Developing Intuitive Thinking in Mathematics Students Engaged In Search Activity

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Abstract: The formation of intuitive thinking is important for the development of the intellect and its successful functioning in modern society. Intuitive thinking is one of the main components of mathematical thinking. It occupies an important place in the process of cognition of any science, and mathematics in particular. Comprehension of mathematical truths by students occurs in the form of a long thought process without reliance on a specific vision. It is derived from their subconscious processing of information received from various sources. This article discusses a study of the introduction of intuitive thinking into the search activities of students seeking solutions to a geometrical problem. The focus is on 1) student acceptance of the task and 2) keeping the student active in the search. This requires students to use heuristic procedures in the analysis of the solution to the problem and to freely choose the direction of the search. The study was conducted at the Physics and Mathematics Lyceum of Dushanbe for several years (2019-2022). The experiment involved 17 teachers and 78 high school students. At the initial stage, the level of intuitive thinking of students in the control and experimental groups was determined. Then the experimental group was trained according to the methodology developed by the authors. At the final stage, the learning outcomes of these two groups were compared. Statistical analysis showed a significant transition to a higher level of thinking among the students of the experimental group. The proposed methodological approaches and the teaching materials developed in the course of their preparation can be used by mathematics teachers for more effective teaching of their students.

Keywords: teaching of mathematics, intuitive thinking, heuristic procedures, geometric problem, students.

1. INTRODUCTION

The mathematical style of thinking in a broad sense can be understood as a person's readiness for mathematical creativity, for the creation of new previously unknown hypotheses, theories and methods in the field of mathematics. In relation to school mathematical education, such novelty is subjective in nature, as perceived by a particular individual. At the same time, the determination of mathematical creativity is closely related to the intuitive component of thinking in solving problems [1].

The intuitive component of thinking usually reveals itself in situations characterized by uncertainty and ambiguity. These characteristics give rise to perceptions that do not correspond to or even contradict the ideas that students have, which they initially evaluate as correct and obvious [2]. Such perceptions emerge in particular when thinking is freed from the shackling influence of techniques and methods of solving problems previously used.

In other words, the intuitive component of thinking is closely related to overcoming stereotypes. Stereotypes are a convenient way to classify and systematize material, a way to make the world around us more accessible to understanding [3]. However, in the processes of mathematical creativity and, in particular, in solving mathematical problems of a search nature, stereotypes become an obstacle: they enslave thinking, create bias, foster conservatism in solving new problems, cause thinking to move along beaten paths that do not lead to the correct solution. Facts and phenomena that go against the established ideas of a person are simply not taken into account, ignored. The student "closes up", ceases to be receptive to the unexpected and loses the desire and ability to be creative [4].

Mathematical content contains rich opportunities for the manifestation of the intuitive component of mathematical thinking [5]. Consider, for example, the duality principle for the projective plane. According to this principle, if we take any true statement formulated in terms of projective geometry and replace each term with its dual, we get a true statement again. The above reasoning illustrates the possibility of varying the semantic components of most mathematical statements.

Turning directly to consideration of the latent potential of school mathematical courses, it is appropriate to mention the founder of Gestalt therapy F. Perls, who in one of his works suggests thought experiments to promote a patient's personal growth, such as "the effect of opposite forces" (presenting a situation opposite to the original one, compiling and comprehending both situations, changing situations from one angle or another and predicting the results of such a change); "attention and concentration" (focusing on a certain visual object, leading to its selection against the surrounding background and further change in this background) [6].

These experiments are easily projected onto school mathematical material. Thus, the "effect of opposite forces" can be realized during the development of the topic of the problem when students change the statement of the problem (composing and solving an inverse problem, generalizing and specializing problems, adding and excluding data and requirements of the problem, etc.) [7]. In the course of such work and subsequent discussion, the students actualize various alternative modifications of the problem situation under consideration. The "attention and concentration" experiment can be implemented when considering geometric problems, when during the search process, students alternately identify key geometric configurations, analyze them, eventually coming to a solution path, and at the same time creating an opportunity to form an intuitive perception of the figure.

Unfortunately, mass school education, as well as specialized education for gifted children, has not focused on the development of intuitive thinking in students [8]. Many students, even in specialized classes, have a rather poorly formed orientation towards such thinking; they hardly switch to new, unusual procedures, and the various methods and techniques of mathematical activity mastered by them are often present in their minds in an isolated form without sufficient correlation with each other. Evidence of this was found in literature data, conversations with teachers, the results of written work and oral surveys we conducted during the search experiment, as well as personal observations of the teaching of mathematics in a number of schools in Tajikistan.

The reason for the current situation, in our opinion, is the recent bias towards "training" students, using the normative format for assessing their success in the field of mathematics, which initially has a negative impact on the full deployment of the search activity process, leading to students using a stereotypical approach when working with mathematical problems. Accordingly, the purpose of this study was to develop a strategy for promoting intuitive thinking in students seeking solutions to a geometric problem.

When organizing work to develop a student's intuitive style of thinking, it is advisable and necessary to consider the following: 1) The task should be interesting and meaningful, both for the student and the teacher, 2) The task should be in conflict with the child's way of thinking in order to encourage him to actually think, and not to work within the range of habitual schemes and ideas. In other words, in order for the joint work of students and teachers to take place quite effectively in the described context, it is necessary to select tasks so that they naturally "draw in" the student and teacher, as something unfamiliar, but at the same time, extremely interesting.

2. METHODOLOGY

This study took into account the specifics of the mathematical content. The principal means of implementing this study is the provision of a series of special mathematical tasks, through the solution of which the actualization of the intuitive style of thinking of students takes place, regulated by the teacher.

During the study, several basic sources were used. Firstly, scientific and scientific-methodological works of leading domestic and foreign experts in the field of developmental psychology, didactics, theory and methods of mathematical education and materials on the history of mathematics were analyzed. As a result of this analysis, it was concluded that the basis of mathematical search activity is an intuitive precognition of its results and ways to achieve them, when the student mentally transforms the studied mathematical object. The vision of how to carry out such a transformation initiates the possibility of choosing the direction of the creative search process on the basis of known concepts, images and methods of activity to create new mathematical content [9].

Obviously, to a certain extent this procedure forms part of the general educational process. According to J. Hadamard, between the work of a student solving a problem in algebra or geometry, and the inventive work of a scientist, the difference is only in the level [10]. Nevertheless, as evidenced by the study of

programs of mathematical courses, existing textbooks, teaching aids and software for educational purposes, at present the problem of forming the intuitive style of thinking of students in mass education has not become a special subject of study for specialists in the field of mathematical education.

Secondly, we carried out a longitudinal study of the course used for teaching mathematics in Tajikistan. The search experiment was carried out during three-year subject mathematical training for schoolchildren at the Physics and Mathematics Lyceum of Dushanbe (2019-2022). In total, the experiment was implemented with 78 students of the basic and senior levels of education. Competence of students was manifested in terms of their readiness to use an intuitive style of thinking when solving mathematical problems of a search nature. As a result of this work, it transpired that more than half of the students surveyed were not ready for a variant consideration of the considered mathematical constructions, limiting themselves to the analysis of only one of the possible alternatives. We see the main reason for this situation in the insufficient attention of teachers to the described component of educational and search mathematical activity.

Finally, materials were created after pedagogical experimentation with the search process to determine the possibilities of introducing the developed mathematical content and assessing its effectiveness in terms of creating students' readiness to update the intuitive style of thinking when solving mathematical problems of a search and research nature. The content of the tested materials is partially reflected in a number of our teaching aids and articles [11-13].

The proposed strategy for stimulating the students' intuitive style of thinking was implemented with the help of special two-level educational and search tasks. The effectiveness of this strategy was determined on the basis of a comparison of the results of the tasks, which were subjected to qualitative and quantitative analysis based on a special quantitative scale. Indicators of the main characteristics of the intuitive style of thinking were considered: readiness to actualize unusual associative connections; the ability to adequately vary their actions when the situation changes; willingness to build alternative designs in order to highlight the most simple, effective and "beautiful" solution.

3. RESULTS

1. The main factors that govern the search activity of students based on their intuitive thinking were identified: acceptance of the task and maintaining search activity.

Acceptance of a task means awareness of its subjective value for the student, which is achieved under the condition of relative freedom of choice to change the task. If the student perceives that he has the "right of free choice", the chosen object or the way of operating with it becomes, as it were, "his own", directly correlated with his value system [13]. The results of the search activity themselves, as well as the ways to achieve them, begin to be recognized, in the case of success, as personal intellectual values. The effective implementation of the act of free choice, on the one hand implies the potential presence of conscious alternatives in relation to one or another element of the mathematical content reflected in the task situation (concept, method, microtask, etc.), and on the other hand, the student's ability to take into account attention to different points of view on the object under consideration and to move flexibly, if necessary, from one mode of action to another.

Another factor that increases the likelihood of a student accepting a task is the significant effort to overcome the difficulties perceived by this student in the course of solving. Awareness of the difficulty of the task is expressed in the understanding of the need for a different vision, a different attitude to the object under study, expressing the need to reconceive certain creative procedures when solving a particular problem. Overcoming this challenge by actualizing his heuristic potential, the student experiences an "exciting feeling of a small discovery", which contributes to the activation of his subsequent search activity. Thus, an important condition for students to realize the nature of the difficulties inherent in the content of mathematical problems and successfully overcome them is the purposeful involvement of heuristic procedures in the analysis of the solution to the problem.

A student develops confidence in his search for a solution to a mathematical problem as he begins to anticipate success. Overcoming difficulties independently, the student persists fearlessly in searching for a solution to the problem and accepts personal responsibility for his own decisions. From a methodological standpoint, the formation of the student's confidence was studied by G. Polya, who

identified a number of factors contributing to this: confirmation of the consequences, analogy with a known fact, analysis of a contradictory assumption, the likelihood of intermediate results, inductive verification, etc. [14,15]. The use of these procedures makes it possible to predict the search result with a sufficiently high degree of probability.

2. At the heart of purposeful involvement in the analysis of the solution of the problem by heuristic procedures are the processes of formation, coding, recognition and subsequent decoding of the corresponding intellectual patterns. These patterns are manifested in the student's behavior when solving mathematical problems and are a kind of guideline for search activities.

Geometric problems, despite their great heuristic potential, do not carry the multi-valued meanings characteristic of text problems, and, accordingly, provide for the conduct of the search process in the "purest" form. The student, performing this or that construction, gets the opportunity to deduce new information about the considered geometric configuration by including it in a new meaningful context. The actualization of each pattern is carried out when performing a chain of tasks which gradually becomes more complex and does not contain an explicit hint for arriving at a specific solution. When moving along a chain of increasingly complex tasks, it is necessary to identify and analyze additional initially hidden geometric dependencies, which act as sources of the corresponding patterns.

3. The inclusion of students in a situation of free choice offers them the opportunity to build their individual educational trajectories. Individual trajectories involve learning at different levels of difficulty and at different paces. In this case, students have the opportunity to proceed at their own pace, without adjusting to a particular average level in studying the topic, while controlling theoretical and practical skills at each stage of the work, the transition to the next stage being possible only after mastery of the previous stage.

In particular, to put forward hypotheses and test them experimentally, students can be asked to work with several interactive models. These models can be offered to students in finished form, or they can be created by them relatively independently. It all depends on the availability of study time and the ability of students to navigate in the specified software environment. Self-construction of models requires a

certain amount of time, but at the same time, in the process of work, the student inevitably has to remember and apply a whole range of knowledge in geometry in a non-standard situation, and sometimes get acquainted with new information, which is more methodologically valuable than working with ready-made material. In particular, students can create geometric configurations and then explore their properties by moving the objects they contain on the display. At the same time, some relationships laid down during construction are preserved, while others are changed, which leads students to generalizations in the most natural way.

Individual educational trajectories are based on a gradual change in the task material. This allows us to formulate and prove a large number of statements of varying degrees of complexity compared to the initial problem. Depending on the level of training and intellectual development of the student, the final result can be a system of tasks of various sizes and quality content. At each stage, the teacher must first make sure that the identified trajectories of search and research work are available and productive.

This approach to the organization of research activities of students engaged in the modification and subsequent analysis of the task material creates a space for a free choice of areas of work for students of various levels of training.

4. CONCLUSIONS

Our study showed that the quality and variety of solutions to mathematical problems are conditional on the development of intuitive mathematical thinking. Its principal characteristics are: readiness to make decisions based on unusual associative connections; the ability to respond flexibly to a change in the task situation; willingness to build alternative designs in order to highlight the most simple, effective and "beautiful" solution.

Acceptance of the task and maintenance of search activity are essential elements in the development of intuitive mathematical thinking. In the first case, ensuring the freedom of choice for students to change the task comes to the fore, and in the second, mastering heuristic procedures. In response to a variety of search activities and procedures, new intellectual patterns emerge.

As the results of the experiment show, the modification of task materials to create challenges for

individual students significantly enhances the developmental and didactic potential of geometry teaching at school. It provides students with the opportunity to engage freely in search activity, exercise initiative and develop mathematical intuition.

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