A Study of Domination in Graphs and Application

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Abstract: Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (e.g., hospitals, fire stations) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problems involving finding sets of representatives, in monitoring communication or electrical networks, and in land surveying (e.g., minimizing the number of places a surveyor must stand in order to take height measurements for an entire region).

Key Words: Domination set, Networking, Biological networks, Minimal Dominating set.

INTRODUCTION

Domination in graphs has been extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics. The last 30 years have with one and spectacular growth of graph theory due to its wide application to discrete optimization problems, combinatorial problems and classical algebraic problem. It has wide range of physical, social and biological sciences; linguistic etc, the theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to the variety of new parameters that can be developed from the basic definition of domination.

The rigorous study of dominating set in graph theory began around 1960, In 1977 Cockayne and extensive survey of results know at that time about dominating set in graph. They have used the notation γ(G) for the domination number of graph, which has become very popular since then. The survey paper of Cockayne and Hedetnieme has generated a lot of interest in study of domination in graphs. In a span about twenty years after the survey more than 1200 research papers have been published on this topic. In this chapters describes about domination in sets, more about varieties of domination, common minimal domination etc. and application of domination in graphs.

Basic Definitions: We cover some basic definitions and notations here. We will define others when necessary. A graph G=(V, E) consists of a vertex set V and edge set E. Let n=|V(G)| denote the order of G. In a graph G, the degree of a vertex v is the number of vertices adjacent to v, denoted by deg(v) or d(v). The minimum and maximum degree of a graph are denoted by δ(G) and Δ(G) respectively. A vertex v is an isolated vertex if and only if d(v)= 0. A graph is connected if for every pair of vertices u and v there is a uv path in the graph. If G is connected, then the distance between two vertices u and v is the minimum length of a uv path in G, denoted by d(u,v). Let $N_G(v)$ denote the set of neighbours of a vertex vϵV(G), and let $N_G[v] = N_G(v) \cup \{v\}$ be the closed neighbourhood of v in G.

Dominating set: A dominating set D is a set of vertices such that each vertex of V-G is adjacent to some vertex in D. The minimum cardinality of such a set is called the domination number of G, γ(G).

APPLICATIONS IN GRAPH THEORY

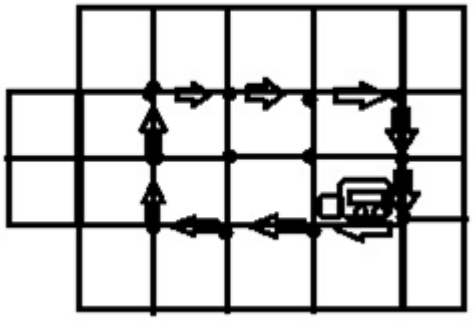
Domination in graphs has applications to several fields. Domination arises in facility location problems, where the number of facilities (eg.. hospitals. tire stations.) is fixed and one attempts to minimized the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that everyone is serviced. Concepts from domination also appear in problem invoking finding sets of representatives, in monitoring communication or electrical network and in land survey.

School bus routing:

Most school in the country provides school buses for transporting children to and from school. Most also operate under certain rules, one of which usually states that no students shall have to walk farther than, say one quarter km to a bus pickup point.

Thus, they must construct a route for each bus that gets within one quarter km of every student in its assigned area No bus ride can take more than some specified number of minutes. And limits on the number of students that a bus can carry at any one time.

Let us say that the following figure represents a street map of part of to be picked up by a school bus. Construct a route for a school bus that leaves the school gets within two blocks of every child and returns to the school.



Locating radar stations problem:

The problem was discussed by Berge. A number of strategic locations are to be kept under surveillance. The goal is to locate a radar for the surveillance at as few of these locations as possible. How a set of locations in which the radar stations are to be placed can be determined.

Modeling biological networks:

Using graph theory as a modeling tool in biological networks allows the utilization of the most graphical invariants in such a way that it is possible to identify secondary RNA (Ribonucleic acid) motifs numerically. Those graphical invariants are variations of the domination number of a graph. The results of the research carried out in show that the variations of the domination number can be used for correctly distinguishing among the trees that represent native structures and those that are not likely candidates to represent RNA.

Modeling social networks:

Dominating sets can be used in modeling social networks and studying the dynamics of relations among numerous individuals in different domains. A social network is a social structure made of individuals (or groups of individuals), which are connected by one or more specific types of interdependency. The choice of initial sets of target individuals is an important problem in the theory of social networks. In the work of Kelleher and Cozzens, social networks are modeled in terms of graph theory and it was shown that some of these sets can be found by using the properties of dominating sets in graphs.

Coding theory:

The concept of domination is also applied in coding theory as discussed by Kalbfleisch, Stanton and Horton and Cockayne and Hedetniemi . If one defines a graph, the vertices of which are the n-dimensional vectors with coordinates chosen from 1,..., p, p >1, and two vertices are adjacent if they differ in one coordinate, then the sets of vectors which are (n,p)covering sets, single error correcting codes, or perfect covering sets are all dominating sets of the graph with determined additional properties.

MULTIPLE DOMINATION PROBLEM

An important role is played by multiple domination. Multiple domination can be used to construct hierarchical overlay networks in peer-to-peer applications for more efficient index searching. The hierarchical overlay networks usually serve as distributed databases for index searching, e.g. in modern file sharing and instant messaging computer network applications. Dominating sets of several kinds are used for balancing efficiency and fault tolerance as well as in the distributed construction of minimum spanning trees. Another good example of direct, important and quickly developing applications of multiple domination in modern computer networks is a wireless sensor network.

CONCLUSION

The main aim of this project is to present the importance of graph theoretical ideas in various areas on science and engineering for domination in graphs theoretical concepts for the research. An overview is

presented especially to project the idea of graph theory. so, the graph theory section of each paper is given importance than to other sections. Researches may get some information related to graph theory and its application some ideas related to their research field. So I conclude this project by introducing an application of domination in real life situations.

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