© November 2024 | IJIRT | Volume 11 Issue 6 | ISSN: 2349-6002

Dynamics equation for 2R planar manipulator using Lagrange dynamics

Arasu S[1], Suhail Farooq mir [2], and M.P Anbarasi [3]
[1,2,3] *Member, PSG College of Technology Coimbatore, Tamil Nadu*

Abstract—The dynamics equation for 2R planar manipulator using the Lagrange method. The Lagrange equation of motion provides a systematic approach to obtaining robot dynamics equations. This paper provides an introduction to lagrangian mechanics and the importance of the lagrangian method, a comparison of Newton Euler method and the lagrangian method, steps to be followed to derive an equation of motion, derivation, simulation result, Application of lagrangian in various domains.

Index Terms—lagrangian, 2R Planar manipulator, kinetic energy, potential energy, joints, link, revolute joint, coordinates.

I. INTRODUCTION

The kinematic equations describe the motion of the robot without consideration of the forces and moments producing the motion, the dynamic equations explicitly describe the relationship between force and motion. The equations of motion are important to consider in the design of robots, as well as in simulation and animation, and the design of control algorithms. We introduce the so-called Euler-Lagrange equations, which describe the evolution of a mechanical system subject to holonomic constraints. To determine the Euler-Lagrange equations in a specific situation, one has to form the Lagrangian of the system, which is the difference between the kinetic energy and the potential energy. Lagrange's equations of motion are a handy tool in classical mechanics, and provide a methodical approach to derive the equations for a mechanical system's dynamics. Joseph-Louise Lagrange formulated this method in the 18th century, and it frames the study of dynamics in energy rather than force, particularly useful in complex systems. In the robotic area of study, Lagrange's equations of motion are an effective method of examining the dynamics of a robotic system, and in particular manipulators. The 2R planar manipulator is a very simple model in the field of robotics; it consists of 2 rotational joints and 2 links and serves as a model for motion analysis, control, and design principles for multi-joint systems. The 2R manipulator consists of 2 connecting links that can rotationally join together to position its end effector at various points in a Cartesian two-dimensional plane. In this project, we are interested in applying the dynamics of a 2R manipulator which will serve as a model to represent the dynamics of the mechanical system under study. In this case, we will employ Lagrangian mechanics to derive the manipulator's behavior. Lagrangian mechanics is concerned with discussing motion in terms of kinetic and potential energy rather than in terms of forces. The Lagrangian L is defined as:

$$L = K - V$$

$$Q_r = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) - \frac{\partial L}{\partial q_r}$$

Where, K represents the kinetic energy of the manipulator, encompassing the motion of both links, and V represents the potential energy due to gravitational forces acting on the links. Lagrange's equations of motion provide a systematic and versatile approach to analyzing the dynamics of mechanical systems. The function L, which is the difference of the kinetic and potential energy, is called the Lagrangian of the system, and the Equation is called the Euler-Lagrange Equation.

The Euler-Lagrange equations provide a formulation of the dynamic equations of motion equivalent to those derived using Newton’s Second law. ease of use is particularly evident in the following aspects: Lagrange's method focuses on kinetic and potential energy rather than forces, simplifying the analysis of complex systems. Concentrating on energy allows for an intuitive understanding of motion and behavior. Using generalized coordinates enables the representation of complex systems with fewer variables. This flexibility allows for the easy incorporation of constraints, such as fixed lengths in robotic arms, and accommodates non-linear systems that may be challenging to handle using traditional methods. As a variational method specialized in multi-body systems, the Lagrangian method is important to the methods of robotics because it has solved robot dynamics problems.

The Lagrangian method is rooted in classical mechanics and modern physics so it provides a systematic way for practitioners to investigate dynamic systems. Instead of forces, the Lagrangian method studies motion by appealing to energy principles to provide a clearer interpretation of motion, particularly for constrained motion in complex systems. The reason why the Lagrangian method is useful as applied to problem-solving in physics is that the mathematics can quite consequentially be simplified using generalized coordinates to allow the equations of motion to be more easily derived using the least action. The Lagrangian formulation also provides the basis for more sophisticated areas, including Hamiltonian mechanics and Quantum Field Theory, bridging cases involving classical models to more complicated forms. In summary, the Lagrangian method enhances our ability to model, predict, and comprehend the actions of physical systems coherently and elegantly.

II. IMPORTANCE OF THE LAGRANGIAN METHOD

As a variational method specialized in multi-body systems, the Lagrangian method is important to the methods of robotics because it has solved robot dynamics problems. The Lagrangian method is rooted in classical mechanics and modern physics so it provides a systematic way for practitioners to investigate dynamic systems. Instead of forces, the Lagrangian method studies motion by appealing to energy principles to provide a clearer interpretation of motion, particularly for constrained motion in complex systems. The reason why the Lagrangian method is useful as applied to problem solving in physics is that the mathematics can quite consequentially be simplified using generalized coordinates to allow the equations of motion to be more easily derived using least action. The Lagrangian formulation also provides the basis for more sophisticated areas, including Hamiltonian mechanics and Quantum Field Theory, bridging cases involving classical models to more complicated forms. In summary, the Lagrangian method enhances our ability to model, predict, and comprehend the actions of physical systems coherently and elegantly.

III. . COMPARISON BETWEEN NEWTON-EULER METHOD AND LAGRANGIAN METHOD

In the Newton-Euler formulation, the equations of motion are derived from Newton's Second Law, which relates force and momentum, as well as torque and angular momentum. The resulting equations involve constraint forces, which must be eliminated to obtain closed-form dynamic equations. In the Newton-Euler formulation, the equations are not expressed in terms of independent variables and do not include input joint torques explicitly. Arithmetic operations are needed to derive the closed-form dynamic equations. This represents a complex procedure that requires physical intuition. An alternative to the Newton-Euler formulation of manipulator dynamics is the Lagrangian formulation, which describes the behavior of a dynamic system in terms of work and energy stored in the system rather than of forces and momenta of the individual members involved. The constraint forces involved in the system are automatically eliminated in the formulation of Lagrangian dynamic equations. The closed-form dynamic equations can be derived systematically in any coordinate system.

IV. STEPS TO BE FOLLOWED TO DERIVE THE EQUATION OF MOTION USING THE LAGRANGIAN METHOD

STEP1: Recognize the configuration of the manipulator (joint types, link types).
STEP 2: Indicate the number of degrees of freedom, and any constraints.
STEP 3: Choose the generalized coordinates (qi) to describe (joint angles, link lengths) the manipulator configuration.
STEP 4: For each link, we have to derive the kinetic energy based on the mass and velocity of the center of mass.
STEP 5: Compute the potential energy due to gravity for each link, respectively. We combine kinetic energy and potential energy to derive the lagrangian L as follows.

$$L = K-V$$

STEP 6: For each generalized coordinate qi, apply the Euler-Lagrange equation. That is described as,

$$Q_r = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) - \frac{\partial L}{\partial q_r} \qquad r = 1,2,\ldots\ldots n$$

V. DERIVATION OF EQUATION OF MOTION FOR 2R PLANAR MANIPULATOR USING THE LAGRANGIAN METHOD

The 2R planar manipulator, which is equipped with two revolute joints and two links, stands as an essential model in robotics and mechanical engineering. It demonstrates a foundational model necessary for grasping the principles of motion, control, and dynamics in robotic mechanisms. An ideal manipulator is to be free of friction in its joints and to possess rigid links, which simplifies the analysis and focuses on the fundamental dynamics.

An ideal model of a 2R planar manipulator is illustrated in the below Figure. It is called ideal because we assumed that the links are massless and there is no friction. The masses m1 and m2 are the mass of the second motor to run the second link and the load at the endpoint. We take the absolute angle θ1 and the relative angle θ2 as the generalized coordinates to express the configuration of the manipulator.

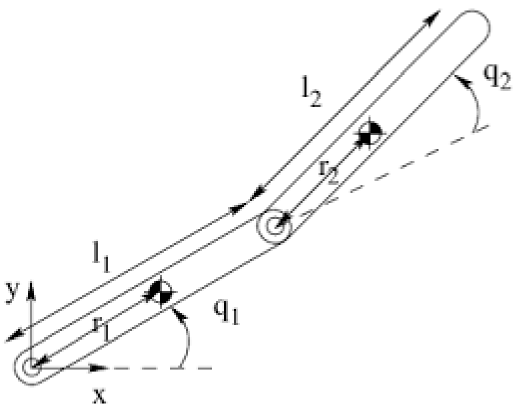


Fig.1

The global position of m1 and m2 are:

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} l_1 cosq_1 \\ l_1 sinq_1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} = \begin{bmatrix} l_1 cosq_1 + l_2 \cos\,(q_1+q_2) \\ l_1 sinq_1 + l_2 \sin\,(q_1+q_2) \end{bmatrix}$$

Therefore, the global velocity of the masses are

$$\begin{bmatrix} \dot{X}_1 \\ \dot{Y}_1 \end{bmatrix} = \begin{bmatrix} -l_1 \dot{q}_1 sinq_1 \\ l_1 \dot{q}_1 cosq_1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{X}_2 \\ \dot{Y}_2 \end{bmatrix} = \begin{bmatrix} -l_1 \dot{q}_1 sinq_1 - l_2(\dot{q}_1+\dot{q}_2)\sin(q_1+q_2) \\ l_1 \dot{q}_1 cosq_1 + l_2(\dot{q}_1+\dot{q}_2)\cos\,(q_1+q_2) \end{bmatrix}$$

The kinetic energy of this manipulator is made of kinetic energy of the masses and is equal to:

$$K = k1 + k2 \qquad (1)$$

Where k1 is the kinetic energy of the first link

Where k2 is the kinetic energy of the second link

$$K = \frac{1}{2} m_1 (\dot{X}_1^2 + \dot{Y}_1^2) + \frac{1}{2} m_2 (\dot{X}_2^2 + \dot{Y}_2^2) \qquad (2)$$

$$K = \frac{1}{2} m_1 l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 (l_1^2 \dot{q}_1^2 + l_2^2 (\dot{q}_1 + \dot{q}_2)^2$$

$$+ 2 l_1 l_2 \dot{q}_1 (\dot{q}_1 + \dot{q}_2) cosq_2) \qquad (3)$$

The potential energy of the manipulator is:

$$V = V_1 + V_2 = m_1 g Y_1 + m_2 g Y_2$$

Where V1 is the potential energy of the link 1

Where V2 is the potential energy of the link 2

$$V = m_1 g l_1\, sinq_1 + m_2 g \big(l_1\, sinq_1 + l_2 sin(q_1+q_2)\big) \qquad (4)$$

The Lagrangian is then obtained from Equations (3) and(4)

$$L = K - V$$

$$L = \frac{1}{2} m_1 l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 (l_1^2 \dot{q}_1^2 + l_2^2 (\dot{q}_1 + \dot{q}_2)^2$$

$$+ 2 l_1 l_2 \dot{q}_1 (\dot{q}_1 + \dot{q}_2) cosq_2)$$

$$- (m_1 g l_1 sinq_1 + m_2 g (l_1\, sinq_1$$

$$+ l_2 sin(q_1+q_2))) \qquad (5)$$

Which provides the required partial derivatives as follows:

$$\frac{\partial L}{\partial q_1} = -(m_1+m_2) g l_1 cosq_1 - m_2 g l_2 cos(q_1+q_2) \qquad (6)$$

$$\frac{\partial L}{\partial \dot{q}_1} = (m_1+m_2) l_1^2 \dot{q}_1 + m_2 l_2^2 (\dot{q}_1+\dot{q}_2)$$

$$+ m_2 l_1 l_2 (2\dot{q}_1 + \dot{q}_2)\, cosq_2 \qquad (7)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = (m_1+m_2) l_1^2 \ddot{q}_1 + m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2)$$

$$+ m_2 l_1 l_2 (2\ddot{q}_1 + \ddot{q}_2) cosq_2$$

$$- m_2 l_1 l_2 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2) sinq_2 \qquad (8)$$

$$\frac{\partial L}{\partial q_2} = -m_2 l_1 l_2 \dot{q}_1 (\dot{q}_1 + \dot{q}_2) sinq_2$$

$$- m_2 g l_2 cos(q_1+q_2) \qquad (9)$$

$$\frac{\partial L}{\partial \dot{q}_2} = m_2 l_2^2 (\dot{q}_1 + \dot{q}_2) + m_2 l_1 l_2 \dot{q}_1 cosq_2 \qquad (10)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) = m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 \ddot{q}_1\, cosq_2$$

$$-m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 sinq_2 \quad (11)$$

Therefore, the equations of motion for the 2R manipulator are:

$$Q_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1}$$

$$Q_1 = (m_1 + m_2) l_1^2 \ddot{q}_1 + m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 (2\ddot{q}_1 + \ddot{q}_2) cosq_2 - m_2 l_1 l_2 \dot{q}_2 (2\dot{q}_1 + \dot{q}_2) sinq_2 + (m_1 + m_2) g l_1 cosq_1 + m_2 g l_2 cos(q_1 + q_2) \quad (12)$$

$$Q_2 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) - \frac{\partial L}{\partial q_2}$$

$$Q_2 = m_2 l_2^2 (\ddot{q}_1 + \ddot{q}_2) + m_2 l_1 l_2 \ddot{q}_1 \, cosq_2 - m_2 l_1 l_2 \dot{q}_1 \dot{q}_2 sinq_2 + m_2 l_1 l_2 \dot{q}_1 (\dot{q}_1 + \dot{q}_2) sinq_2 + m_2 g l_2 cos(q_1 + q_2) \quad (13)$$

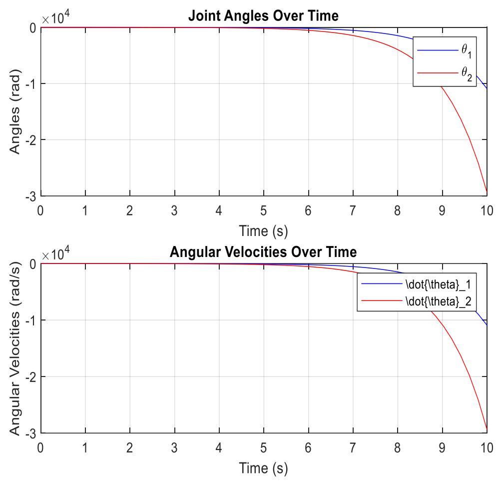
The generalized forces Q1 and Q2 are the required forces to drive the generalized coordinates. In this case, Q1 is the torque at the base motor and Q2 is the torque of the motor at m1. The equations of motion can be rearranged to have a more systematic form.

$$Q_1 = \left((m_1 + m_2) l_1^2 + m_2 l_2 (l_2 + 2 l_1 \, cosq_2)\right) \ddot{q}_1 + m_2 l_2 (l_2 + l_1 \, cosq_2) \ddot{q}_2 - 2 m_2 l_1 l_2 sinq_2 \dot{q}_1 \dot{q}_2 - m_2 l_1 l_2 sinq_2 \dot{q}_2^2 + (m_1 + m_2) g l_1 cosq_1 + m_2 g l_2 cos(q_1 + q_2) \quad (14)$$

$$Q_2 = m_2 l_2 (l_2 + l_1 \, cosq_2) \ddot{q}_1 + m_2 l_2^2 \ddot{q}_2 + m_2 l_1 l_2 sinq_2 \dot{q}_1^2 + m_2 g l_2 cos(q_1 + q_2) \quad (15)$$

VII. SIMULATION RESULTS AND DISCUSSION

Define the link's masses and lengths, as well as the gravitational acceleration. All of these are important in determining kinetic and potential energy. There are 10 seconds of simulation which starts with the initial conditions of both joint angles initialized at π/4radians and all angular velocities set to 0. The function manipulator_eq is defined as an anonymous function. This function will calculate the derivatives of the state variables (angles and angular velocities). The ode45 function will solve the ordinary differential equations (ODEs), which are defined in manipulator_eq, and output a time vector t and solution matrix sol, which contains the angles and angular velocities at each time. Plots both the joint angles and their respective velocities concerning time. There are two subplots showing angles in the first, and angular velocities in the second subplot. The main function is to calculate the derivatives of the state variables. It takes in the current state y (which contains angles and angular velocities) along with the system parameters. The links' center of mass positions are computed with trigonometric functions based on the given joint angles. The kinetic energy T for each link is calculated, with the second link's energy including contribution from both translational and rotational motion. Potential energy V related to the force of gravity for both links is computed from their height. Calculate whatever derivatives of the Lagrangian needed for the Euler-Lagrange equation.



Joint Angles Over Time: The graph containing two curves that plot two angles, θ1 (in blue) and θ2 (in red), of the manipulator joints over the 10 seconds of simulation. The angles will likely oscillate (or advance) due to the initial conditions, meaning θ1 is always at the same measure of change for θ2 at any time.

Angular Velocities Over Time: A second graph shows the angular velocities (in blue) and (in red), over the same period. This graph shows how the speeds of each joint change to the dynamics of the manipulator.

VIII. APPLICATION OF THE LAGRANGIAN FORMULATION IN VARIOUS DOMAIN

Since Lagrangian mechanics leads to deriving the equations of motion for robotic arms and mobile robots it makes it possible to design control systems to ensure precise motion and manipulation. It allows for the simulation of robotic systems to forecast the outputs under different conditions.

This applies to the study of dynamics in mechanical systems, which involve structural and machine vibration modes. Develop models of multiple interacting elements, such as vehicles and machinery, to simulate their motion and interactions.

In space or aircraft design, Lagrangian methods help to optimize flight paths and strategies of control. It is applied in the investigation of stability of flight dynamics and design of the control systems ensuring stabilization of aircraft.

This formulation is fundamental to modern physics, with special references to the derivation of equations of motion in classical field theories and quantum mechanics.

Lagrangian mechanics picks out symmetries; it gives rise to conservation laws by Noether's theorem.

Lagrangian principles find utility in using engineers to optimize their designs for optimum performance and efficiency in structures and mechanical systems. It is used to explain the dynamics of materials under different forces and motions

Modeling of the movement of humans and animals. This is applicable in designing prosthetics and understanding locomotion dynamics. This may help in studying the efficacy of different movements and postures, which can lead to more ergonomic design in the workplace.

The present approach to Lagrangian formulation lends itself to the construction of controllers for dynamic systems with desirable behavior and stability.

The theory is used to observe wave motion in fluids and the dynamics of fluid bodies. It has been employed in the design of controlling strategies for engineering applications related to flows.

IX. CONCLUSION

In summary, the present study investigates the major aspects of a motion equation for a 2R robot manipulator using Lagrangian mechanics; specifically, Lagrangian dynamics derivation of the equations of motion for a 2R planar manipulator. This method will enable an understanding of the influences of consideration of kinetic and potential energy on the behavior of the manipulator under various conditions. It also enables the use of the results for simulation, control design, and further study of even more complex robotic systems. Knowledge of these dynamics is crucial in propelling the research of robotics, strategies of optimization in control designs, and even enhancing methodologies of design in future applications.

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