Dynamic and Gyroscopic stability of spin-stabilized projectile with Modified Point Mass Trajectory Modeling

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Abstract: The paper presents the study of Dynamic and Gyroscopic stability of a spin stabilized projectile using Modified Point Mass Model (MPMM). In the developed Trajectory Model Physical properties and aerodynamic coefficients of a typical M107 projectile is used. The result parameters, such as spin rate, Gyroscopic stability factor and Dynamic stability factor of the projectile are shown and compared for high angle and low angle launch configurations for different Muzzle velocities. A conclusion is drawn, highlighting the important points based on the discussions.

Keywords: Gyroscopic Stability, Dynamic Stability, Aerodynamic Coefficients, Trajectory Model

I. NOMENCLATURE

$\overrightarrow{\alpha_R}$ yaw of repose approximation vector
$\vec{V}$ Vector velocity of the projectile with respect to the earth fixed coordinate system
$\vec{v}$ Vector velocity of the projectile with respect to the air
t Time
ρ Air density
d Projectile reference diameter
S Projectile reference area S
m Projectile mass
Ix Axial Moment of Inertia
Iy Lateral Moment of Inertia
C_D Drag force coefficient
$C_{L\alpha}$ Lift force coefficient
$C_{Np\alpha}$ Magnus force coefficient
$C_{l\delta}$ Rolling moment coefficient
δ_F Fin cant angle
C_{lp} Spin damping moment coefficient
$C_{Mq} + C_{M\dot{\alpha}}$ Pitch damping moment coefficient
$C_{Mp\alpha}$ Magnus moment coefficient
$C_{M\alpha}$ Pitching moment coefficient
$\vec{g}$ Vector acceleration due to gravity
$\vec{\Lambda}$ Vector Coriolis acceleration

II. INTRODUCTION

Any projectile symmetric or non-symmetric has three forms of stability: static, dynamic and gyroscopic and they are measured by their corresponding stability factors Ss, Sd & Sg respectively. A projectile is stabilized by using combination of any two of these or all three ways provided the associated stability factors are within their required range. When a body is statically stabilized, the small disturbing force makes the body to retain its original alignment. When a body is dynamically stabilized, the oscillatory motions induced by disturbances damps out until the body attains its initial alignment. When a body is Gyroscopically stabilized then under any influence of a disturbing force, its initial axial alignment is retained.

III. PROJECTILE CONFIGURATION AND LAUNCH CONDITIONS

The developed MPMTM model in this work uses the physical properties of a typical 155mm spin-stabilized M107 projectile. These physical properties used are listed in Table 1.

Parameters	Value
Caliber	D = 155 mm
Length	l = 902 mm
Total Mass	m = 43.2 kg
Center of Gravity from the Nose	XCG = 558 mm
Axial Moment of Inertia	Ix = 0.144 Kg. m2
Lateral Moment of Inertia	Iy = Iz = 1.216 Kg.m2

Table 1

Trajectory simulations have been performed for the following flight conditions:
Muzzle velocity, V0 = 750 m/sec
Elevation angle, $\theta = 20^0$ and 70^0.
According to McCoy definition [1], the initial muzzle spin rate has been estimated by

$$p = \frac{2\pi\, V0}{nd} \qquad (1.0)$$

IV. AERODYNAMIC COEFFICIENTS

The body non-dimensional aerodynamic coefficients of an M107 projectile has been taken from Ref [2] and are shown as a function of Mach number in Fig. 1.

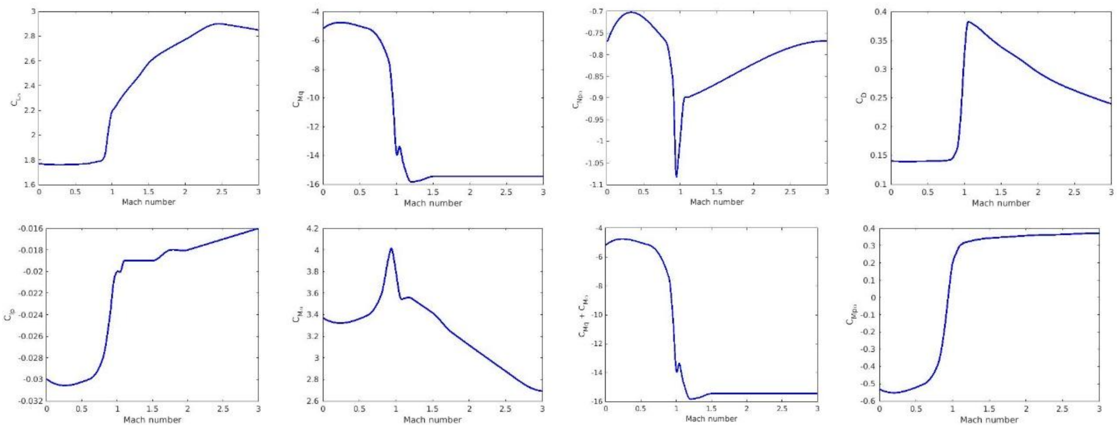


Fig. 1 Body aerodynamic parameters versus Mach number

V. MODIFIED POINT MASS TRAJECTORY MODEL

The modified point-mass model (MPMM), is a reduced form of the 6-DOF model with four degrees of freedom instead of six [3]. In the 6-DOF model all the angular motions are calculated but in the MPMM they are approximated as a total yaw of repose. In the trajectory calculations the additive influence of the drift producing aerodynamic forces, lift force and Magnus force is calculated using the yaw of repose which acts as the fourth degree of freedom.

The MPMM consists of three equations, first equation explaining the projectile's linear acceleration, second explaining the axial spin deceleration of the projectile and third approximating the projectile's yaw of repose [1]. The MPMM equations are as following:

$$\frac{d\vec{V}}{dt} = -\frac{\rho S\, C_D}{2m} v\vec{v} + \frac{\rho S\, C_{L\alpha} d}{2m} v^2 \overrightarrow{\alpha_R} + \frac{\rho S\, d\, C_{Np\alpha}}{2m}\, p(\vec{v} \times \overrightarrow{\alpha_R}) + \vec{g} + \vec{\Lambda} \qquad (1.1)$$

$$\frac{dp}{dt} = \frac{\rho S d^2 v}{2Ix} p C_{lp} + \frac{\rho S d v^2}{2Ix} \delta_F C_{l\delta} \qquad (1.2)$$

$$\overrightarrow{\alpha_R} = \frac{2Ixp(\vec{g} \times \vec{v})}{\rho S d v^4 C_{M\alpha}} \qquad (1.3)$$

In equation (1.1) explaining the projectile's linear acceleration is the sum of five accelerations due to the drag force, lift force, Magnus force, gravity force and the Coriolis force respectively.

In second equation (1.2) the axial spin deceleration of the projectile has two terms one is proportional to the aerodynamic spin damping moment and second term is proportional to the rolling moment due to canted fin of a finned projectile hence is not considered.

And the yaw of repose approximation vector $\overrightarrow{\alpha_R}$ is computed by the vector equation (1.3). This equation is based on the amount equality of an aerodynamic overturning moment and a gyroscopic moment caused by the rotation of the projectile along its longitudinal axis.

VI. GYROSCOPIC STABILITY

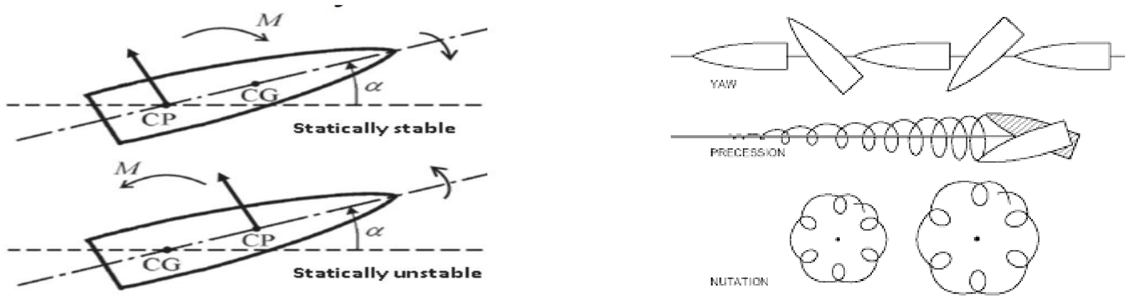
Gyroscopic stabilisation is the method used to stabilise most of the gun launched projectiles the another method is usage of fins at the base of the projectiles. Generally, a spin stabilized projectile is statically unstable as the center of pressure lies forward of the center of gravity due to its aerodynamic shape. Hence, when a projectile ejects from the muzzle of the Gun tube, aerodynamic forces acting about the center of mass causes an overturning moment which tends to increase the yaw angle and destabilizes the projectile. In the absence of any stabilizing factor, the moment would cause the projectile to tumble (see Fig. 2). Gyroscopic stability ensures that a projectile will not immediately tumble upon leaving the muzzle of the rifle [1]. To Gyroscopically stabilize the projectile, it requires to be spun very rapidly so that the aerodynamic over turning moment acting on it can be neutralised by its angular momentum.

The measure of the gyroscopic stability of a projectile is gyroscopic stability factor, Sg and McCoy [1] defines the gyroscopic stability factor in the following generalized form:

$$Sg = \frac{Ix^2 p^2}{2\rho Iy Sd V^2 C_{M\alpha}} \qquad (1.4)$$

Where, the condition 1.5 for gyroscopically stable projectile flight throughout the trajectory is as follow

$$Sg > 1 \qquad (1.5)$$



(a) (b)

Fig. 2

For a stable flight gyroscopic stability is a necessary requirement, but is not the only requirement. The other requirement is of dynamic stability which must be fulfilled.

VII. DYNAMIC STABILITY

Most of the projectiles when launched they have very small angle of yaw induced at the muzzle and the initial yaw rate, which causes yawing motion of nutation and precession (see Fig. 3 nutation and precession) known as Dynamic instability. Dynamic stability requires that both yawing motion of nutation and precession is damped out with time, which means that angle of yaw induced at the muzzle (the initial yaw) should decrease with time. And hence it is always undesirable to launch a projectile at a Mach number where it is dynamically unstable, because the nutation and precession motion can then quickly grow to a large amplitude [4]. The measure of the Dynamic stability of a projectile is Dynamic stability factor, Sd and is defined as following:

$$Sd = \frac{2C_{L\alpha} + (\frac{md^2}{Ix})C_{Mp\alpha}}{C_{L\alpha} - C_D - (\frac{md^2}{2Iy})(C_{Mq} + C_{M\dot{\alpha}})} \qquad (1.6)$$

Any spinning or non-spinning symmetric projectile must have to satisfy the generalized dynamic stability criteria 1.7 to fulfil the condition of Gyroscopic stability as well as Dynamic Stability.

$$\frac{1}{Sg} < Sd\,(2 - Sd) \qquad (1.7)$$

A stability diagram (Fig. 3) is utilized to illustrate different regimes of stability. A projectile is perfectly dynamically stable when Sd = 1 and when Sd < 1 the precession is undamped and if Sd > 1 the nutation is undamped. A sufficient spin is provided to the projectile to ensure it is dynamically as well as Gyroscopically stable.

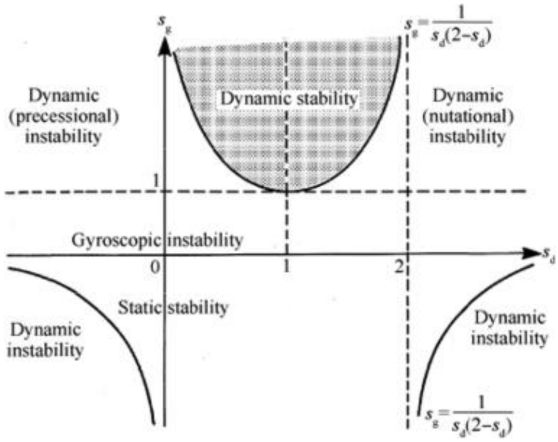
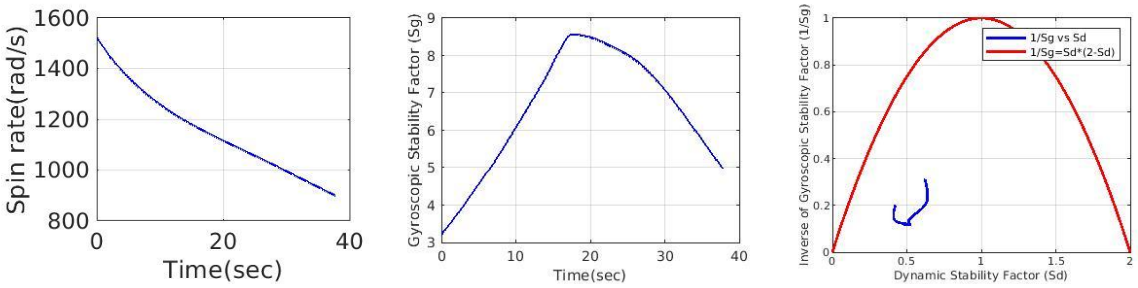


Fig. 3 Stability relations [4].

VIII. RESULTS AND DISCUSSION

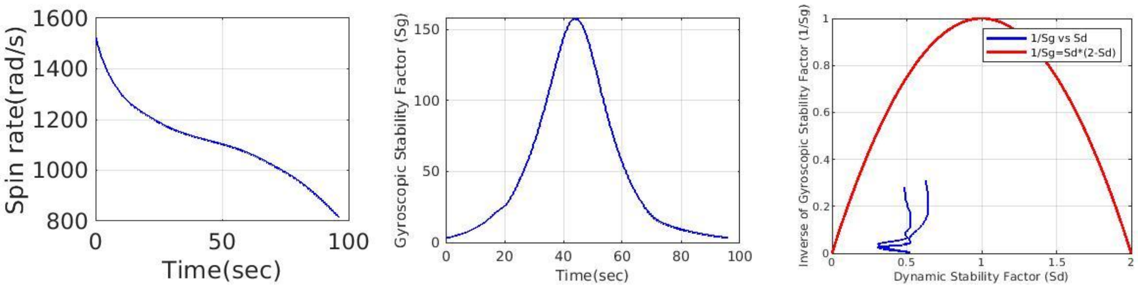
The trajectory parameters are computed for muzzle velocity of 750 m/s at quadrant elevation of 200 and 700 with the MPMM model and shown in Fig. 4(a) and 4(b) respectively. Fig. 4(a)-i. 4(b)-iv. shows the spin rate of the given projectile as a function of flight time, where the spin rate at the firing point is 1545 rad/sec and then decreases all flight due to the friction that acting on the projectile body until reached to 860 rad/sec at the impact point. Fig. 4(a)-ii. 4(b)-v. shows gyroscopic stability factor as a function of flight time for the entire flight duration. The gyroscopic stability factor starts to increase as the projectile leaves the muzzle and reaches its maximum at the apogee of the trajectory and again decreases until it hits the ground. The computational result presented in Fig. 4(a)-ii.

4(b)-iii. shows the projectile has been made gyroscopically stable by appropriate selection of the spin rate (Sg>1). Fig. 4(a)-iii. 4(b)-vi. depicts Gyroscopic stability as well as Dynamic Stability criteria, and it shows for both the elevation angles the projectile is stable gyroscopically as well as dynamically.



i. ii. iii.

Fig. 4(a)



iv. v. vi.

Fig. 4(b)

IX. CONCLUSIONS

An MPMM model is developed to examine and analyse the gyroscopic and dynamic stability of an M107 projectile in flight. The results show that the MPMM is highly capable for finding the stability of a projectile for high angle and low angle launch configurations for different Muzzle velocities covering its entire flight. And, MPMM can be advantageous at initial stage of projectile designing to estimate gyroscopic and dynamic stability without having to use 6- dof trajectory model, which is time and resource consuming.

REFERENCES

[1] R. McCoy, Modern Exterior Ballistics. Schier Publishing Ltd, 2012.

[2] Henry E. Hudgins, Jr., Aerodynamics, Dimensions, Inertial Properties, and Performance of Artillery Projectiles, Picatinny Arsenal, Dover, New Jersey, January 1977.

[3] R. Lieske and M. Reiter, Equations of motion for a modified point mass trajectory, Ballistic Research Laboratories, 1966.

[4] Textbook of ballistics and gunnery, 1987.