An EOQ Model for Weibull Distributed Deteriorating Items with Stochastic Ram type Demand and shortages.

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Abstract: In the present paper, we discussed an EOQ (Economic Order Quality) model for deteriorating items with stochastic demand in which shortages are allowed with fully backlogged. Here rate of deterioration follows a two-parameter Weibull (Swedish engineer Wallodi Weibull) distribution. The demand pattern is assumed to be ramp type demand rate on to time with a stochastic error. The model is minimized to the total average cost by finding optimal values. The developed model is illustrated by a numerical example and finally the sensitivity analysis for the optimal solutions towards the changes in the values of key parameters has been presented.

Key Words: EOQ model, Weibull deteriorating items, ramp type demand and shortages.
Subject classification: AMS Classification No. 90B05

1. INTRODUCTION

In the classical inventory model, the depletion of inventory is caused by a constant demand and also due to deterioration. The rate of deterioration of such items like steel, hardware, glassware etc is very small that there is hardly any need to assume the effect of deterioration. But many items like fruits, vegetables, milk, fish, fashion goods, medicine, electronic goods, alcohol, food grains etc having deterioration considerably high and therefore this realistic factor should be considered in inventory modelling. It is noticed that

- Deterioration is defined as falling from a higher to a lower level in quality, it also simply implies a change, decay, obsolescence, collapse, spoilage, loss of utility or loss of marginal value of goods that results in a decrease of the usefulness of the original item
- Deterioration is always considered over time
- The rate of deterioration is nearly negligible for commodities like hardware, glassware, toys and steel, but it is very much effective for products such as fruits, vegetables, medicines etc
- The rate of consumption are very large for highly deteriorated items like gasoline, alcohol, turpentine etc, and they deteriorate continuously through the process of mortification.
- The deteriorated items like radioactive substances, electronic goods, grain, photographic film etc., deplete over time through a process of evaporation and also they deteriorate through a gradual loss of potential or utility from one time to another.

Therefore deterioration of items plays a vital role in the determination of an inventory model and has to be taken into account. These assumptions were followed by many researchers like Ghare and Schrader [1], Covert & Philip [2] , A. K.Jalan et al [3], Mandal B[4], Dr. Biswaranjan Mandal [5], P. R. Tadikamalla [6], R.KavithaPriya1 et al [7], A. Hatibaruah et al [8] and many more.

The assumption of constant demand rate may not be always appropriate for many inventory theories. For examples, milk items, vegetables, fruits, cosmetics etc have a negative impact on demand due to loss of confidence of consumers on the quality of such products for their age of inventory. Also we noticed that the demand of seasonal foods and garments is highly dependent on time. So it can be concluded that demand for items varies with respect to time. However, in order to match with real-life criteria, many authors have developed new types of inventory models with a variable demand rate. Also the acceptance of some constant demand rate is not reasonable for many inventory items such as electronic goods, fashionable garments, tasty foods, volatile liquids etc, as they fluctuate in the demand rate. Again many researchers like Giri-et-al [9], Mandal-and-Pal [10], Biswaranjan-

Mandal[11], [12], Datta-&-Pal [13] etc. were engaged to develop the inventory models assuming the demand of the items to be constant, linear trended, exponential increasing or decreasing, power demand pattern, alternating demand with time. Later we observed that such type of demands do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles, mobiles, computers etc. for which the demand increases as launched the items into the market, and after some times, it becomes constant. This ramp type demand is developed by Mishra-&-Singh [14], Mandal-&-Pal [15], Agarwal.et.al [16], Singh.et.al. [17], Meena Kumai et al [18] and many other researchers. Here we discussed the ramp type demand which is combination of a linear and quadratic function of time. As a result, ramp type demand rate on to time with a stochastic error has a prominent role in inventory control system.

The word shortage means a state or situation in which the needed items cannot be obtained in sufficient amounts or totally absent. It has a great importance for many models, especially when delay in payment is considered. When a shortage occurs but the company offers delay in payment, it can gain more orders from the customers. So shortages have an important role on optimization in inventory theory.

In view of the above sort of situations and facts, the present paper deals with an inventory model for Weibull deteriorating items having ramp type demand rate on to time with a stochastic error in which shortages are allowed and fully backlogged. The model is minimized to the total average cost by finding optimal values and finally it is illustrated by a numerical example along with the sensitivity analysis for the optimal solutions towards the changes in the values of system parameters.

2. ASSUMPTIONS AND NOTATIONS

2.1 Assumptions:

The present inventory model is developed on the basis of the following assumptions

i. Lead time is zero.
ii. Replenishment rate is infinite but size is finite.
iii. The time horizon is finite.
iv. There is no repair of deteriorated items occurring during the cycle.
v. Rate of deterioration follows a two parameter Weibull distribution. .
vi. The demand rate is a ramp type demand with stochastic error.
vii. Shortages are allowed and completely backlogged.

2.2 Notations:

The following notations are used in the proposed model:

i. Q : On hand inventory at time t.
ii. $\theta(t) = \alpha\beta t^{\beta-1}$, $t \geq 0$ is the two parameter Weibull distribution deterioration rate function where α $(0 < \alpha << 1)$ is a scale parameter and $\beta (> 0)$ is a shape parameter.
iii. t_1 : The time length in which the stock is completely diminished.
iv. T : The fixed length of each production cycle.
v. d_C : The deterioration cost per unit item.
vi. c_S : The shortage cost per unit item.
vii. h_C : Inventory holding cost per unit time.
viii. D(t) : Demand rate $D(t) = R(t) + \varepsilon$, where $R(t) = D_0[t - (t-\mu)H(t-\mu)]$, $D_0 > 0$ is assumed to be a ramp type function of time, and the well-known Heavysides' function $H(t-\mu)$ defined as

$$H(t-\mu) = \left| \begin{matrix} 1, t \leq \mu \\ 0, t > \mu \end{matrix} \right\rangle$$

so that the demand is positive throughout the demand, ε (stochastic error). Here the shape of the demand curve is deterministic while the scaling parameter representing the size of the market is random.. We assume that F(.) and f(.) represent the cumulative distribution and probability density function of ε, respectively having mean m and standard deviation δ.

ix. q(t) : The level of inventory $q(t) = \left| \begin{matrix} q_1(t), 0 \leq t \leq \mu \\ q_2(t), \mu \leq t \leq t_1 \\ q_3(t), t_1 \leq t \leq T \end{matrix} \right\rangle$
x. ATC : Average total cost per unit time.
xi. <ATC>: Expected average total cost per unit time.

3. MODEL DEVELOPMENT

Let Q be the total amount of inventory produced or purchased at the beginning of each period. The inventory level is depleted only due to demand and deterioration and ultimately falls to zero at t = t_1.

The shortages occur during time period [t_1 , T] which are completely backlogged. The differential equations pertaining to the above situations are given by

$$\frac{dq(t)}{dt} + \theta(t)q(t) = -D(t), 0 \le t \le t_1 \quad (3.1)$$

And $$\frac{dq(t)}{dt} = -D(t), t_1 \le t \le T \quad (3.2)$$

In this model, we assume $\mu < t_1$, and therefore the above two governing equations become

$$\frac{dq_1(t)}{dt} + \alpha\beta t^{\beta-1} q_1(t) = -(D_0 t + \varepsilon), 0 \le t \le \mu \quad (3.3)$$

$$\frac{dq_2(t)}{dt} + \alpha\beta t^{\beta-1} q_2(t) = -(D_0 \mu + \varepsilon), \mu \le t \le t_1 \quad (3.4)$$

And $$\frac{dq_3(t)}{dt} = -(D_0 \mu + \varepsilon), t_1 \le t \le T \quad (3.5)$$

The initial conditions are $q_1(0) = Q$, $q_2(t_1) = 0$ and $q_3(t_1) = 0$ (3.6)

Since $\alpha (0 < \alpha << 1)$, we ignore the terms $O(\alpha^2)$, then the solutions of the equations (3.3), (3.4) and (3.5) using (3.6) are given by the following

$$q_1(t) = Q(1 - \alpha t^{\beta}) - \varepsilon(t - \frac{\alpha\beta}{\beta+1} t^{\beta+1}) - \frac{D_0}{2}(t^2 - \frac{\alpha\beta}{\beta+2} t^{\beta+2}), \ 0 \le t \le \mu \quad (3.7)$$

$$q_2(t) = (D_0\mu + \varepsilon)[(t_1 - t)(1 - \alpha t^{\beta}) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})], \ \mu \le t \le t_1 \quad (3.8)$$

And $$q_3(t) = (D_0\mu + \varepsilon)(t_1 - t), \quad t_1 \le t \le T \quad (3.9)$$

Since $q_1(\mu) = q_2(\mu)$, we get the following expression of on-hand inventory from the equations (3.7) and (3.8), neglecting second and higher order powers of α ,

$$Q = (D_0\mu + \varepsilon)[t_1 - \mu + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - \mu^{\beta+1})] + \varepsilon[\mu + \frac{\alpha}{\beta+1}\mu^{\beta+1}] + \frac{D_0}{2}[\mu^2 + \frac{2\alpha}{\beta+2}\mu^{\beta+2}] \quad (3.10)$$

4. COST COMPONENTS

The total cost over the period [0, T] consists of the following cost components:

- Holding cost for carrying inventory **(HC)** over the period [0,T]

$$\text{HC} = c_h \int_0^{t_1} q(t)dt = c_h[\int_0^{\mu} q_1(t)dt + \int_{\mu}^{t_1} q_2(t)dt]$$

Putting the values of $q_1(t)$ and $q_2(t)$ from (3.7) and (3.8), and then integrating we get the following expression after neglecting second and higher order powers of α

$$\text{HC} = c_h \Big[Q(\mu - \frac{\alpha}{\beta+1}\mu^{\beta+1}) - \varepsilon(\frac{\mu^2}{2} - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\mu^{\beta+2}) - \frac{D_0}{2}(\frac{\mu^3}{3} - \frac{\alpha\beta}{(\beta+2)(\beta+3)}\mu^{\beta+3}) + (D_0\mu + \varepsilon)[t_1(t-\mu) - (\frac{t_1^2}{2} - \frac{\mu^2}{2}) - \frac{\alpha t_1}{\beta+1}(t_1^{\beta+1} - \mu^{\beta+1}) + \frac{\alpha}{\beta+2}(t_1^{\beta+2} - \mu^{\beta+2}) + \frac{\alpha}{\beta+1}(\frac{\beta+1}{\beta+2}t_1^{\beta+2} - \mu t_1^{\beta+1} + \frac{1}{\beta+2}\mu^{\beta+2})] \Big] \quad (4.1)$$

- Cost due to deterioration **(CD)** over the period [0,T]

$$\text{CD} = d_c[Q - \int_0^{t_1} D(t)dt] = d_c[Q - \{\int_0^{\mu}(D_0 t + \varepsilon)dt + \int_{\mu}^{t_1}(D_0\mu + \varepsilon)dt\}]$$

Integrating and neglecting second and higher order powers of α , we get the following

$$\text{CD} = d_c\alpha[(D_0\mu + \varepsilon)\frac{1}{\beta+1}t_1^{\beta+1} - \frac{D_0}{(\beta+1)(\beta+2)}\mu^{\beta+2}] \quad (4.2)$$

- Cost due to shortage **(CS)** over the period [0,T]

$$\text{CS} = c_s \int_{t_1}^{T} q_3(t)dt$$

Putting the value $q_3(t)$ from (3.9), and then integrating we get the following

$$\text{CS} = -c_s \frac{D_0\mu+\varepsilon}{2}(T-t_1)^2 \quad (4.3)$$

Thus the average total cost per unit time of the system during the cycle [0,T] will be

$$\text{ATC}(Q,\ t_1) = \frac{1}{T}\ \textbf{[HC+ CD-CS]}$$

$$= \frac{c_h}{T}\Bigg[Q(\mu - \frac{\alpha}{\beta+1}\mu^{\beta+1}) - \varepsilon(\frac{\mu^2}{2} - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\mu^{\beta+2}) - \frac{D_0}{2}(\frac{\mu^3}{3} - \frac{\alpha\beta}{(\beta+2)(\beta+3)}\mu^{\beta+3})$$

$$+ (D_0\mu+\varepsilon)[t_1(t-\mu) - (\frac{t_1^2}{2} - \frac{\mu^2}{2}) - \frac{\alpha t_1}{\beta+1}(t_1^{\beta+1} - \mu^{\beta+1}) + \frac{\alpha}{\beta+2}(t_1^{\beta+2} - \mu^{\beta+2})$$

$$+ \frac{\alpha}{\beta+1}(\frac{\beta+1}{\beta+2}t_1^{\beta+2} - \mu t_1^{\beta+1} + \frac{1}{\beta+2}\mu^{\beta+2})] \Bigg]$$

$$+ \frac{d_c\alpha}{T}[(D_0\mu+\varepsilon)\frac{1}{\beta+1}t_1^{\beta+1} - \frac{D_0}{(\beta+1)(\beta+2)}\mu^{\beta+2}] + c_s\frac{D_0\mu+\varepsilon}{2T}(T-t_1)^2 \quad (4.4)$$

Let us suppose that

$$f(\varepsilon) = \frac{1}{k_1 - k_0},\ k_1 \le \varepsilon \le k_0$$

$$= 0, \text{ elsewhere}$$

Where (m, σ) are mean and standard deviation.

Therefore the expected average total cost (EATC) is given by

$$\text{EATC} = \langle ATC(Q,t_1)\rangle = \frac{c_h}{T}\Bigg[\langle Q\rangle(\mu - \frac{\alpha}{\beta+1}\mu^{\beta+1}) - m(\frac{\mu^2}{2} - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\mu^{\beta+2}) - \frac{D_0}{2}(\frac{\mu^3}{3} - \frac{\alpha\beta}{(\beta+2)(\beta+3)}\mu^{\beta+3})$$

$$+ (D_0\mu+m)[t_1(t-\mu) - (\frac{t_1^2}{2} - \frac{\mu^2}{2}) - \frac{\alpha t_1}{\beta+1}(t_1^{\beta+1} - \mu^{\beta+1}) + \frac{\alpha}{\beta+2}(t_1^{\beta+2} - \mu^{\beta+2})$$

$$+ \frac{\alpha}{\beta+1}(\frac{\beta+1}{\beta+2}t_1^{\beta+2} - \mu t_1^{\beta+1} + \frac{1}{\beta+2}\mu^{\beta+2})] \Bigg] + \frac{d_c\alpha}{T}[(D_0\mu+m)\frac{1}{\beta+1}t_1^{\beta+1} - \frac{D_0}{(\beta+1)(\beta+2)}\mu^{\beta+2}]$$

$$+ c_s\frac{D_0\mu+m}{2T}(T-t_1)^2 \quad (4.5)$$

where the expected value of Q is given by

$$\langle Q\rangle = (D_0\mu+m)[t_1 - \mu + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - \mu^{\beta+1})] + m[\mu + \frac{\alpha}{\beta+1}\mu^{\beta+1}] + \frac{D_0}{2}[\mu^2 + \frac{2\alpha}{\beta+2}\mu^{\beta+2}] \quad (4.6)$$

The necessary condition for minimization of the cost is $\frac{d\langle ATC(t_1)\rangle}{dt_1} = 0$

This gives $c_d\alpha t_1^{\beta} + c_h(t_1 + \frac{\alpha\beta}{\beta+1}t_1^{\beta+1}) + c_s(t_1 - T) = 0 \quad (4.7)$

Let $g(t_1) = c_d\alpha t_1^{\beta} + c_h(t_1 + \frac{\alpha\beta}{\beta+1}t_1^{\beta+1}) + c_s(t_1 - T)$

Since $g(0) < 0$ and $g(T) > 0$, then there exits one solution $t_1 = t_1^* \in (0,T)$ of the equation (4.7).

For minimum, the sufficient condition $\frac{d^2\langle ATC(t_1)\rangle}{dt_1^2} > 0$ would be satisfied.

The optimal values $\langle Q^*\rangle$ of <Q> and $\langle ATC^*\rangle$ of <ATC> are obtained from the expressions (4.6) and (4.5) by putting the value $t_1 = t_1^*$.

5. SOME SPECIAL CASES:

(a). Absence of deterioration :

If the deterioration of items is switched off i.e. α=0 , then the expressions (4.6) and (4.5) of average on-hand inventory(<Q>) and expected average total cost per unit time (<ATC(1 *t*)>) during the period [0,T] become

$$< Q >= (D_0\mu + m)(t_1 - \mu) + m\mu + \frac{D_0}{2}\mu^2 \tag{5.1}$$

And $$\text{EATC} = < ATC(Q,t_1) >= \frac{c_h}{T}\Big[[(D_0\mu + m)(t_1 - \mu) + m\mu + \frac{D_0}{2}\mu^2]\mu - \frac{m\mu^2}{2} - \frac{D_0\mu^3}{6} + (D_0\mu + m)[t_1(t-\mu) - (\frac{t_1^2}{2} - \frac{\mu^2}{2})\ ]\Big] + c_s\frac{D_0\mu + m}{2T}(T - t_1)^2 \tag{5.2}$$

The equation (4.7) becomes

$$c_h t_1 + c_s(t_1 - T) = 0 \text{ or } t_1 = \frac{c_s T}{c_h + c_s} \tag{5.3}$$

This gives the optimum value of t_1.

(b). Constant deterioration rate:

If the demand rate is constant in nature i.e. β=1, then the expressions (4.6) and (4.5) of expected average on-hand inventory (<Q>) and expected average total cost per unit time (<ATC (1 *t*)>) during the period [0,T] become

$$< Q >= (D_0\mu + m)[t_1 - \mu + \frac{\alpha}{2}(t_1^2 - \mu^2)] + m[\mu + \frac{\alpha}{2}\mu^2] + \frac{D_0}{2}[\mu^2 + \frac{2\alpha}{3}\mu^3] \tag{5.4}$$

And $$< ATC(Q,t_1) >= \frac{c_h}{T}\Big[< Q > (\mu - \frac{\alpha}{2}\mu^2) - m(\frac{\mu^2}{2} - \frac{\alpha}{6}\mu^3) - \frac{D_0}{2}(\frac{\mu^3}{3} - \frac{\alpha}{12}\mu^4) + (D_0\mu + m)[t_1(t-\mu) - (\frac{t_1^2}{2} - \frac{\mu^2}{2}) - \frac{\alpha t_1}{2}(t_1^2 - \mu^2) + \frac{\alpha}{3}(t_1^3 - \mu^3) + \frac{\alpha}{2}(\frac{2}{3}t_1^3 - \mu t_1^2 + \frac{1}{3}\mu^3)]\Big] + \frac{d_c\alpha}{T}[(D_0\mu + m)\frac{1}{2}t_1^{\beta+1} - \frac{D_0}{6}\mu^3] + c_s\frac{D_0\mu + m}{2T}(T - t_1)^2 \tag{5.5}$$

The equation (4.7) becomes

$$c_d\alpha t_1 + c_h(t_1 + \frac{\alpha}{2}t_1^2) + c_s(t_1 - T) = 0 \tag{5.6}$$

This gives the optimum value of t_1.

6. NUMERICAL ANALYSIS

To exemplify the above model numerically, let the values of parameters be as follows:

c_d = \$5 per unit; c_h = \$ 3 per unit c_s = \$15per unit;

$\alpha = 0.001$, $\beta = 2$, $D_0 = 100$, $\mu = 0.12$ and T = 1 year

Also we assume that a uniformly-distributed random demand component exhibited an error span of u= 20 with [k_0, k_1]=[10,20], and a mean m =20.

Solving the equation (4.7) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.833 year; $< Q^* >$ = 25.95 units and $< ATC^* >$ = Rs. 39.98

It is checked that this solution satisfies the sufficient condition for optimality.

For Special Cases:

Nature of deterioration	t_1^*	$< Q^* >$	$< ATC^* >$
Absence of deterioration	0.833	25.94	39.91
Constant deterioration rate	0.833	25.96	40.00

7. SENSITIVITY ANALYSIS AND DISCUSSION

We now study the effects of changes in the system c_d , c_h , c_s ; α , β , m and D_0 on the expected average on- hand inventory (< Q^* >) and expected average total cost per unit time (< ATC^* >) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in	
		< Q^* >	< ATC^* >
c_d	-50	0.009	- 0.04
	-20	0.004	- 0.02
	+20	- 0.004	0.02
	+50	- 0.009	0.04
c_h	-50	9.34	- 45.37
	-20	3.54	- 17.21
	+20	- 3.32	16.09
	+50	- 7.91	38.37
c_s	-50	-14.69	- 14.36
	-20	-4.12	-04.02
	+20	2.94	04.87
	+50	6.06	15.92
α	-50	- 0.0002	-0.08
	-20	- 0.00006	-0.03
	+20	0.00006	0.03
	+50	0.0002	0.08
β	-50	0.009	0.07
	-20	0.002	0.02
	+20	- 0.001	- 0.02
	+50	- 0.006	- 0.05
m	-50	- 32.12	- 31.32
	-20	- 12.85	- 12.53
	+20	12.85	12.53
	+50	32.12	31.32
, D_0	-50	- 17.88	- 18.68
	-20	- 7.15	- 7.47
	+20	7.15	7.47
	+50	17.88	18.68

Analyzing the results of table A, the following observations may be made:

(i) The optimum expected average on-hand inventory (< Q^* >) increase or decrease with the increase or decrease in the values of the system parameters c_s , α , m and D_0 . On the other hand < Q^* > increase or decrease with the decrease or increase in the values of the system parameters c_d , c_h and β . The results obtained show that < Q^* > is very highly sensitive to changes in the value of parameters m ; moderate sensitive for the parameters c_h , c_s and D_0 ; and less sensitive to the changes of parameters c_d , α and β .

(ii) The optimum expected average total cost (< ATC^* >) increase or decrease with the increase or decrease in the values of the system parameters, c_d , c_h , c_s ; α , m and 0 D . On the other hand < ATC^* > increase or decrease with the decrease or increase in the values of the system parameters β . The results obtained show that < ATP^* > is very highly sensitive to changes in the value of parameters h c and m; moderate sensitive for s c and 0 D ; and less sensitive to the changes of parameters c_d , α .and β .

From the above analysis, it is seen that m is very sensitive parameter in the sense that any error in the estimation of this parameter result in significant error in the optimal solution. Hence estimation of this parameter needs adequate attention.

CONCLUDING REMARKS

In the present paper, we consider an EOQ inventory model where the inventory is depleted not only by ramp type demand, but also by two-parameter Weibull distribution deterioration. The stochastic demand pattern is assumed to be ramp type with a stochastic error. Shortages are fully backlogged. The model is minimized to the expected average total cost by finding optimal values. This model can be further extended considering trade credits, two warehouses under shortages which are partially backlogged.

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