Analysis of Some Cordial Labelings in Bistar Graph

S.Bala(1), S. Saraswathy(2), K.Thirusangu(3)
1,2,3 Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai-73.

Abstract: In 1987[4],the concept of cordial labeling was introduced by Cahit. Let G be a simple graph. A function is defined as S: δ(G) → $\{0,1\}$ is said to be cordial labeling, if an induced function S^*: β(G) → $\{0,1\}$ defined by $S^*(bc) = |s(b) - s(c)|$, ∀ $bc \in \beta$(G) satisfies the conditions $|\delta_{S^*}(0) - \delta_{S^*}(1)| \leq 1$ and $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. In this paper, we investigate the existence of some cordial labelings in bistar graph.

Keywords: Graph labeling, Bistar graph, Triplicate graph, Cordial labeling.

1. INTRODUCTION

In 1967 [8], Rosa introduced the concept of graph labeling. Graph labeling is assigning an integer to the edges or vertices or to both on certain conditions. In 2023[3], Bala .et.al. introduced the concept of Extended triplicate graph of star ETG($k_{1,p}$).

In 2004 [9], the concept of product cordial labeling was introduced by Sundaram.et.al.,. Let G be a simple graph. A injective function S: δ(G) → {0,1 } is said to be product cordial labeling, if an induced function S^*: β(G) → {0,1} defined by $S^*(bc) =$ s(b).s(c) satisfies the condition $|\delta_S(0) - \delta_S(1)| \leq 1$ and $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a product cordial labeling is called as product cordial graph.

In 2006 [10], the concept of total product cordial labeling was introduced by Sundaram.et.al.,. Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A injective function S: δ(G) → {0,1 } is said to be total product cordial labeling, if an induced function S^*: β (G) → {0,1} defined by $S^*(bc) =$ S(b).S(c) satisfies the condition $\left|(\delta_S(0) + \beta_{S^*}(0)) - (\delta_S(1) + \beta_{S^*}(1))\right| \leq 1$. A graph which admits a total product cordial labeling is called a total product cordial graph.

In 2016 [7], the concept of Integer cordial labeling was introduced by Nicholas.et.al.,. Let G be a simple graph with p vertices and q edges. An injective function S: δ(G) → $\{\left(-\frac{p}{2}, \dots, +\frac{p}{2}\right) or - \left\lfloor\frac{p}{2}\right\rfloor, \dots\dots, + \left\lfloor\frac{p}{2}\right\rfloor\}$ as p is even or odd, said to be an Integer cordial labeling, if an induced function S^*: β (G) → {0,1} defined by $S^*(bc) = \begin{cases} 1 \ ; & if\ s(b) + s(c) \geq 0 \\ 0 \ ; & otherwise \end{cases}$ satisfies $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits an Integer cordial labeling is called as Integer cordial graph.

In 2018 [1], an idea of Divided square difference cordial labeling was initiated by Alfred Leo.et.al.,. Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, |p|\}$ said to be a divided square difference cordial labeling, if an induced function S^*: β (G) → {0,1} is defined by $S^*(bc) = \begin{cases} 1 \ ; \ if \left|\frac{s(b)^2 - s(c)^2}{s(b) - s(c)}\right| is\ odd \\ 0 \ ; \qquad otherwise \end{cases}$ satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits a divided square difference cordial labeling is called as divided square difference cordial graph.

In 2015[5], Abhirami.et.al., introduced the conceptualization of even sum cordial labeling. Let $G = (\delta(G), \beta(G))$ be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1,2,3, \dots, p\}$ is said to be Even sum cordial labeling. If an induced function S^*: β(G) → $\{0, 1\}$ is defined by $S^*(bc) = \begin{cases} 1 \ ; & if\ s(b) + s(c) is\ even \\ 0 \ ; & otherwise \end{cases}$ satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$. A graph which admits Even sum cordial labeling is called as Even sum cordial graph.

In 2012[2], the concept of extended triplicate graph of a path p_p was introduced by Bala.et.al.,. Let G be a path graph with p vertices and q edges. Let $\delta'(G) = \{b_1, b_2, b_3, \dots\dots, b_{p+1}\}$ and $\beta'(G) = \{c_1, c_2, c_3, \dots\dots, c_p\}$ be the vertex and edge set of a path p_p. For every $b_i \in \delta'(G)$,construct an ordered triple $\{b_i, b_i', b_i''; 1 \leq i \leq p+1\}$ and for every edge $b_i b_j \in \beta'(G)$, construct four edges $b_i b_j', b_j' b_i'', b_j b_i'\ and\ b_i' b_j''$ where $j = i + 1$, then the graph with this vertex set and edge set is called as Triplicate graph of p_p. It is denoted as $TG(p_p)$. Clearly, the triplicate graph $TG(p_p)$ is disconnected. Let $\delta(G) = \{b_1, b_2, b_3, \dots\dots, b_{3p+1}\}$ and $\beta(G) = \{c_1, c_2, c_3, \dots\dots, c_{4p}\}$ be the vertex set and edge set of $TG(p_p)$. If p is odd, include a new edge $\{b_{p+1}, b_1\}$ and if p is even, include a new edge $\{b_p, b_1\}$ in the edge set of $TG(p_p)$. This graph is called the Extended triplicate graph of the path p_p and it is denoted by $ETG(p_p)$.

Motivated by the above works, we investigate the existence of Product cordial labeling, Total Product cordial labeling, Integer cordial labeling, Divided square difference cordial labeling and Even sum cordial labeling in context of the Extended Triplicate of bistar graph.

2. MAIN RESULT

In this section, we investigate the existence of Product cordial labeling, Total Product cordial labeling, Integer cordial labeling, Divided square difference cordial labeling and Even sum cordial labeling in the context of Extended Triplicate of bistar graph.

2.1 STRUCTURE OF EXTENDED TRIPLICATE OF BISTAR GRAPH

Let G be a bistar graph $B_{(p,l)}$ .The triplicate of bistar graph with the vertex set $\delta'(G)$and edge set $\beta'(G)$is given by $\delta'(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_j \cup d_j' \cup d_j'' / 1 \leq i \leq p, 1 \leq j \leq l\}$ and $\beta'(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_j' \cup b_1''d_j' \cup b_1'd_j \cup b_1'd_j''/1 \leq i \leq p, 1 \leq j \leq l\}$. Clearly, Triplicate of bistar graph TG($B_{p,l}$) with $3(p+l+2)$ vertices and $4(p+l+1)$ edges is disconnected. To make this as a connected graph include a new edge $b'b_1'$ to the edge set of $\beta'(G)$.Thus, we get an Extended triplicate of bistar graph with vertex set $\delta(G) = \delta'(G)$ and edge set $\beta(G) = \beta'(G) \cup b'b_1'$ denoted by ETG($B_{p,l}$). Clearly, ETG($B_{p,l}$) has $3(p+l+2)$ vertices and $4(p+l)+5$ edges.

THEOREM 2.1: Extended triplicate of bistar graph is a product cordial graph.

PROOF: Extended triplicate of bistar graph ETG($B_{p,p}$) has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that: ETG($B_{p,p}$) is a product cordial graph.

Define an injective function $S: \delta(G) \to \{0, 1\}$ to label the vertices as below.

$s(b) = s(b'') = s(b_1') = 1$	$s(b') = s(b_1) = s(b_1'') = 0$
For, $1 \leq i \leq p$	$s(c_i) = s(c_i'') = s(d_i') = 0$
	$s(d_i) = s(d_i'') = s(c_i') = 1$

We get, $\delta_s(0) = \delta_s(1) = 3(p+1)$, Thus $|\delta_s(0) - \delta_s(1)| = |3(p+1) - (3(p+1))| \leq 1$.

It is clear, that the condition $|\delta_s(0) - \delta_s(1)| \leq 1$ is satisfied.

Now, Define an induced function $S^*: \beta(G) \to \{0, 1\}$ such that $S^*(bc) = S(b)S(c)$ to label the edges as below.

$S^*(bb_1') = S^*(b''b_1') = 1$	$S^*(b'b_1') = S^*(b'b_1'') = S^*(b'b_1) = 0$
For, $1 \leq i \leq p$	$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1d_i') = S^*(b_1''d_i') = 0$
	$S^*(bc_i') = S^*(b''c_i') = S^*(b_1'd_i) = S^*(b_1'd_i'') = 1$

we get, $\beta_{s^*}(0) = 4p+3$ and $\beta_{s^*}(1) = 4p+2$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p+3) - (4p+2)| \leq 1$

The condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

It is clear, that the vertex labelings and edge labelings satisfies the condition $|\delta_S(0) - \delta_S(1)| \leq 1$ and $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$.

Hence, Extended triplicate of bistar graph is product cordial graph.

EXAMPLE 2.1 : ETG($B_{3,3}$) and its product cordial labeling is shown in figure 1.

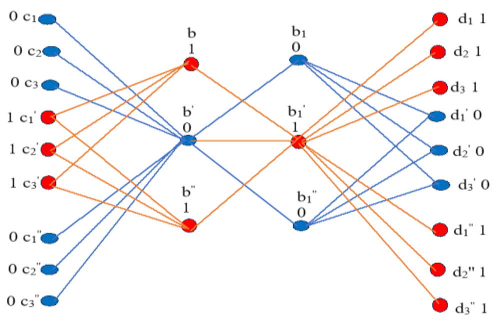

FIGURE – 1

THEOREM 2.2: Extended triplicate of bistar graph is a total product cordial graph.

PROOF: Extended triplicate of bistar graph $ETG(B_{p,p})$ has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that: $ETG(B_{p,p})$ is a total product cordial graph.

Define an injective function $S:\delta(G) \to \{0,1\}$ to label the vertices as below.

$s(b) = s(b'') = s(b_1') = 1$		$s(b') = s(b_1) = s(b_1'') = 0$
For, $1 \le i \le p$		$s(c_i) = s(c_i'') = s(d_i') = 0$
		$s(d_i) = s(d_i'') = s(c_i') = 1$

Here $\delta_s(0) = \delta_s(1) = 3(p+1)$.Thus $|\delta_s(0) - \delta_s(1)| = |3(p+1) - (3(p+1))| \le 1$

It is clear, that the condition $|\delta_s(0) - \delta_s(1)| \le 1$ is satisfied.

Now, Define an induced function $S^*:\beta(G) \to \{0,1\}$ such that $S^*(bc) = S(b)S(c)$ to label the edges as below.

$S^*(bb_1') = S^*(b''b_1') = 1$	$S^*(b'b_1') = S^*(b'b_1'') = S^*(b'b_1) = 0$
For, $1 \le i \le p$	$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1d_i') = S^*(b_1''d_i') = 0$
	$S^*(bc_i') = S^*(b''c_i') = S^*(b_1'd_i) = S^*(b_1'd_i'') = 1$

we get, $\beta_{s^*}(0) = 4p+3$ and $\beta_{s^*}(1) = 4p+2$

This implies $|(\delta_s(0) + \beta_{s^*}(0)) - (\delta_s(1) + \beta_{s^*}(1))|$

$$= |((3(p+1) + (4p+3)) - ((3(p+1) + (4p+2))| \le 1$$

It is clear, that the condition $|(\delta_{s^*}(0) + \beta_{s^*}(0)) - (\delta_{s^*}(1) + \beta_{s^*}(1))| \le 1$ is satisfied.

Hence, Extended triplicate of bistar graph is total product cordial graph.

EXAMPLE 2.2 : $ETG(B_{3,3})$ and its total product cordial labeling is shown in figure 2.

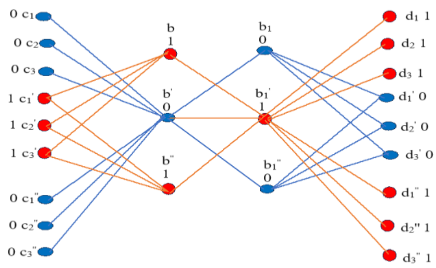

FIGURE – 2

THEOREM 2.3: Extended triplicate of bistar graph is an integer cordial graph.

PROOF: Extended triplicate of bistar graph $ETG(B_{p,p})$ has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that: $ETG(B_{p,p})$ is an integer cordial graph.

Define an injective function $S:\delta(G) \to \begin{cases} \left(-\frac{p}{2}, \dots\dots, +\frac{p}{2}\right) \; if\ p\ is\ even \\ \left(-\left\lfloor\frac{p}{2}\right\rfloor, \dots\dots, +\left\lfloor\frac{p}{2}\right\rfloor\right) \; if\ p\ is\ odd \end{cases}$ to label the vertices as below.

$s(b) = -2(p+1)$	$s(b_1) = -2(p+3)$	
$s(b') = 2(p+2)$	$s(b_1') = 0$	
$s(b'') = -2(p+2)$	$s(b_1'') = 2(p+1)$	
For, $1 \leq i \leq p$	$s(c_i) = 2i$	$s(d_i) = -(2i+5)$
	$s(c_i') = 2i-1$	$s(d_i') = -2i$
	$s(c_i'') = -(2i-1)$	$s(d_i'') = 2i+5$

Now, Define an induced function $S^*: \beta(G) \rightarrow \{0, 1\}$ such that

$S^*(bc) = \begin{cases} 1 & if\ s(b) + s(c) \geq 0 \\ 0 & otherwise \end{cases}$ to label the edges as below.

$S^*(b'b_1') = S^*(b'b_1'') = 1$	$S^*(bb_1') = S^*(b''b_1') = S^*(b'b_1) = 0$
. For, $1 \leq i \leq p$	$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1''d_i') = S^*(b_1'd_i'') = 1$
	$S^*(bc_i') = S^*(b''c_i') = S^*(b_1'd_i) = S^*(b_1d_i') = 0$

Here we get, $\beta_{s^*}(0) = (4p+3)$ and $\beta_{s^*}(1) = (4p+2)$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p+3) - (4p+2)| \leq 1$

It is clear, that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate of bistar graph is an Integer cordial graph.

EXAMPLE 2.3: ETG($B_{3,3}$) and its integer cordial labeling is shown in figure 3.

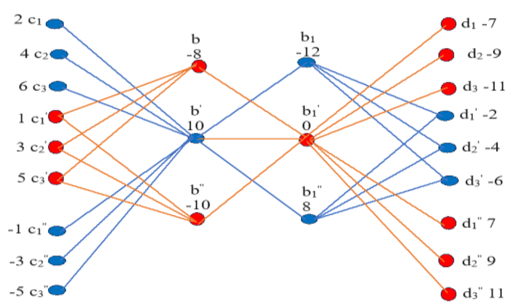

FIGURE – 3

THEOREM 2.4: Extended triplicate of bistar graph is a Divided square difference cordial graph.

PROOF: Extended triplicate of bistar graph ETG($B_{p,p}$) has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that:
ETG(
$B_{p,p}$) is a Divided square difference cordial graph.

Define the bijective function $S: \delta(G) \rightarrow \{1, 2, 3, \ldots\ldots, p\}$ to label the vertices as below.

$s(b) = 4p+5$	$s(b_1) = 2(2p+1)$	

$s(b') = 1$	$s(b_1') = 3$	$s(c_p') = 6(p+1)$
$s(b'') = 4(p+1)$	$s(b_1'') = 4p+7$	
For, $1 \leq i \leq p$	$s(c_i) = 2i$	$s(d_i) = (2i+3)$
	$s(c_i'') = 2(i+p)$	$s(d_i') = 2(i+2p+2)$
		$s(d_i'') = 2(i+p)+3$
For, $1 \leq i \leq p-1$	$s(c_i') = 2(i+2p)+7$	

Now, Define an induced function $S^*: \beta(G) \rightarrow \{0, 1\}$ such that

$S^*(bc) = \begin{cases} 1 & if \left|\frac{s(b)^2 - s(c)^2}{s(b)-s(c)}\right| \; is\ odd \\ 0 & otherwise \end{cases}$ to label the edges as follows.

$S^*(b'b_1') = S^*(b'b_1'') = S^*(bb_1') = S^*(b''c_p') = 0$	$S^*(b''b_1') = S^*(b'b_1) = S^*(bc_p') = 1$	
For, $1 \leq i \leq p$	$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1''d_i') = 1$	$S^*(b_1'd_i) = S^*(b_1d_i') = S^*(b_1'd_i'') = 0$
For, $1 \leq i \leq p-1$	$S^*(bc_i') = 0$	$S^*(b''c_i') = 0$

Here we get, $\beta_{s^*}(0) = 4p+3$ and $\beta_{s^*}(1) = 4p+2$

This implies $|\beta_{s^*}(0) - \beta_{s^*}(1)| = |(4p+3) - (4p+2)| \leq 1$

It is clear that the condition $|\beta_{s^*}(0) - \beta_{s^*}(1)| \leq 1$ is satisfied.

Hence, Extended triplicate of bistar graph is a Divided square difference cordial graph.

EXAMPLE 2.4: ETG$(B_{3,3})$ and its divided square difference cordial labeling is shown in figure 4.

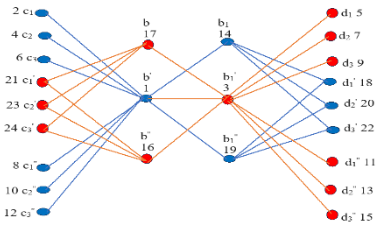

FIGURE - 4

THEOREM 2.5: Extended triplicate of bistar graph is a Even sum cordial graph.

PROOF: Extended triplicate of bistar graph ETG$(B_{p,p})$ has $6(p+1)$ vertices and $(8p+5)$ edges.

To show that: ETG$(B_{p,p})$ is a even sum graph.

Define the bijective function $S: \delta(G) \rightarrow \{1, 2, 3, \ldots\ldots, p\}$ to label the vertices as below.

$s(b) = 4p+5$	$s(b_1) = 2(2p+1)$	$s(c_p') = 6(p+1)$
$s(b') = 1$	$s(b_1') = 3$	

$s(b'') = 4(p+1)$	$s(b_1'') = 4p+7$	
For, $1 \le i \le p$	$s(c_i) = 2i$	$s(d_i) = (2i+3)$
	$s(c_i'') = 2(i+p)$	$s(d_i') = 2(i+2p+2)$
		$s(d_i'') = 2(i+p)+3$
For, $1 \le i \le p-1$	$s(c_i') = 2(i+2p)+7$	

Define an induced function $S^*\colon \beta(G) \to \{0,1\}$ such that $S^*(bc) = \begin{cases} 1 & ;\ if\ s(b)+s(c)\ is\ even \\ 0 & ;\ \qquad otherwise \end{cases}$ to label the edges as follows.

$S^*(b'b_1') = S^*(b'b_1'') = S^*(bb_1') = S^*(bc_p') = 0$	$S^*(b''b_1') = S^*(b'b_1) = S^*(b''c_p') = 1$	
For, $1 \le i \le p$	$S^*(b'c_i) = S^*(b'c_i'') = S^*(b_1''d_i') = 0$	$S^*(b_1'd_i) = S^*(b_1d_i') = S^*(b_1'd_i'') = 1$
For, $1 \le i \le p-1$	$S^*(bc_i') = 1$	$S^*(b''c_i') = 0$

Here we get, $\beta_{S^*}(0) = 4p+2$ and $\beta_{S^*}(1) = 4p+3$

This implies $|\beta_{S^*}(0) - \beta_{S^*}(1)| = |(4p+2) - (4p+3)| \le 1$

It is clear, that the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$ is satisfied.

Hence, Extended triplicate of bistar graph is a Even sum cordial graph.

EXAMPLE 2.5: ETG($B_{3,3}$) and its Even sum cordial labeling is shown in figure 5.

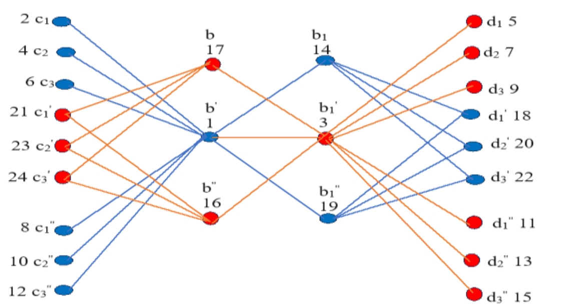

FIGURE – 5

CONCLUSION

In this paper, we have investigated that the Extended triplicate of bistar graph admits the Product cordial labeling, Total Product cordial labeling, Integer cordial labeling, Divided square difference cordial labeling and Even sum cordial labeling.

REFERENCE

[1] Alfred Leo . A , Vikramaprasad . R, Dhavaseelan. R, Divided square difference cordial labeling graph, International journal of mechanical engineering and technology (IJMET), vol.9,Issue 1(2018),pp.1137-1144.

[2] Bala.E, Thirusangu.K, Some graph labelings in Extended triplicate graph of a path p_n, International review in Applied Engineering research,Vol.1.No(2011),pp.81-92.

[3] Bala.S, Saraswathy.S, Thirusangu.K, Some labelings on extended triplicate graph of star, Proceedings of the international conference on recent in application of mathematics 2023,pp.302-305.

[4] Cahit.I, Cordial graphs:A Weaker version of graceful and harmonious graph. ARS combinatorial,1987, Volume 78,pp 179-199.

[5] Dhanaseelan R, Vikramaprasad R and Abhirami S, A New notion of cordial labeling graphs, Global Journal of pure and applied mathematics,vol.11(4),(2015), pp.1767-1774.

[6] Gallian. J, A dynamic survey of graph labeling, the Electronic journal of combinatories,1996-2005.

[7] Nicholas. T and Maya.P, Some results on integer cordial graph, Journal of progressive research in mathematics(JPRM),Vol 8(2016),Issue 1,pp.1183-1194.
[8] Rosa.A, On certain valuation of the vertices of graph. Theory of graphs(International symposium , Rome), July 1996,Gordan and Breach.N.Y and Dunad paris(1967),pp.349-335.
[9] Sundaram .M, Ponraj. R and Somasundaram. S, Product cordial labeling of graph, Bulletin of pure and applied science,2004, Volume 23, pp 155-163.
[10] Sundaram .M, Ponraj. R and Somasundaram. S, Total product cordial labeling of graphs, Bulletin of pure and applied science, Section E: Mathematics and statistics, 2006, Volume 25,Issue 1, pp 199-203.