Micromechanics of Tensile Modulus of Random Fiber Composite: A review

A. K. Issa[1], J. I. Unwaha[2], M. I. Tikau[3]

[1,2, 3]Department of Mechanical Engineering Technology, Federal Polytechnic Mubi, Adamawa State, Nigeria

***Abstract*- Randomly fiber reinforced composites (RFRC) are becoming warming area of research because of its usefulness in structural applications. However, selection of micromechanics models that can predict well the mechanical properties of these materials become problematic due to randomly orientation nature of the fiber. This discourages many researchers from its utilization. This paper reviewed few available literatures of micromechanics models Viz: Series and Parallel, Modified rule of mixtures (ROM)models, Hirsh, Bowyer and Bader, Halpin-Tsai and Tsai Pagano, Lewis Nielson, Manera, Cox-krenchel, Pan 2D and 3D and Christensen and wall, of elastic modulus as employed by different researchers for these materials. It was discovered that Halpin, Neilson, Christensen-waal and manera are good for prediction of elastic modulus at low fiber volume fraction. While, Halpin micromechanics model is for high fiber volume fraction of random fiber composite. Generally, Hirsh micromechanics model emerged as overall best predictors of elastic modulus of random fiber composite as it appeared to fit many random fiber composites reported in this work.**

***Index Terms*- Model, Modulus, Random, Reinforced, Volume fraction.**

I. INTRODUCTION

Micromechanics is the study of composite materials properties wherein the interaction of the constituent of materials is examined in detail as part of the definition of the behavior of the heterogeneous composite material to enable the determination of effect of properties of each constituent on the properties of the resultant composite materials. The constituent material properties needed in micromechanics are not certainly determined at the microscopic level but are frequently obtained by testing the fibers, matrix materials, and other constituents in the same manner the resultant composite material properties are tested.

Prediction models are used to lessen the cost, energy and timeframe involved in developing and producing composite materials [1]. Among the prediction models is analytical method, this has been used by researchers to predict the mechanical properties of composite materials before and after development of composite [2]. The most two important mechanical properties of composite materials are elastic modulus and tensile strength. The Elastic modulus is a measure of the stiffness of the materials. It is numerical equal to the slope of the stress-strain curve in the elastic range (linear), as represented by Hooke's Law while tensile strength of a material is the ability to withstand a load without undue deformation, or rupture or failure. Present studies for predicting the elastic modulus values of random fiber composites are erratic and the selection of the preferred models for random fiber reinforced composites are difficult relatively to oriented fiber composite. To know which model that can carefully forecast the elastic modulus at different levels of volume or mass fraction of the fiber are not easy because the well-known method called the rule of mixtures for predicting the elastic moduli of composite materials could not deliver suitable predictions in that regard. This is associated to lack of fiber orientation and geometric parameters in the so-called rule of mixture. The most generally used parameters to theoretically calculate elastic modulus of composites are fiber and matrix materials properties, fiber volume ratio. But for random fiber composite, Fiber length distribution and orientation must be included for appropriate prediction. Therefore, it is necessary to investigate which micromechanics is more suitable to predict the mechanical properties at different mass/volume fraction of the composite. Therefore, this work compiles the comparison of the ten predicted theoretical model in different composites materials and volume fractions.

II. MICROMECHANICS MODELLING THEORY AND EQUATION

A. *Series and Parallel Models*

Series model is rule of mixture which based on the Voigt theory (iso-strain model) and parallel is inverse rule of mixture which based on Reuss theory (iso-stress model). The tensile strength models of both are given in equation 1 and 2.

$$E_c = E_f V_f + E_m V_m \qquad (1)$$

$$E_c = \frac{E_m E_f}{E_m V_f + E_f V_m} \qquad (2)$$

B. *Modified Rule of Mixtures (ROM)*

This also modifies rules of mixture by including additional parameter that account for the weakening of the composite due to fiber orientation and is less than1 as shown in equation 3

$$E_c = \eta E_f V_f + E_m(1 - V_f) \qquad (3)$$

C. *Hirsch Model*

This is the combination of series and parallel models with introduction of x. Parameter (x) determines the stress transfer between fiber and matrix. The tensile strength is shown in Equation 4

$$E_c = x\left[E_m V_m + E_f V_f\right] + (1 - x)\,\frac{E_m E_f}{E_m V_f + E_f V_m} \qquad (4)$$

D. *Bowyer and Bader's Model*

This modifies series model by introducing fiber orientation factor (K_1) and fiber length factor (K_2). The tensile strength model is given as in equation 5

$$E_c = K_1 K_2 E_f V_f + E_m V_m \qquad (5)$$

E. *Halpin-Tsai and Tsai Pagano Model*

This is simple semi-empirical equations because it involves parameters from the curve fitting that are based on elasticity. These parameters are aspect ratio fiber geometry (ξ) and Reinforcing efficiency (η_1). Equation 6, 7 and 8 are for Halpin-Tsai and Tsai Pagano

$$E_{3c} = \left(\frac{3}{8}E_1 + \frac{5}{8}E_2\right) \qquad (6)$$

$$E_1 = E_m\left(\frac{1+\xi_{\eta_1} V_f}{1-\eta_1 V_f}\right) \qquad (7)$$

$$E_2 = E_m\left(\frac{1+\xi_{\eta_2} V_f}{1-\eta_2 V_f}\right) \qquad (8)$$

F. *Lewis Nielson Model*

This modifies Halpin–Tsai model by introducing particle packing factor'. The model used here for prediction of tensile strength and modulus is as given in 9.

$$E_c = E_m\left(\frac{1+\xi_\eta V_f}{1-\eta\psi V_f}\right) \qquad (9)$$

G. *Manera Model*

This assumes that the mechanical properties of randomly oriented fiber composite are the same in all direction and include a high fiber orientation ratio, two-dimensional random-fiber range, as shown in equation (10)

$$E_c = V_f\left[\frac{16}{45}E_f + 2E_m\right] + \frac{8}{9}E_m \qquad (10)$$

H. *Cox-krenchel Model*

This model modifies rule of mixture and includes fiber length distribution η_l and orientation factors $\eta_0$respectively. In this model, stiffness or elastic modulus can be written as shown in equation and the fibre orientation factor is expressed as in equations 11, 12, 13 and 14.

$$E_c = \eta_l \eta_0 E_f V_f + E_m V_m \qquad (11)$$

$$\eta_0 = \sum_n a_n cos^4 \phi_n \qquad (12)$$

$$\eta_l = 1 - \frac{tanh\left({}^{\beta l}/_{2}\right)}{\left({}^{\beta l}/_{2}\right)} \qquad (13)$$

$$\beta = {}^{2}/_{d}\left(\sqrt{\frac{E_m}{E_f(1-p_m)\ln\frac{\pi}{x_i V_f}}}\right) \qquad (14)$$

I. *Pan 2D and 3D Model*

This uses rule of mixtures for the parts where the fibers were not unidirectional; and relationship between the fiber volume fraction and the fiber-area ratio. The elastic modulus/stiffness of 2- and 3-dimensional case can be obtained in equation 15 and 16 respectively.

$${E_c}^{2D} = E_f\frac{V_f}{\pi} + E_m\left(1 - \frac{V_f}{\pi}\right) \qquad (15)$$

$${E_c}^{3D} = E_f\frac{V_f}{2\pi} + E_m\left(1 - \frac{V_f}{2\pi}\right) \qquad (16)$$

J. *Christensen and Wall Model*

This recognize both fiber/matrix interaction and the fiber orientation and the fiber orientation is in three dimensional directions. The elastic modulus equation for this model is shown below

$$E_{c^{3D}} = \frac{V_f}{6}E_f + \left[1 + (1 + V_m)V_f\right]E_m \qquad (17)$$

III. FINDINGS

Table 1. Shows the list of different composite materials developed by different researchers together with micromechanics models employed in the prediction of tensile modulus, their fiber volume fraction and length.

Table1: Name of Composite Materials, Micromechanics and Fiber Length

S/N	Composite Material	Micromechanical Model	Fiber Volume Fraction	Fiber Length (mm)	Reference
1	Cotton/Low density Polyethylene	MRoM Bowyer–Bader’s model Tsai–Pagano, Cox–Krenchel’s, Pan’s 3-D, model Christensen waal,model Cox model, Manera model, Shear lag model	6.5, 13, 19.6	1.18-3.67	[3]
2	chicken-feather fiber /PLA	Pan’s 2-D, Pan’s 3-D, inverse rule of mixtures, Halpin-Tsai, Nielsen-Chen, Manera, Christensen-Waal	0, 5, 10, 15	20-30	[4]
3	Kenaf/epoxy	Rule of mixture, Haplin-Tsai, Nielson Chen and Manera models	25, 30, 35, 40	5-50	[5]
4	Zalacca fibre/low-density polyethylene	Tsai–Pagano, Manera and Cox–Krenchel’s model.	0.1, 0.2, 0.3, 0.4, 0.5, 0.6	40	[6]
5	sansevieria cylindrica/biochar tailored vinyl ester	Series, Hirsch and Halpin Tsai	0, 2, 4, 6, 8, 10	Particulate	[7]
6	Coir/propylene	Parallel, Series Halpin-Tsai, Nielson, Modified Bowyer and Bader’s	0, 1, 2, 3, 4	110-120/ 20%	[8]
7	zalacca fibre/high-density polyethylene	Christensen, Tsai-Pagano, and Cox-Krechel	0, 0.1, 0.2, 0.3 and 0.4	1, 3, 6 and 9	[9]

IV. DISCUSSION

Yashwant and Ravindra [3] produced randomly oriented coir reinforced propylene (coir/PP) composite, the developed composites were tested to obtain the values for some properties of the composite. Consequently, experimental results of elastic modulus were compared with predicted results from Parallel, Series, Halpin-Tsai, Nielson, and Modified Bowyer and Bader’s micromechanics model. It was observed in Fig. I that, at low volume fraction of fiber, elastic modulus properties showed reasonably good concord with all models. While Hirsch model concord very well with experimental data than the others.

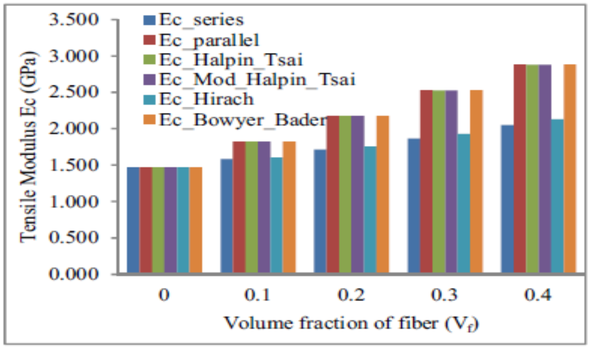

Fig. I: Comparison of Experimental Values of Coir/PP Composite with Prediction Models [3]

Ozmen and Baba [4] carried out the prediction of the elastic modulus of randomly chicken-feather-reinforced Pla (CF/PLA) composite using six different micromechanical models namely: Pan’s 2-D, Pan’s 3-D, inverse rule of mixtures, Halpin-Tsai, Nielsen-Chen, Manera, and Christensen-Waal. The elastic moduli of the CFF/PLA composites with different samples were obtained and compared with the experimental results. The comparison results indicated that, Pan’s 2-D, IROM (the inverse rule of mixtures), Nielsen-Chen and Halpin-Tsai models gave more converging results for the prediction of the elastic moduli of the CFF/PLA composites than the other. At low volume fraction of fiber (2%) the Christensen-Waals model gave close result to the experimental and diverged very well with respect to increment in volume of fiber. IROM and Halpin diverged at low volume fraction of fiber of 2% then converge very well for the remaining volume fraction of fiber including high volume. Pan’ s 2-D model gave close results to the experimental but it did not fit very well. Pan’ s 3-D model is higher than experimental at all volume fraction of the fiber. Manera’s model did not fit all with the experimental results. Nielsen-Chen model was the best model that predicted well the elastic behavior of this composite as illustrated in fig. II below.

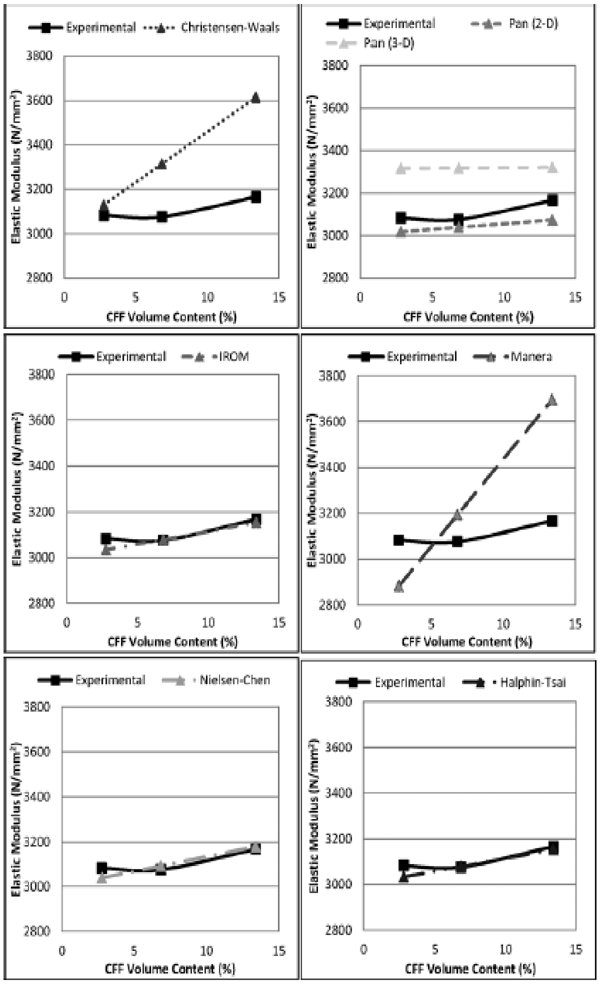

Fig. II: Comparison of experimental values of CF/PLA composite with prediction models (Ozmen & Baba, 2016)

Kumar, Hashmi, Nimanpure and Naik [5] fabricated randomly distributed kenaf fiber reinforced epoxy resin (KF/ERC). The experimental results of elastic modulus were compared with some important micromechanics models such as rule of mixture, Haplin-Tsai, Nielson Chen and Manera models. Manera model was found most suitable with the experimental data, followed by Nielson, then Halpin and maximum deviation was recorded in rule of mixture model. It was also noted from the comparison graph that at low fiber volume fraction, Manera was the closest at low and medium fiber fraction while Halpin became the closest at high fiber volume fraction as illustrated in fig.III. It was suggested that micromechanics model can be successfully used to forecast properties of natural fiber composites to lessen the cost and time consumed in experiments.

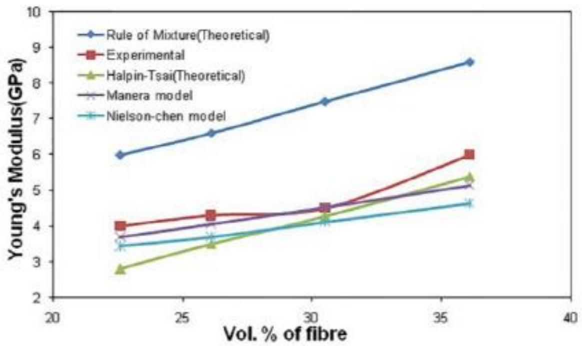

Figure III: Comparison of experimental values of (KF/ERC) with prediction models [4].

Raharjo et al [6] developed randomly oriented zalacca fiber/low-density polyethylene composites (ZF/LDPE). Tsai–Pagano, Manera and Cox–Krenchel's model were used to predict the elastic modulus of the composites. The experimental results of elastic modulus of the composites was closer to the Tsai–Pagano model result than Manera and Cox–Krenchel's model results. The inability of manera model to perform better was associated to lack of fiber length and orientation distribution in the model as shown in fig. IV. While that of cox-Krenchel considers the fiber length and fiber orientation factors but its failure was attributed to non-recognition of the present of void in the composite and the limitation in the fiber orientation angle.

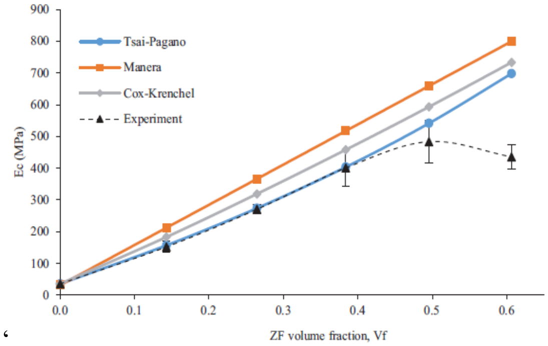

Figure IV: Comparison of experimental values of ZF/LDPE composite with prediction models [6]

Deepak, Arumugaprabu, and Tae [8] developed randomly Sansevieria cylindrica reinforced biochar vinyl ester (SC/BVS) composite. Theoretical models namely Series model, Hirsch model and Halpin Tsai model were employed to predict the elastic modulus property of the fabricated material. Empirical elastic modulus properties were compared with the obtained micromechanics models' elastic modulus properties to estimate the conformity between them. It can be also observed from the comparison graph that the results of elastic modulus for all the models follow similar trend with the empirical. Furthermore, at low and high-volume fraction of fiber, Halpin-Tsai closest to the empirical. But generally, the elastic modulus predicted from the Hirsh Model showed very good prediction with empirical values compared to the other followed by series model then Halpin -Tsai model as shown in fig.V below.

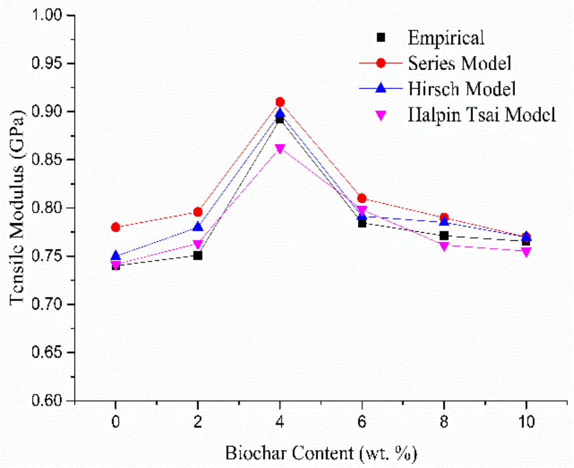

Fig.V: Comparison of experimental values of SC/BVS with prediction models [8]

Bodur and Bakkal [7] randomly oriented cotton fiber reinforced low density polyethylene composites (CF/LDPE) then, they compared the experimental results with micromechanics model namely MRoM Bowyer–Bader’s model, Tsai–Pagano, Cox–Krenchel’s, Pan’s 3-D model, Christensen waal model, Cox model, Manera model, Shear lag model. The comparison results showed that there are variances in the models that best fit the experiment at different volume fraction of fiber. At low volume fraction Halpin-Tsai, Nielson, Hirsch and Shear Lag models offered the closest fitness to experimental. At the 65% volume fraction of the fiber, MROM, Halpin-Tsai, Nielson and Hirsch models became the closest, but at all volume fraction of fiber are MRoM and Hirsch models give the closest results to experimental data. It was also observed that as the volume fraction of fiber increases, the difference between experimental data and analytical models increases. This was due to increment in the void size. Cox, Halpin Tsai, Hirsch and Nielson models showed higher value than the experimental value because they disregard the random distribution of the fiber while those that take it into consideration showed lower value than experimental as shown in fig.VI.

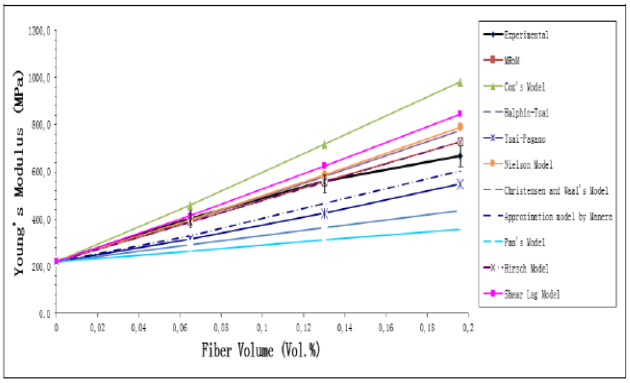

Fig.VI: Comparison of experimental values of CF/LDPE composite with prediction models [7]

Ariawan et al [9] utilized zalacca random fiber to reinforced high-density polyethylene for the production of the composite. Different volume fractions such as 0, 0.1, 0.2, 0.3 and 0.4 were employed and compression method was used. The tensile modulus of the composites were predicted using Christensen, Tsai-Pagano, and Cox-Krechel micromechanics models. Thereafter tensile modulus tests were carried out on the composites to obtain the experimental results. This was later compared with the predicted results of different models with respect to different volume fractions as shown in the fig.VII in their prediction of composites elastic modulus. The results showed that all models over predicted the tensile modulus of the composite, although Tsai-Pagano, and Cox-Krechel results were better than Christensen throughout the prediction especially at lower fiber volume fraction. The prediction of Cox-Krenchel's model was the best compared to the other two models. This was associated to inclusion of fiber length and fiber orientation factors in the equation of the model.

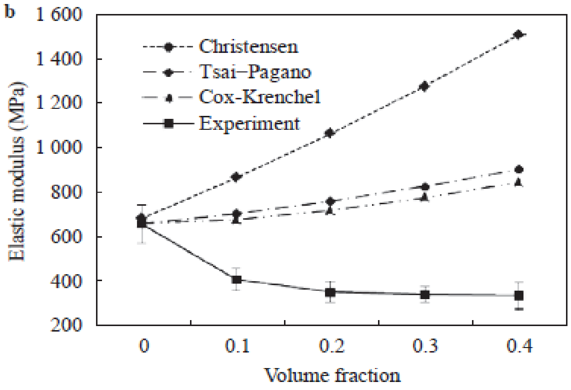

Fig.VII. Comparison of experimental tensile modulus values of zalacca fiber/high-density polyethylene with prediction models [9].

V. RECOMMENDATION

Halpin, Neilson, Christensen-waal and manera are recommended to predict the composite elastic property at low fiber volume fraction. While, Halpin micromechanics model is for high fiber volume fraction of random fiber composite.

REFERENCE

[1] M. R. Mansori, S. M. Sapuan, E. S. Zainudin, A. A. Nuraini and A. Hambali. Stiffness Prediction of Hybrid Kenaf/Glass Fiber Reinforced Polypropylene Composites using Rule of Mixtures (ROM) and Rule of Hybrid Mixtures (RoHM). *Journal of Polymer Materials*. Vol. **30**, no3, pp. 321-334, 2013.

[2] N. Bouhfid, M. Raji, R. Boujmal, H. Essabir, M. O. Bensalah, R. Bouhfid and A. K. Qaiss, Numerical Modeling of Hybrid Composite Material. In *Modelling of Damage Processes in Biocomposites, Fibre-Reinforced Composites.* Elsevier Ltd. doi:doi.org/10.1016/B978-0-08-102289-4.00005-9. 2019.

[3] S. M. Yashwant, and B. I. Ravindra. Theoretical Modeling and Experimental Verification of Mechanical Properties of Natural Fiber Reinforced Thermoplastic. *8th International Conference of Interdisciplinarity in Engineering. 19*, pp. 320-326. 2015.India: Elsevier Ltd. doi:10.1016/ j.protcy. 2015.02.046

[4] U. Özmen, and O. B. Baba, Prediction of the Elastic Moduli of Chicken-Feather-Reinforced PLA and a Comparison with Experimental Results. *Materials and technology*, vol.50, no.1, pp. 141–146, 2016.

[5] S. A. R. Kumar, S. A. R. Hashmi, S. Nimanpure and A. Naik, Enhanced Dynamic Mechanical Properties of Kenaf Epoxy Composites. *Advanced Materials Proceedings*, pp.749-757, 2017.

[6] P. W. Raharjo, R. Soenoko, A. Purnowidodo and A. M. Choiron, Experimental and micromechanical modelling of randomly oriented zalacca fibre/ low-density polyethylene composites fabricated by hot-pressing method. *Cogent Engineering,* vol.5, no.1, pp. 1-14. 2018.

[7] M. S. Bodur, and M. Bakkal, Young's Modulus Estimation Techniques for the Randomly Oriented Natural Ffiber Reinforced Composites: Finite element analysis and analytical models. *Pamukkale University Journal of Engineering Sciences*, vol.25, no.4, pp.454-461. doi:10.5505/pajes.2019.88864, 2019.

[8] R. D. J.J. Johnson, V. Arumugaprabu and T. J. Ko,. Theoretical and Experimental Evaluation of Tensile Property of Sansevieria Cylindrica Reinforced Wood Waste Biochar Tailored Vinyl Ester Composite. *International Journal of Engineering and Advanced Technology (IJEAT),* vol 9, no.14. doi:10.35940/ijeat.A1082.1291S419. 2019.

[9] D. Ariawan, E. Surojo, J. Triyono, I. F. Purbayanto, A. F. Pamungkas, Micromechanical analysis on tensile properties prediction of discontinuous randomized zalacca fibre/high-density polyethylene composites under critical fibre length. *Theoretical & Applied Mechanics Letters* 10, pp. 57-65, 2020.