theta_2}{dX^2}-N_r\left(\begin{array}{l}\left(4\theta_0^{\,3}\theta_2+6\theta_0^{\,2}\theta_1^{\,2}\right)+B\left(5\theta_0^{\,4}\theta_2+10\theta_0^{\,3}\theta_1^{\,2}\right) \\ -B\theta_s^{\,4}\theta_2-B\theta_s\left(4\theta_0^{\,3}\theta_1+6\theta_0^{\,2}\theta_1^{\,2}\right)\end{array}\right)\end{array}\right]=0 \qquad (12a)$$

$$\frac{d\theta_3}{dX}=0 \;\; at \;\; X=0, \quad \theta_3=0 \;\; at \;\; X=0 \qquad (12b)$$

$p^4:$

$$\frac{d^2\theta_4}{dX^2}+\left[\begin{array}{l} A\left(\theta_0\frac{d^2\theta_3}{dX^2}+\theta_1\frac{d^2\theta_2}{dX^2}+\theta_2\frac{d^2\theta_1}{dX^2}+\theta_3\frac{d^2\theta_0}{dX^2}\right) \\ +A\left(2\frac{d\theta_0}{dX}\frac{d\theta_3}{dX}+2\left(\frac{d\theta_1}{dX}\right)\left(\frac{d\theta_2}{dX}\right)\right)-A\theta_a\frac{d^2\theta_3}{dX^2} \\ -N_r\left(\begin{array}{l}\left(4\theta_0^{\,3}\theta_3+12\theta_0^{\,2}\theta_1\theta_2+4\theta_0\theta_1^{\,3}\right)+B\left(5\theta_0^{\,4}\theta_3+20\theta_0^{\,3}\theta_1\theta_2+10\theta_1^{\,3}\theta_0^{\,2}\right) \\ -B\theta_s^{\,4}\theta_3-B\theta_s\left(4\theta_0^{\,3}\theta_3+12\theta_0^{\,2}\theta_1\theta_2+4\theta_0\theta_1^{\,3}\right)\end{array}\right)\end{array}\right]=0 \qquad (13a)$$

$$\frac{d\theta_4}{dX}=0 \;\; at \;\; X=0, \quad \theta_4=0 \;\; at \;\; X=0 \qquad (13b)$$

By increasing number of terms in the solution higher accuracy will be obtained. Solving (9a), (10a), (11a), (12a) and (13a) results $\theta(X)$. When $p \rightarrow 0$,we have the solution for the first five in the series as follows.

Summing these terms, the final temperature filed θ , is calculated as follows

$$\theta(X)=\sum_{0}^{\alpha}\theta_m=\theta_0(X)+\theta_1(X)+\theta_2(X)+\theta_3(X)+\theta_4(X)+\cdots\cdots \qquad (14)$$

Now the temperature field, θ can be evaluated if the fin tip temperature C is known whose value lies in the interval (0, 1). Using an arbitrary initial guess value for C, for the temperature field θ computed from the above equation (16) and applying Newton-Rapson method satisfying the boundary conditions (6) the actual temperature field can be obtained.

4. RESULTS AND DISCUSSION

The governing equation (5) is validated in the limiting conditions and results are compared with the previous results available in literature Table 1. The present works consider five terms in the solutions and the presents results are compared with the previous work available in literature.

Table 1 Comparison of present results with the results available in literature

$N_r=1,\theta_s=0,\theta_a=0,A=0.4,B=0$				
Present				NM
0.8129	0.8122	0.8133	0.8145	0.8133

The variation of temperature, θ of the fin material is affected by environmental temperature, θ_s with different values of surface emissivity parameters B. For the constant emissivity B=0, the temperature is independent of radiation sink temperature up to $\theta_s = 0.3$ and then increases. Figure 2 shows variation of temperature along the length of the fin with two values of conduction radiation parameter Nr=0.75 & 0.25, while surface emissivity parameter B maintaining at three values at -0.2, 0 and +0.2 respectively while thermal conductivity parameter A and sink temperature kept at constant values. It has been observed that with the lower value of N_r, the tip temperature is higher and therefore rate of heat conduction will be higher.

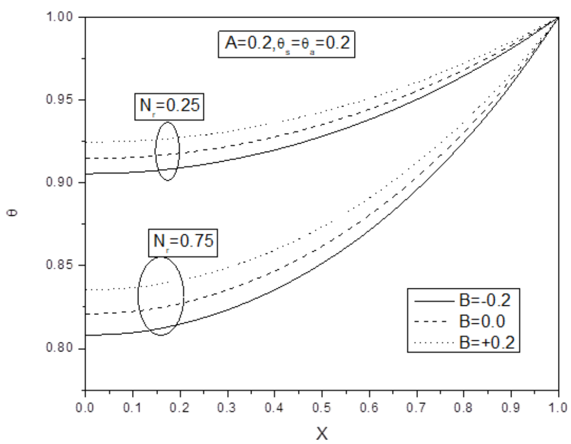

Figure 2. The temperature distribution obtained by HPM with respect to the radiation sink temperature and for various values of surface emissivity parameter.

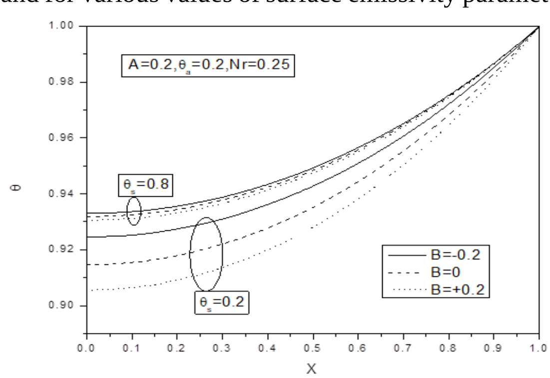

Figure 3. The temperature distribution obtained by HPM with respect to the radiation sink temperature and for various values of surface emissivity parameter.

Figure 3 shows variation of tip temperature for two values of sink temperature while surface emissivity parameter B maintaining at three values at -0.2, 0 and +0.2 respectively for thermal conductivity, radiation conduction parameter and convection sink temperature kept at constant values. The higher the sink temperature, high is the tip temperature and therefore higher is the heat transfer rate of radiative fin.

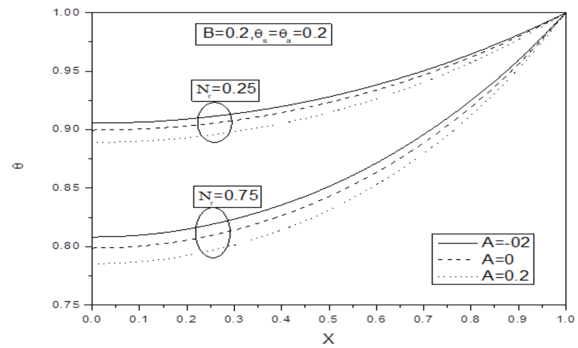

Figure 4. The temperature distribution obtained by HPM for various values conduction radiation parameter and thermal conductivity parameter.

Figure 4 shows variation of conductive-radiative parameter for three values of thermal conductivity parameter while the surface emissivity parameter and sink temperature kept at constant values. The lower the radiation-conduction parameter, higher is the tip temperature and therefore better is the rate of heat transfer. Again the negative values of thermal conductivity parameter provides the higher heat transfer rates.

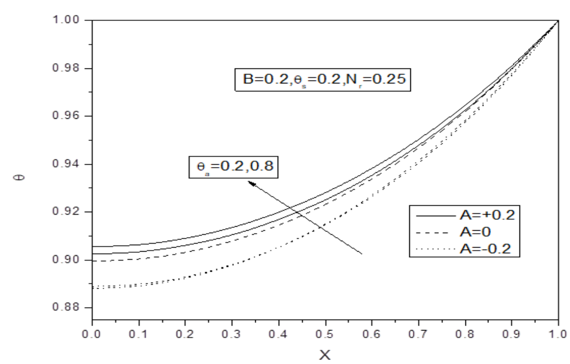

Figure 5. The temperature distribution obtained by HPM for various values convection sink temperature and for thermal conductivity parameter.

Figure 4 demonstrate the variation of convection sink temperature for three different values of thermal conductivity parameter while radiation sink temperature, surface emissivity parameter and radiation conduction parameter remains at constant values. The higher the convective sink temperature, higher is the tip temperature and rate of heat transfer is higher.

CONCLUSION

In this work, the HPM has employed the closed form solution of conductive-radiative fin with temperature dependent thermal conductivity and surface emissivity. The results of HPM is validated in the limiting condition with the results other methods validated in the literature such as ADM, DTM, VIM and NM. The effects of various thermo-physical parameters are discussed and presented.

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