Compactified Milne Space and Big Bang/Big Crunch Transition in M Theory.

Rakesh Sharma*#
*National Defence Academy Khadakwasla Pune Maharashtra India 411023
#Sunrise University Alwar Rajasthan 301026

ABSTRACT: In present paper analysis on various computational approaches to explain Big Bang/ Big Crunch transition is given. In the investigation a deep review of BB-BC model of compactified Milne space and M theory is given with mathematical analysis. The cosmic singularity plays a vital role to explain transition from big crunch to big bang state. In this work we are giving detailed discussion on singularity. Various models developed to explain the BB-BC transition are also suitable in other analytical work also.

KEY WORDS: Big-Bang, Big-Crunch, Milne Space, M Theory.

I.INTRODUCTION

The study of evolution of our universe is still in young age as so many unexplored and unexplained mysteries are yet to be solve. In theory Einstein's field equations are firstly inspired by the concept of static universe given by Newton but the study of various nebulas and galaxies by Hubble suggested that our universe is not static but expanding. Friedmann used the field equations proposed by Einstein with Λ_{CDM} model to explain the evolution of our universe. The work of Lemaitre was also in the same line and he proposed the idea of Big-Bang first time with mathematical approach. The Friedmann proposed that destiny of our universe depends on its density that whether it will expand forever or will contract after a particular limit. Recently various researches proposed that the universe will end in: (i) Big-Rip, (ii) Big- Freeze or (iii) Big-Crunch. In the present paper analysis on various computational approaches to explain Big Bang/ Big Crunch transition is given.

II.MATHEMATICAL BACKGROUND

In the data analysis of WMAP collaboration it was suggested that the universe is emerged from high density physical field called cosmic singularity [1,2]. For BB-BC modelling it is essential to understand the nature of singularity. The cosmic singularity is quite acceptable to explain BB-BC transition as it is supposed that either Big Bang originated from singularity and Big Crunch will be the end of universe in to the form of singularity. In fact according to Einstein's model of cyclic universe fit well with BB-BC transition. In the following figure 1 the difference of Inflationary model of universe and Cyclic model of universe is explained. The figure 2 present size of universe with respect to its age according to concept of open, closed or flat model of universe.

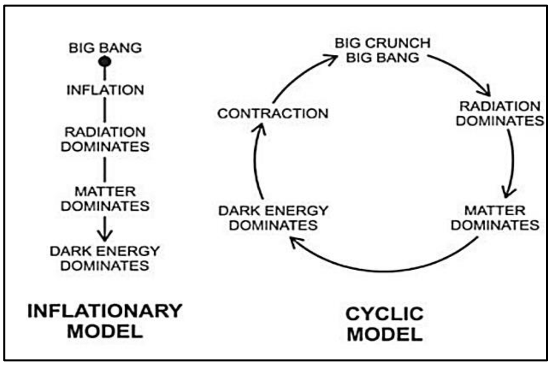

Fig: 1 A schematic overview of the inflationary theory vs the Cyclic Model Source: (Prof. G Venkataraman)

Since cosmic singularity consist of contraction and expansion phase hence it suggests the concept of cyclic universe [3,4,5,6,7,8,9]. Singularity can be explained by two different approaches such as General Relativity and String/M theory the de Sitter space in GR used to explain inflation on the other hand anti-de Sitter used to show contraction.

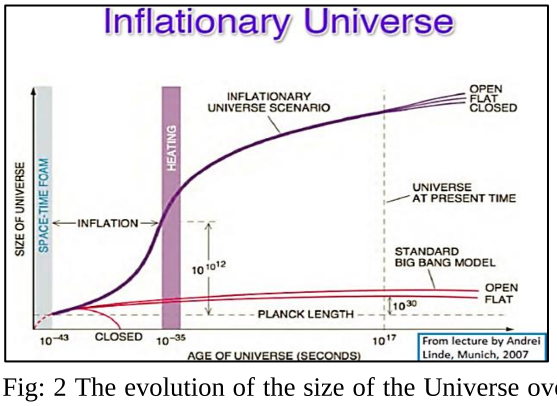

Fig: 2 The evolution of the size of the Universe over time Source: (M Strassler)

The Milne space in SM theory is fit well to explain BB-BC type singularity.

III.MILNE SPACE

In two dimensional Minkowski space two quadrants $\mathcal{M}$ are:

$$\mathcal{M}= \{(x^+, x^-) \in\}\mathbb{R}^2 | x^+x^- > 0 \vee x^+ = 0 = x^-\},\ \ x^\pm : x^0 \pm x^1 \qquad 1$$

Here x^0 and x^1 are time and space coordinates of Minkowski space respectively. The line element is defined as

$$ds^2 = -(dx^0)^2 + (dx^1)^2 \qquad 2$$

Now we introduce finite boost transformation B on $\mathcal{M}$ as

$$B : (x^+, x^-) \rightarrow (e^{2\pi r}x^+, e^{-2\pi r}x^-) \qquad 3$$

In above equation r define the boost and ratio B/ $\mathcal{M}$ called compactified Milne space $\mathcal{M}_C$. The compactified Milne space depend on r hence this gives uncountable values hence new coordinates t and Θ are introduced which may be defined as :

$$x^0 =: t cosh\theta, \qquad x^1 =: t sinh\theta \qquad 4$$

Hence from 2 and 4 the line element in $\mathcal{M}_C$ read as :

$$(ds)^2 = -(dt)^2 + t^2 d\theta^2) \qquad 5$$

Here $(t, \theta)\epsilon\mathbb{R}^1 \times \mathbb{S}^1$.$\mathcal{M}_C$ In 3 dimensional Minkowski space defined by the mapping

$$y^0(t,\theta) = t\sqrt{1+r^2},\ \ y^1(t,\theta) = rt \sin(\theta/r), y^2(t,\theta) = \ rt \cos(\theta/r) \qquad 6$$

And we have:

$$\frac{r^2}{1+r^2}(y^0)^2 - (y^1)^2 - (y^2)^2 = 0 \qquad 7$$

Above equation is of great significance as it gives common vertex of two cones at $(y^0, y^1, y^2) = (0,0,0)$ it is easy to verify that induced metric on 7 coincides with metric 5.

The compactified Milne space coincides with Minkowski space at t=0. The neighborhood of this point is not homeomorphic to the open circle in $\mathbb{R}^2$, thus $\mathcal{M}_C$ is not manifold but orbifold.

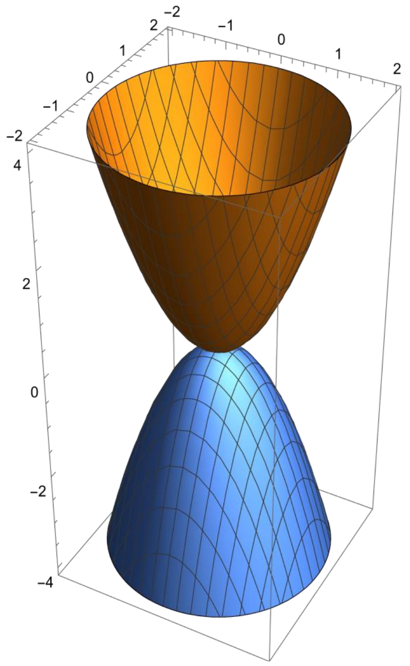

Fig 3: The compactified orbifold Milne space showing transition between BC-BB when cone is in 3d space for every x,y>0

The fig 3 shows BC-BB transition the upper cone shows Big Crunch lower shows Big Bang between two cones cosmic singularity is obtained when two coordinates $(y^0, y^1, y^2) = (0,0,0)$ coincides at vertex. Coefficient $d\theta^2$ disappear at $t \rightarrow 0$ thus one can use compactified Milne space to model BC-BB singularity.

IV.M THEORY

The M theory very often called String theory it was developed by Edward Witten [10]. Above discussion shows that biggest challenge for M theory is to show time dependent singularity. The direct product of $d-1$ dimensional flat Euclidian space R^{d-1} with two dimensional compactified Milne space-time , with line element 5 is presented. The value of θ is between 0 and θ_0 for t between $-\infty$ and $+\infty$.

V.BIG BANG - BIG CRUNCH SPACE TIME

The d+1 space time is direct product of d-1 *dimensional Euclidian space* R^{d-1} *and compactified Milne space* $\mathcal{M}_C$.The line element for $\mathcal{M}_C\times R^{d-1}$:

$$(ds)^2 = -(dt)^2 + t^2(d\theta^2) + d\overrightarrow{x^2}, \qquad 8$$

the value of θ is between 0 and θ_0 for t between $-\infty$ and $+\infty$. This equation is quite different then the equation 5. In equation 8, x is Euclidian coordinate on R^{d-1}, θ parameterized the compact dimension and t represent the time.

The line element in 8 is of great interest as it is flat locally and is not only an exact solution of d+1dimensional Einstein gravity but also is solution of higher dimensions with or without cosmological constant. In detail the toy model's winding string are discussed [11]. It is further to be noted that since translation invariance exist hence canonical (constant) momentum $\vec{p}$ remain constant of the motion. Thus the general solution for 8 is :

$$\vec{x} = \overrightarrow{x_0} + \frac{\vec{p}}{\mu\theta_0}\, sinh^{-1}(\mu\theta_0 t/|\vec{p}|), -\infty < t < \infty \quad 9$$

Here μ is string tension, according to above equation 9 particles can move smoothly through the singularity. This is very interesting as at early stage or at late time the particle with large mass almost crawl but at $t = 0$ particle's mass vanished and they start to speed up with velocity of light.

VI.CONCLUSION

The above discussion shows the behavior of particles in M theory the Einstein's gravity holds well by using the M theory in compactified Milne space even at t=0. The code developed at Mathematica is used to show the BC-BB transition. In further work we will bring out analysis of various coding methods to understand the BC-BB transition.

ACKNOWLEDGEMENT

The author gives his generous and sincere thanks to competent authorities of National Defence Academy. Principal Dr O P Shukla, Faculty and staff of Department of Physics NDA. My sincere thanks to Prof K N Pandey VC of Sunrise University, Management and Faculty of School of Science and Research Cell. This work has no conflict of interest with any other and not funded by any agency.

REFERENCE

[1] D. N. Spergel et al. [WMAP Collaboration], "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters", Astrophys. J. Suppl. 148 (2003) 175 [arXiv:astro-ph/0302209].
[2] N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, "The Cosmic Triangle: Revealing the State of the Universe", Science 284 (1999) 1481 [arXiv:astro-ph/9906463].
[3] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, "From big crunch to big bang", Phys. Rev. D 65 (2002) 086007 [arXiv:hep-th/0108187].
[4] P. J. Steinhardt and N. Turok, "A cyclic model of the universe", Science 296 (2002) 1436 [arXiv:hep-th/0111030].
[5] P. J. Steinhardt and N. Turok, "Cosmic evolution in a cyclic universe", Phys. Rev. D 65 (2002) 126003 [arXiv:hep-th/0111098].
[6] P. J. Steinhardt and N. Turok, "The cyclic model simplified", New Astron. Rev. 49 (2005) 43[arXiv:astro-ph/0404480].
[7] M. Bojowald, R. Maartens and P. Singh, "Loop quantum gravity and the cyclic universe", Phys. Rev. D 70 (2004) 083517 [arXiv:hep-th/0407115].
[8] M. Gasperini and G. Veneziano, "The pre-big bang scenario in string cosmology", Phys. Rept.373 (2003) 1 [arXiv:hep-th/0207130].
[9] J. E. Lidsey, D. Wands and E. J. Copeland, "Superstring cosmology", Phys. Rept. 337 (2000)343 [arXiv:hep-th/9909061].
[10] E Wittan, "String Theory Dynamics in Various Dimensions". Nucl.Phys.B443:PP85-126,(1995), arXiv:hep-th/9503124v2.
[11] N. Turok, M. Perry, P.J.Steinhardt, "M Theory Model of a Big Crunch/Big Bang Transition", Phys.Rev.D70:106004,2004. arXiv:hep-th/0408083.

APPENDIX: PICTORIAL CONCEPT OF CYCLIC UNIVERSE

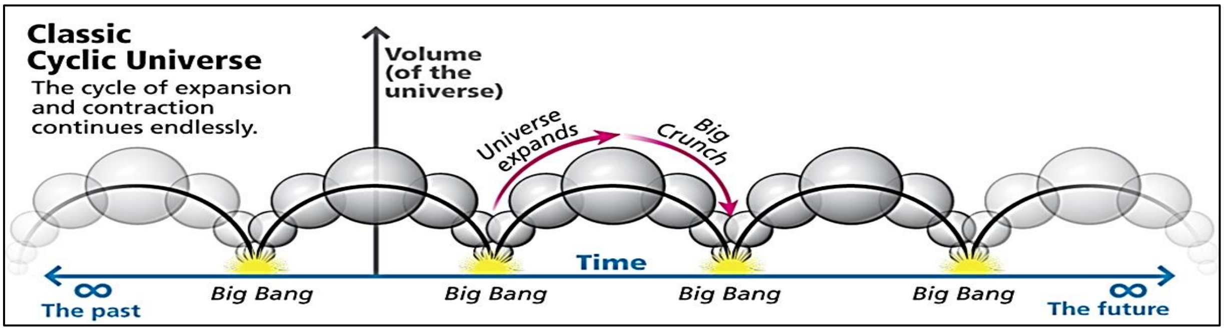

Fig4: Pic Source: Ali Ovgun (Övgün)