Exploring the Applicability of Power Law in Management Theories and Practices

Kunjal Mehta
Research Scholar, Osmania University, Hyderabad

Abstract- Power law or Scale-free distributions are widespread distributions. Power laws have two features that make them strikingly different from normal-type distributions. First unlike a normal distribution power law does not have to peak at its average value. Second, the rate at which power law decays is much slower than the decay rate of normal distribution. The key characteristic of a power law distribution is a quantity called exponent, which in essence describes how the distribution changes as a function of the underlying distribution. Many Real-life functions follow this distribution. Particular it is evident in wealth and success, the rich always seem to get richer. Military an organization that reflects the power of a nation seems to follow this distribution in multiple facets. Several interesting power law graphs concerning the world wide weapon industry are analysed in this paper. Further multiple usages where power law can be used by Armed forces are also discussed. The study aims to provide an insight into the power law applications by the Armed forces worldwide.

Keywords: Scale-free networks, Power laws, Military applications

1. INTRODUCTION

A network is called scale-free if the characteristics of the network are independent of the size of the network, i.e. the number of nodes. A scale-free network follows power laws. That means that one quantity varies as a power of another. For instance, considering the area of a square in terms of the length of its side, if the length is doubled, the area is multiplied by a factor of four. A scale-free network is visualized when the fraction of nodes are placed on the vertical axis and the distribution of the number of edges is placed on the horizontal axis, follows a typical power-law curve. This same graph on log-log distribution becomes a straight line.

A power law is often represented by an equation with an exponent:

Y ~ x^ ג

Y= MX^ג

Each letter represents a number. Y is a function (the result); K is the variable (the thing you can change); ג (Lambda) is the order of scaling (the exponent), and M is a constant (unchanging).

If M is equal to 1, the equation is then Y=k^ג . Ifג =2, the equation becomes Y=X^2 (Y=K squared). If X is 1, Y is also 1. But if X=2, then Y=4; if X=3, then Y=9, and so on. A small change in the value of X leads to a proportionally large change in the value of Y.

2. SPECIAL CHARACTERISTICS OF NETWORKS FOLLOWING POWER LAW

(a) Small world principle

In large scale-free networks, the small world principle often holds. This principle says that any node is on average connected to any other node in a small number of steps, say around 6. The nodes with many connections act as a kind of hub between all the other nodes.

(b) Security

Large scale-free networks are not vulnerable to random attacks at nodes. They are vulnerable to targeted attacks at the few highly connected nodes. But the network will often not lose connectedness if one hub fails, because other hubs remain. Connectedness means that any node is connected to any other node. Only when the nodes are attacked strategically will the hub lose the connectivity

(c) Stability

The hubs play a crucial role in the stability of a network. For example, in the financial system, the focus is on systemically relevant financial institutions. These may be the large banks that are crucial for the stability of the financial system as a whole. If they fail many other financial institutions will fail too.

(d) Spreading information and contagion

The fact that in many real-world networks a small number of highly connected hubs exist has important consequences for how information and diseases travel through the network. This is relevant to very different disciplines like epidemiology, public relations, marketing, and military warfare, etc.

3. DIFFERENCE BETWEEN POWER-LAW AND NORMAL DISTRIBUTIONS

A power law distribution is different from the normal distribution. The latter is very common in real life. The Central Limit Theorem explains the fact that the normal distribution is found so often in many situations in practice, like for instance the distribution of body length among males or females in a given population. In a somewhat simplified way, the Central Limit Theorem says that if we take any sequence of small independent random effects, then their sum or average will be approximated by the normal distribution. A crucial difference between the normal and power-law distribution is that the number of nodes with really high numbers of edges is much higher in the power-law distribution than in the normal distribution. But generally, well-connected nodes are more common in a normal distribution. This means that in power-law networks you will often find a small number of very highly connected nodes. Their having multiple connections would not occur if the distribution was normal.

The normal distribution is less frequently observed in real-life networks where the distribution of edges in a network may not result from the sequence of independent quantities. Networks grow over time. In real life Nodes that already have a high number of edges are more likely to see new edges to them established compared with nodes with a lower number of edges. That is the idea behind preferential attachment or the "rich-gets-richer" principle. Preferential attachment is a plausible hypothesis for real-life networks in particular. It is attractive to be connected to people who are already highly connected. Think about celebrities, sporters, and politicians in social networks or for that matter countries as part of their diplomacy would like to get connected to more influential and linked nations. An example of preferential attachment following a scale-free network is the chart indicating connectivity of the number of nodes in a road network vis a vis Air network and is placed at Fig 1.

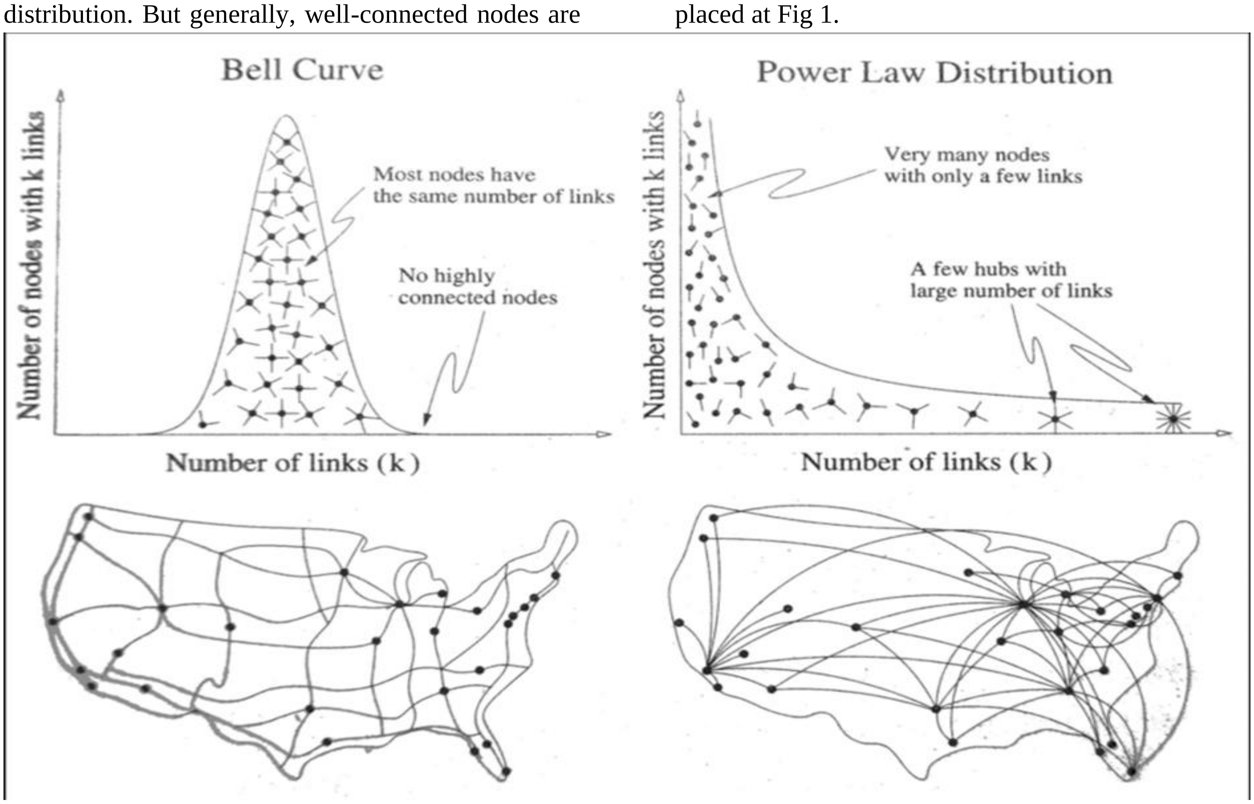

Fig 1: Sourced from online resource placed at Reference [2]

4. REVIEW OF LITERATURE

a) In 1896 Pareto introduced a distribution to describe income, which was known as Pareto law. It was demonstrated that the relative number of individuals with an annual income larger than a certain value x was proportional to the power of x. Since then a considerable amount of work has been done in the study of income and wealth distribution, and expenditure in distinct economic communities.

b) In all known languages, a small percentage of words make up the majority of its usage. This is known as Zipf's law, after George Kingsley Zipf, who first identified the phenomenon. The most used word in a language may make up as much as 7% of all words used, while the second-most-used word is used half as much, and so on. As few as 135 words can together form half of a language (as used by native speakers).

c) In recent times the most influential network papers have been —"Emergence of scaling in random networks" by Barabási and Albert and Watts and Strogatz's paper on small-world networks. These two papers drew much attention in this field and have revolutionized network study connecting researchers from different backgrounds making it an interdisciplinary area.

d) Scale-free networks are also rigorously studied as a new emerging concept of complexity science. This new science studies emergent patterns as consequences of basic symmetries and behavioral rules of the constituents. Barabási and Albert have argued that the scale-freeness of networks is a universal phenomenon and the study of one system can share its emergent properties with the entirely different and unconnected system. For example, the pattern of movement of Birds in a flock versus how metropolitan cities grow tends to geographically.

e) Barabasi et al proposed Preferential attachment as a plausible mechanism to explain the power-law degree distributions in real networks. In the growth mechanism, of preferential attachment, new vertices attach to the old ones with a probability proportional to the degree (number of neighbors) of the old vertices already present in the network.

5. STUDY OF POWER LAW OBSERVED IN MILITARY ARMS TRADE

The military Arms industry is observed preferential attachment in its growth mechanism. To understand this phenomenon a comprehensive study of the worldwide weapons industry was undertaken to establish its connection to Power law. The data obtained was obtained from reliable open source websites and plotted in multiple Histograms charts. The trend line was established using Microsoft excel and the Power-law equation with associated R square value for the trendline was calculated and placed alongside the Histogram charts for ready reference as shown below.

(a) The United States is the established leader in the Arms industry. The curve at Fig 2 is the Histogram plot of 15 countries. The curve is observed to fit in the power-law distribution with exponent lambda for the trendline at -1.17 and R square at 0.92.

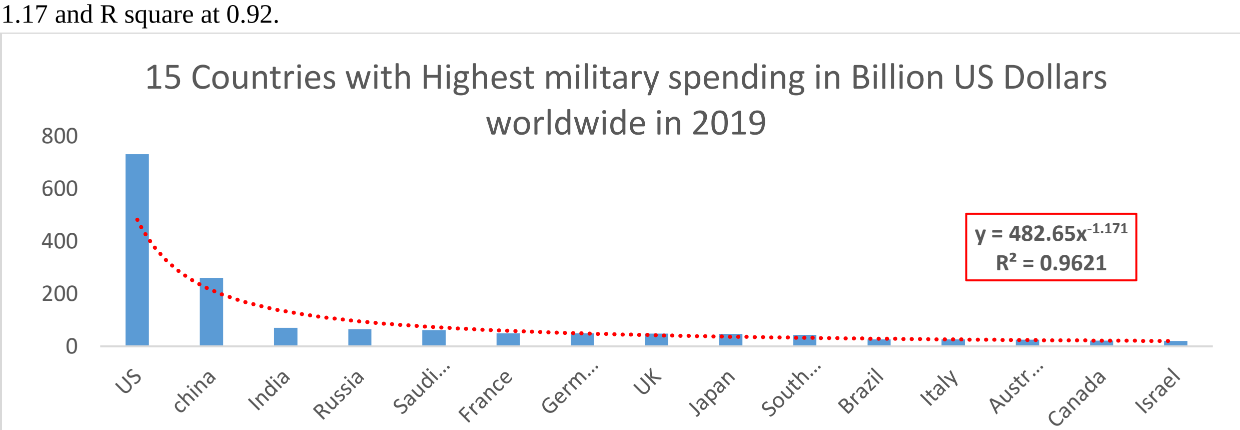

Fig 2: Countries with highest military spending in US billion dollars for the year 2019 (Source: Author compilation based on open source data)

(b) Similarly the military expenditure of the countries as a percentage yielded power-law with lambda raised at -0.752 and the trendline demonstrating a high R square value. The histogram plot with the associated trendline is placed at Fig 3.

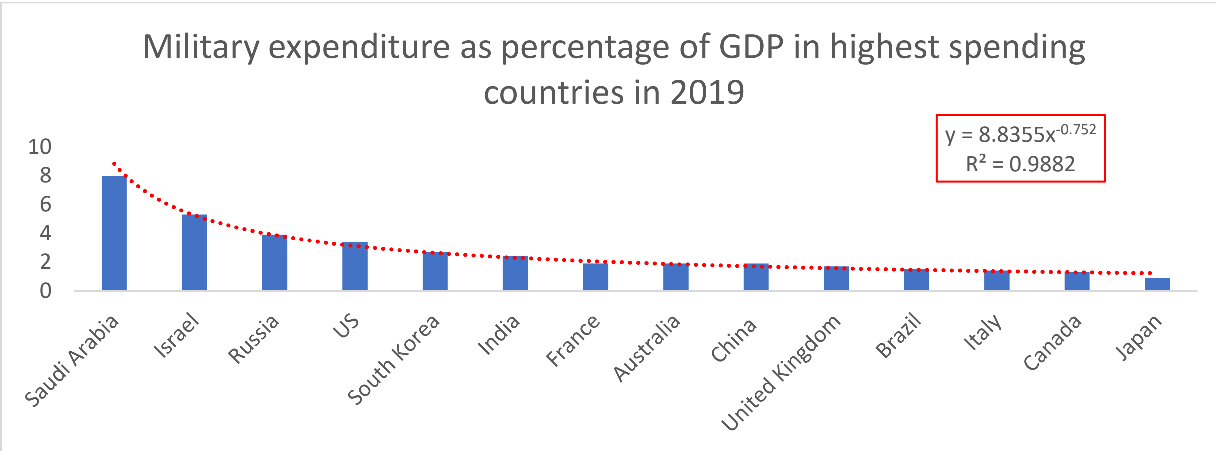

Fig 3: Histogram indicating Military expenditure of countries as a percentage of GDP for the year 2019 (Source: Author compilation based on open source data)

(c) The data obtained for exporters of major weapon systems also yielded power law. The Histogram plot placed at Fig 4 indicates the market share in percentage terms of leading exporters in the worldwide Arms industry. Here the curve follows a rapidly declining power-law with lambda at -1.47 and a high R square of 0.96 for the trend line.

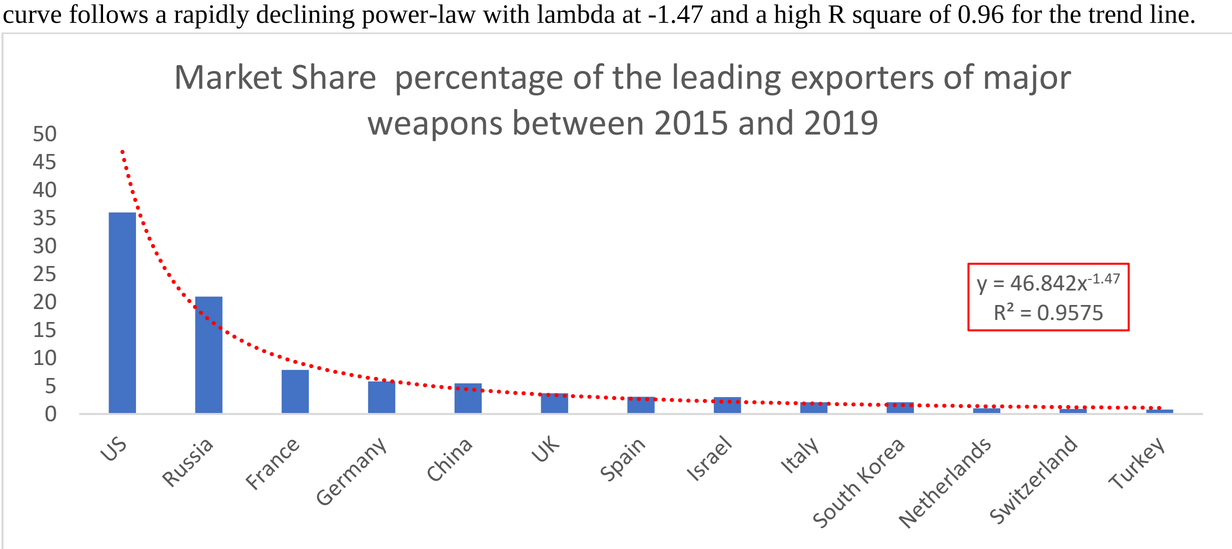

Fig 4: Histogram indicating Market share percentage of the leading exporters of major weapons between 2015 and 2019 (Source: Author compilation based on open source data)

(d) Detailed analysis of the US, the market leader of the Arms industry was undertaken to understand the growth patterns of its Military-industrial complex. Histogram plot indicating country-wise Arms export for the year 2019 in US million dollars is placed at Fig 5. Power law was confirmed with Saudi Arabia as being the prime importer of major weapon systems from the US. The trendline is observed with a lambda of 0.98 with a high R square value of 0.94.

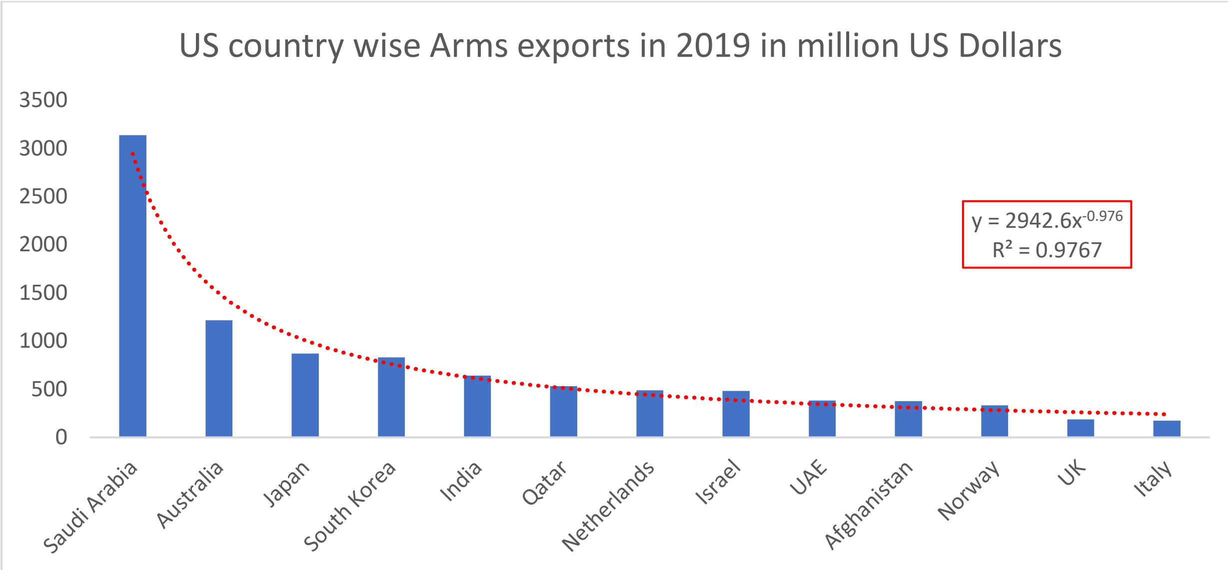

Fig 5: Histogram indicating Arms sale of US companies in million US dollars in 2018 (Source: Author compilation based on open source data)

(e) The study of data from major weapon system manufacturers was undertaken to check power-law affinity. Towards this Histogram with associated trendline of Sales figure of top 20 Arms manufacturing companies is placed at Fig 5. Power law presence is confirmed in the distribution of the sales figure by the Weapon OEMs/ Companies was seen with lambda at 0.865 and R square of 0.93 for the trendline.

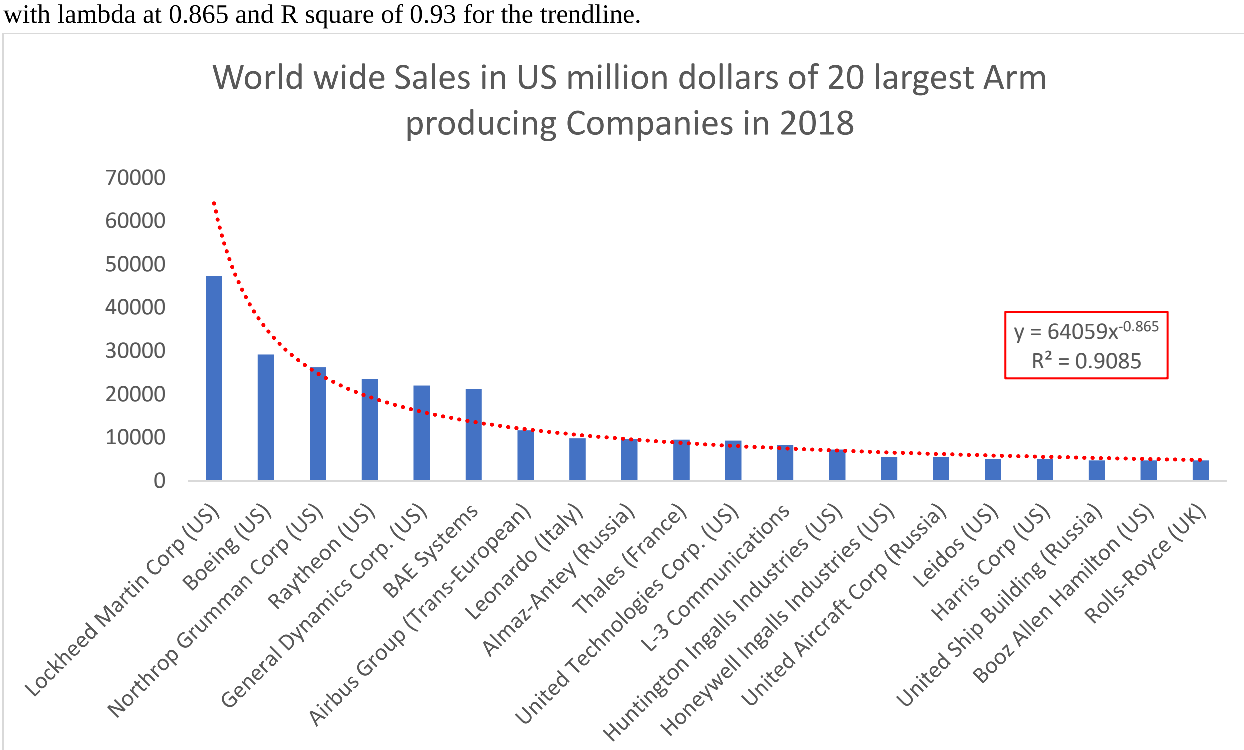

Fig 6: Histogram of world wide sale in US million dollars of 20 largest Arm producing companies (Source: Author compilation based on open source data)

(f) Further the data of OEMs belonging to the United States were segregated to see any association with Power-law inside the leading Arms trade exporting nation. The weapon companies of the US are a subset of the larger arms industry and functions as an integral part of the formidable Military-Industrial complex of the United States. The histogram for worldwide sales of US countries in million dollars is placed at Fig 6. Surprisingly the arms industry in the US confirmed the presence of power-law with lambda as -1.04 and R square of 0.89 for the trend line.

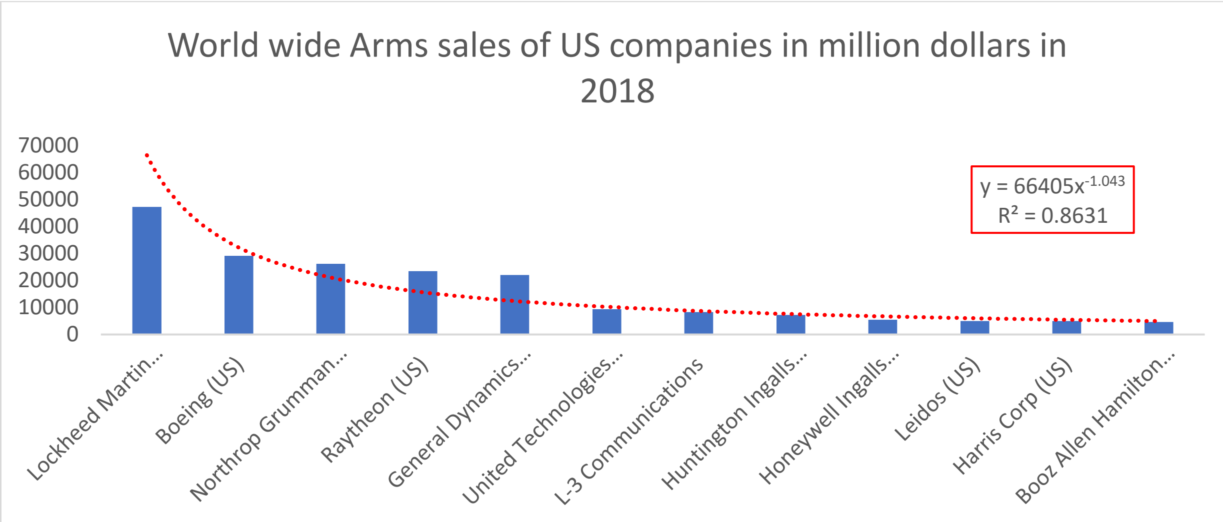

Fig 7: Histogram Arms sale of US companies in million US dollars in 2018 (Source: Author compilation based on open source data)

(g) Histogram indicating Market share in percentage in imports between 2015-19 is placed at Fig 7. The trendline was observed with lambda at 0.6 and a very high trendline R square of 0.96.

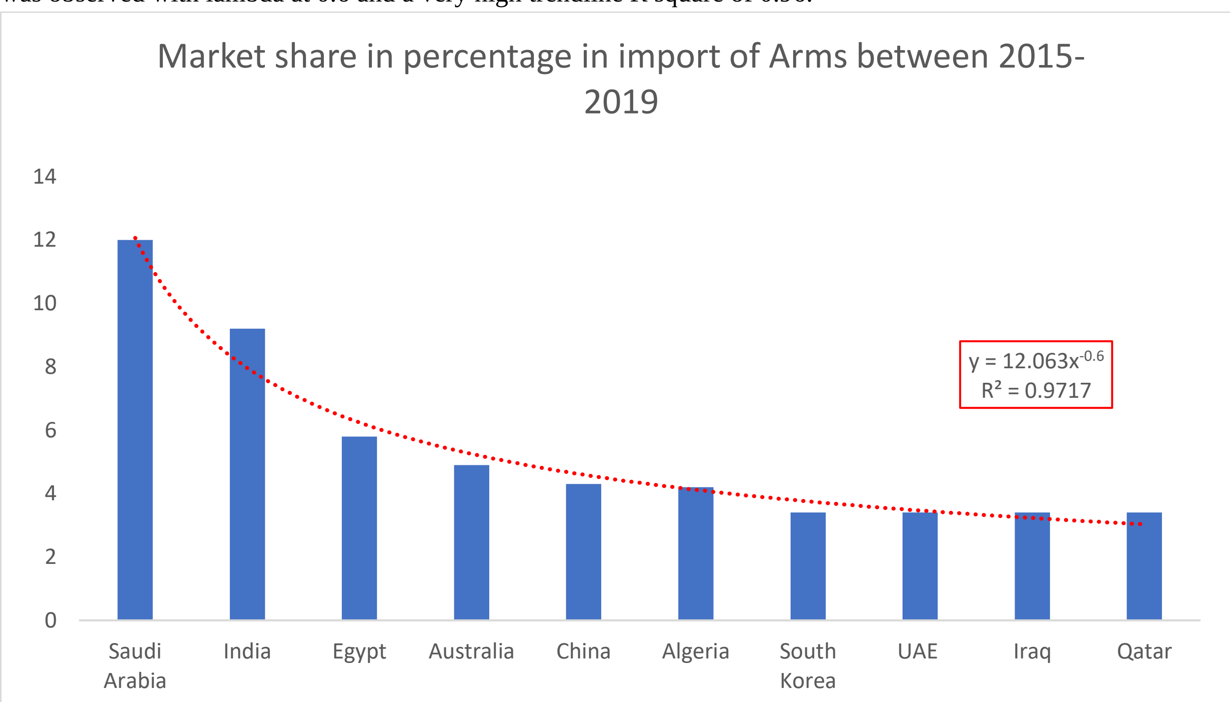

Fig 8: Histogram indicating Market share in percentage in imports of Arms between 2015-2019 (Source: Author compilation based on data available on www.statista.com)

6. INTERPRETATION OF THE RESULTS

(a) Countries engaged in selling and buying of the weapons form a network. In this real-life network, not every node is equal. Multiple Arms treaty Geopolitical considerations, International relations between the countries govern this network. This network shows a high tendency to form strong preferential attachments. It has been established by Barabasi et al that the outcome of such interactions forms a scale-free network governed by the power law.

(b) The property of “Rich getting richer” is clearly visible in the multiple charts of weapon exporting industries. The companies that have established a presence in the defense sector get much more preference over other companies that are looking to establish the connection.

(c) The nation-state is the prime buyer of the weapon systems from the Arms producing firms. This causes the monopsony conditions of a single buyer with multiple sellers (OEMs). This leads to preferential attachment by the military to seek a reliable and established vendor who can deliver without fail when asked from it. This forms a competitive advantage for the OEM and leads to it becoming more successful compared to other players.

(d) The weapon importing countries from the US also forms power law-like distributions. This shows the total dependability and faith that the importing country demonstrates in the exporting country.

(e) International Affairs and Geopolitics play a considerable role in supporting this power-law equation. Multiple treaties regulate and restrict the sale of weapons to other countries and government certification is mandatory for a major sale. This forms a strong basis for preferential attachment. The market share of the exporting countries (placed at Fig. 4) and importing countries (placed at Fig 8) is highly insightful in giving the perspective on how the industry has a strong affinity for the power law.

(f) The country’s government triangulate the Military with the Industrial complex. The private industrial sector employs forceful lobbies to influence government decisions. This forms a strong preferential attachment in the network and is the primary reason behind the scale-free networks in this sector.

7. SUGGESTIONS AND RECOMMENDATIONS

(a) A country that understands power-law or scale-free growth possesses a comparative advantage. As an example, the recent boost in the sales of Turkish origin Bayraktor TB2 drones can be predicted using this power-law model. The sales of drones that were used recently used in Azerbaijan – Armenia conflict has already started to see an upward trend as multiple countries like Ukraine and Kazakhstan vie to import this highly effective and proven technology. The Geopolitics and restrictions in the export of Armed drone technology by the US to other countries play their part in forming strong preferential attachments.

(b) India presently lies at the extremely low end in the tail of the weapon systems export industry. However, this can be changed quickly as demonstrated by the power-law model. With the effective change in policies, the import percentage will tend to come down drastically and exports can also go up fast following the power-law model.

(c) Researchers can undertake further research in this domain especially in the growth curve of the Indian Arms Industry including the potential of Defence PSUs to the power law.

8. CONCLUSION

Power laws are a powerful class of tools that can help us better understand the world around us. Some of the wide arrays of naturally occurring and man-made phenomena can be described through the application of power law. The military Arms industry demonstrates a very high power-law affinity in all its facets and is a very interesting area of research. In the context of the Indian domain, this principle may be exploited to boost the arms growth sale and make India truly self-reliant (Atmanirbhar) in this sector.

REFERENCES

[1] Broido, A. D., & Clauset, A. (2019). Scale-free networks are rare. *Nature Communications, 10*(1), Article 1017. https://doi.org/10.1038/s41467-019-08746-5

[2] Zeng, A., Fan, Y., Di, Z., Wang, Y., & Havlin, S. (2021). Asymmetry in link prediction of scale-

free networks. *Nature Communications, 12*(1), Article 2117. https://doi.org/10.1038/s41467-021-22356-8

[3] Newman, M. E. J. (2003). The structure and function of complex networks. *SIAM Review, 45*(2), 167-256. https://doi.org/10.1137/S003614450342480

[4] Albert, R., & Barabási, A. L. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics, 74*(1), 47-97. https://doi.org/10.1103/RevModPhys.74.47

[5] Dorogovtsev, S. N., & Mendes, J. F. F. (2003). *Evolution of networks: From biological nets to the Internet and WWW*. Oxford University Press. https://global.oup.com/academic/product/evolution-of-networks-9780198515906

[6] Clauset, A., Shalizi, C. R., & Newman, M. E. J. (2009). Power-law distributions in empirical data. *SIAM Review, 51*(4), 661-703. https://doi.org/10.1137/070710111

[7] Pastor-Satorras, R., & Vespignani, A. (2001). Epidemic spreading in scale-free networks. *Physical Review Letters, 86*(14), 3200-3203. https://doi.org/10.1103/PhysRevLett.86.3200

[8] Kaya, M., & Öztürk, İ. (2022). The strategic implications of Turkey's Bayraktar TB2 in modern warfare. *Journal of Defense Strategies and Operations, 10*(2), 135-152. https://www.journalofdefenseoperations.com/2022/strategic-implications-turkeys-bayraktar-tb2

[9] Statista. (n.d.). Defense and arms - Statistics & Facts. Statista. https://www.statista.com/ topics/ 1696/defense-and-arms/

[10] Hagel, J. (2007, May 16). The power of power laws. Edge Perspectives with John Hagel. https://edgeperspectives.typepad.com/edge_perspectives/2007/05/the_power_of_po.html