

Analysis for Best Configuration for Composite Wall Layers and Prediction of Effect on Temperature for Infinite Layers

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Abstract: *Heat Transfer is a very crucial concept in Thermodynamics and understanding the ability to control and use it accurately. Heat Transfer due to conduction has been already known to us but for more than one layer of material, the effect of physical properties of layers, would help understand how they affect heat conduction. We have worked on putting percentage change in thickness of layers for a certain fixed total length to understand the percentage change in temperature through the layers in one equation. This is to give us the best possible solution over thermal resistance of the layers. We used Fourier Law of Heat conduction as a starting point. We used Jupyter to simulate the results for the big dataset depending on the number of layers. We derived a single equation for percentage change in temperature with percentage change in thickness of layers. After plotting the results, we can use it to decide most optimum thickness for the layers depending on the required thermal resistance. We can clearly see that depending on the materials used in the layers the results have appropriate variation in temperature change. For the selected number of layers and materials we found the precise values of thickness of materials which would give us heat conduction as per requirement with accuracy.*

Keywords—*Conduction, Composite Wall, Fourier Law of Conduction, Python, Infinite Layers*

I. INTRODUCTION

Heat transfer is a fundamental concept that governs the movement of thermal energy from one system to another due to temperature differences. This phenomenon plays a pivotal role in our everyday lives, shaping how we perceive comfort, operate technology, and interact with our environment. Heat transfer occurs through three distinct modes: conduction, convection, and radiation. Each mode has unique characteristics, applications, and implications that collectively contribute to our understanding of energy dynamics. In this paper we have studied conduction in particular.

Conduction is the transfer of heat through a solid medium without any bulk movement of the material itself. It relies on the collision and transfer of kinetic energy between adjacent particles. When a region of a solid is heated, its particles gain kinetic energy, leading to increased vibration and more frequent collisions. As these energetic particles interact with neighboring particles, energy is transferred, causing a gradual increase in temperature across the material. Conduction is evident in everyday scenarios, from a metal spoon conducting heat from hot soup to your hand, to the efficient cooling of electronic devices using heat sinks and thermal conductive materials. Fourier's Law of Conduction describes the principle of rate of heat transfer through solid materials, as this law is specifically very useful in analysis of heat transfer through solid materials. Energy transfer according to Fourier's Law of heat conduction states that,

$$Q = -k \times A \times \left(\frac{dT}{dx}\right)$$

Conduction plays an important role in many applications like thermal insulation, temperature regulation. The materials used to achieve the goal are decided according to their thermodynamic properties but also other factors like cost and material availability.

II. METHODOLOGY

A. Theory

First, the problem statement was to find the most optimum configuration of the materials and their thickness, arranged in a stack like a composite wall, to offer thermal resistance according to the requirements. We approached the problem so as to find a generic solution for the composite wall problem with materials, number of layers, thickness, all being variables.

In our approach we have made certain assumptions to achieve conditions similar to ideal conditions. The process of heat transfer by conduction through slabs of materials at steady state with all slabs having equal dimensions. Heat loss due to convection and radiation is considered to be negligible. We have taken 1-D heat flow into consideration so loss from the sides is negligible.

According to Fourier's Law for heat transfer by conduction, the amount of energy that flows is a function of the temperature difference along the thickness and thermal properties of the material. Integrating the equation with respect to thickness and temperature we get a solution for temperature change across certain thickness. Our initial step was to carry out calculations for a single slab and then move onto multiple slabs. We integrated the equation with limits for the thickness variable as 1% change in thickness. Accordingly, the result will give us the percentage change in temperature. The final temperature derived after the first slab was then used as the initial temperature for the second. And similar to the first slab we find percentage change in temperature for 1% change in thickness for the second slab. After physically carrying out these steps for multiple slabs, we generated a generic solution. This solution would give a percentage change in temperature for a user-defined number of slabs and initial conditions. The derived equation follows all proportionalities correctly as well.

B. Mathematics

According to the Fourier's Law of heat conduction,

$$Q = -k \times A \times \left(\frac{dT}{dx}\right)$$

$$Q \times \int_{b_1}^{b_1+0.01b_1} dx = -k \times A \times \int_{T_1}^{T_1 - \frac{x}{100}T_1} dT$$

$$\therefore x\% = \left(\frac{Q}{k_1 \times A}\right) \left(\frac{0.01b_1}{T_1}\right)$$

— (1)

Similarly, we calculate for the second slab as well, we know the effect of 1% change in thickness on the first slab. The next step is calculating the effect of 1% change in thickness in second layer along with results of first layer,

$$Q = -k_2 \times A \times \left(\frac{dT_2}{db_2}\right)$$

$$\therefore (y\%)(1 - x\%) = \left(\frac{Q}{k_2 \times A}\right) \left(\frac{0.01b_2}{T_1}\right)$$

Now we put value of x% from (1), we get

$$y\% = Q \left(\frac{k_1}{k_2}\right) \left(\frac{0.01b_2}{k_1AT_1 - 0.01b_1Q}\right) \quad \text{— (2)}$$

Following this method we derived a general solution.

$$\epsilon_n\% = Q \left(\frac{\prod_{m=1}^{n-1} (k_m)}{k_n}\right) \left(\frac{0.01x_n \times b_n}{((\prod_{a=1}^{n-1} (k_a))A \times T) + ((\sum_{i=1}^{n-1} (\frac{-0.01x_i \times b_i}{k_i}))(\prod_{j=1}^{n-1} (k_j)))}\right)$$

and, a equation of temperature to allow us to measure,

$$T_{n+1} = T_n(1 - \epsilon_n\%)$$

where,

b ⇒ Thickness of a slab

k ⇒ Thermal conductivity of a slab

Q ⇒ Heat energy input to the system(W)

k_n ⇒ Thermal Conductivity of nth slab(W/m°C)

A ⇒ Area of the slabs(m²)

T ⇒ Initial Temperature(°C)

T_n ⇒ Temperature after nth slab(°C)

b_n ⇒ thickness of the nth slab(m)

ε_n ⇒ percentage change in temp after nth slab

C. Simulation Model

One of our objectives was to find the optimum configuration of slabs of any materials and thickness. Equation (1) allows us to calculate the percentage change in temperature for different materials. Using a dataset consisting of percentage change in thickness of the number of slabs defined by the user can be implemented in the equation to get the required output. We used Jupyter, a Python based software, which was simulated. From equation (1) values of Q,n,k_n,T,b_n are all user defined and taken from the user during runtime. The output gives the values of ε_n for all possible values percentage in thickness of all the slabs in the considered composite wall in a graphical form and also the lowest value of ε_n and corresponding values percentage change in thickness for all the slabs. The model has been given constraints to keep the results as realistic as possible. The major issue that arises in a model without constraints is that its results suggest completely removing the materials with higher thermal conductivities from the composite wall setup. This solution, although correct, is not ideal. Hence, the maximum change in thickness the model can calculate up to is 60% of the reference thickness'.

III. RESULT AND DISCUSSION

A. Validation of Equation

We needed to validate the equation derived with experimental data and so we used the existing Composite slab setup available to us. We had access to the previous records of the experiments conducted on that setup. Taking the values from these records we tested our Python model and compared it with hand calculations. The setup available has three layers in the Composite Wall. The materials in the Composite Wall are Steel, pressed Wood, Bakelite, starting from innermost to outermost layer.

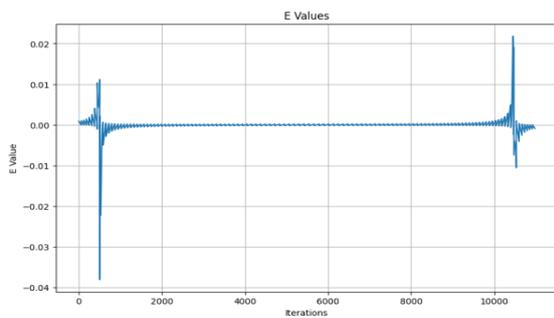
The values when we conducted the experiments one time, the input values for the Python model we got are,

$$Q = 38.48 \text{ W}; A = 0.0176\text{m}^2; T = 300^\circ\text{C};$$

$$k_1 = 1.38 \text{ W/m}^\circ\text{C}; k_2 = 0.268 \text{ W/m}^\circ\text{C}; k_3 = 0.846 \text{ W/m}^\circ\text{C}$$

$$b_1 = 0.015\text{m} \quad ; b_2 = 0.012\text{m} \quad ; b_3 = 0.012\text{m}$$

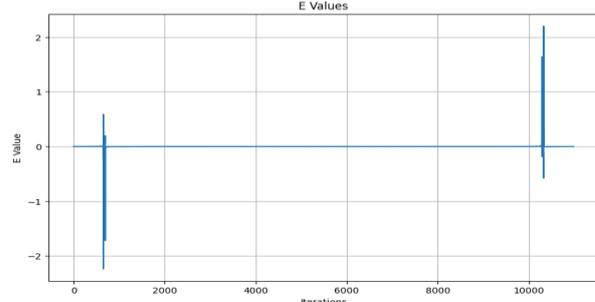
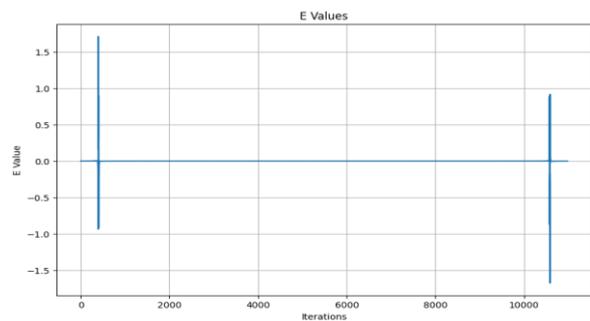
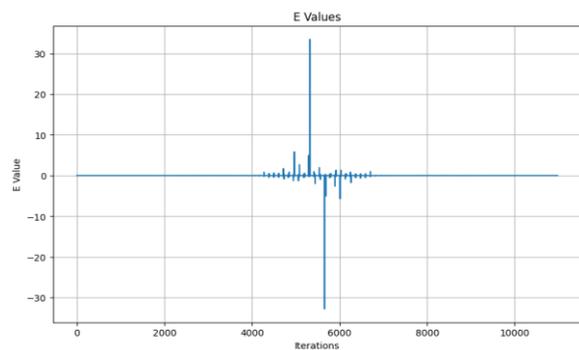
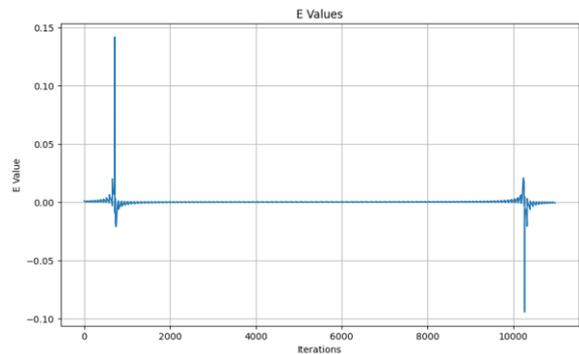
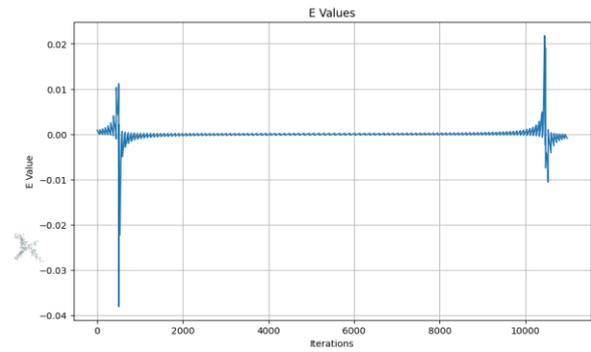
The Python model suggested 47% increase in original thickness for b_1 , 2% increase in original thickness for b_2 , 49% reduction in original thickness for b_3 layer. For this optimised solution the model predicted the reduction in temperature at the outermost surface to be ~7%. Hand calculations show that the temperature reduces by 11%. The reason for the close but not equal outcome is that the heat flow in the Python model in 1-D with no loss which is not the case in real life.

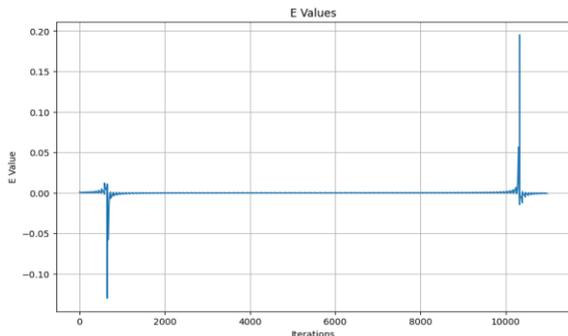
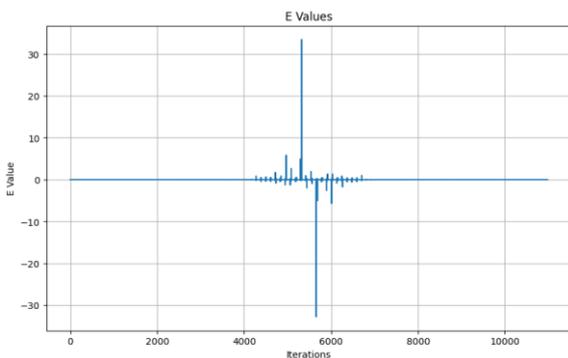
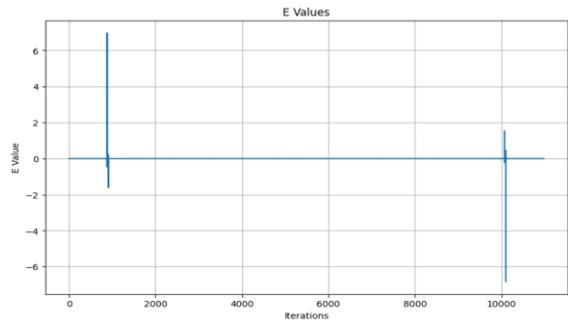
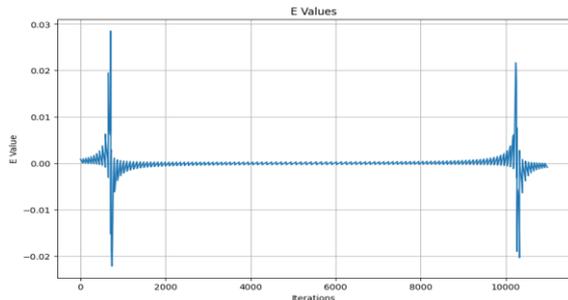


Graph 1: Iteration vs E values

B. Repeatability of the model

After verifying that the equation we derived, the next step is to test the model on multiple data records. As we had access to past records and readings of the Composite Wall experiments. The method to understand whether our equation stands is to see the plot of change in thickness to effect of temperature. For any given situation the graph should follow a similar pattern.





IV. CONCLUSION

Clear Objectives: The project had well-defined objectives, indicating that there were specific goals or outcomes the team aimed to achieve. These objectives were to find the best thickness configuration for composite slab and finding a general solution for this problem.

Correct Equation: As a result of our work, we derived an equation. This equation is described as being "correct," indicating that it accurately represents the relationships between thickness of layers and temperature.. This could be a mathematical formula or

a model representing the behaviour of composite slabs in relation to insulation.

Practical Applications: The derived equation has strong potential in practical applications. This means that the research findings can be applied in real-world scenarios to solve problems related to insulation, particularly in the context of composite slabs.

Optimizing Insulation: The research findings enable the determination of the best configuration of thickness for composite slabs. This optimization is aimed at achieving the most effective insulation in line with specific requirements, which could be related to energy efficiency or temperature control.

Wide Range of Applications: The solution isn't limited to a specific scenario. It's applicable to setups with anywhere from one to an unlimited number of layers in a composite wall. This flexibility implies that the results have a broad range of potential applications in various contexts and industries.

Python Model: To make the calculations required for finding the best insulation solution, the team developed a Python model. Python is a popular programming language known for its efficiency and versatility. This model is capable of performing these calculations efficiently and accurately, which implies that it's a valuable tool for practitioners in the field of composite slab insulation.

REFERENCES

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