Potential Methodologies for Providing Robust Evidence of Fermat's Last Theorem

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***Abstract*—Pierre de Fermat wrote in the margin of his copy of an ancient Greek text that he had discover a truly marvelous proof of the proposition "$X^n + Y^n = Z^n$", but that the margin was too small to contain it. This claim however was never found and Fermat's Last Theorem remained unproven for more than 350 years. This proposition remains open problem in mathematics until 1994, when Prof. Andrew Wiles proved it that $X^n + Y^n \neq Z^n$, for all 'n' > 2. This paper delves into various intriguing facts and methodologies, presenting alternate perspective on this proposition. Three distinct approaches are highlighted, each presenting a unique perspective on this proposition.**

***Index Terms*—Fermat's Last Theorem, Optimization, Modular Pattern, Marvellous Solution**

I. INTRODUCTION

In 1994, a British mathematician Prof. Wiles with the help of his former student Richard Taylor proved that Fermat's Last Theorem ($X^n + Y^n \neq Z^n$) is true for all n > 2, but this proof involved sophisticated techniques from algebra and modular form [4,5]. The techniques that Prof. Wiles used did not be accessible in Fermat's time. Prof. Wiles breakthrough was built upon the previous work of mathematicians, Gerhard Frey, who recommended a connection between Fermat's Last Theorem and the Taniyama-Shimura-Weil conjecture (now known as the Modularity Theorem). Wiles showed that proving a special case of this conjecture would imply Fermat's Last Theorem, and after years of work, he completed the proof. The concepts used by Wiles in proving Fermat last theorem are now more integrated into research on modularity, Diophantine equation and solution of equations in number theory.

If we translate the statement of Fermat's words written on copy of an ancient Greek text it appears to be an enigmatic assertion composed of puzzling or cryptic terminology [3], "I *have a truly marvellous* proofs of this facts, but these margins are frankly too wide to contain it. I mean it's too much white space; you know, it is be like a huge dinner plate with a tiny appetizer on it. Now I do have proof, obviously but let's be real. People know how great I am at math and there's something they don't know how great I am at drawing cats. To that end, I have a truly marvellous cut picture and this margin is just big enough for it".

Many mathematicians believe that the claim made by Fermat having a proof was either an exaggeration or a mistake, some says that Fermat may have been referring to a proof for a specific, limited case of the theorem, the thinking of some mathematicians was that it's possible that this claim was more of a challenge than a genuine assertion of having a valid proof. Regardless of the reasons, if there remains even the slightest possibility of truth, this challenge remains unsolved.

Exploring this preposition, reveals several intriguing insights this theorem, one notable observation is the wonderful pattern in numbers that satisfies this proposition with some small variability. The second concept suggests that employing optimization technique will provide the closest combination in specific group for different values of 'n' and the third concept asserts that the equality holds under specific conditions and diverse patterns. This paper serves as an extension of my earlier work [1].

A. Modular Pattern of Numbers

Prof. Andrew Wiles proof for Fermat's Last Theorem itself is complete, its consequences continue to inspire and influence a wide array of mathematical fields, especially in the areas of number theory, algebraic geometry, and modular forms. There is no doubt about this proof. In this section, a remarkable numerical pattern associated with the proposition ($X^n + Y^n \neq Z^n$) which is visually appealing and may inspire someone to uncover something intriguing within its remarkable structure.

$X^n + Y^n \neq Z^n$

$K^n + (K + 1)^n \approx (K + 2)^n$

$K \geq 2p+1$

$p \geq 2$

$n \geq 3$

X = (n-1) (Mod n) Y = 0(Mod n) Z = 1(Mod n)

B. Optimization Approach

Using the concept of optimization, we can find the least error in the specific group for different values of 'n'.

As per Fermat's theorem

$X^n + Y^n \neq Z^n$

$A^n + B^n + k = Cn$ k – any natural number

`$p = (k/C^n) = 1 - (A/C)^n - (B/C)^n$

To identify the precise value for different powers of 'n' the concept of optimization has been employed, a mathematical programming problem is framed [2]. This approach facilitates the determination of the values of the variables at minimum value of 'p'.

Minimize $p = 1 - (A/C)^n - (B/C)^n$

Subject to

$1 - (A/C)^n - (B/C)^n \geq 0$

A, B, C (Natural Number)

A, B, C > 1

A, B, C < L

(L is any upper limit so that number of iterations and run time remains limited)

n	A	B	C	p
3	94	64	103	0.9151×10^{-6}
4	36	21	37	0.34×10^{-5}
5	16	17	19	0.30960×10^{-2}

C. Equality Series.

It is true that "$X^n + Y^n \neq Z^n$" for all values of n > 2 so equality never holds for this proposition for n > 2. The only possible way for the equality is to modify this proposition, the modification is as under:

$(x_1)^n + (x_2)^n + (x_3)^n + \dots (x_{m-1})^n + (x_m)^n = Cn$

$x_1 \leq x_2 \leq x_3 \leq \dots x_{m-1} \leq x_m$

n > 2 and m > 2

n	$(x_1)^n + (x_2)^n + (x_3)^n + \dots (x_{m-1})^n + (x_m)^n = Cn$
3	$1^3 + 6^3 + 8^3 = 9^3$
4	$510^4 + 2040^4 + 4624^4 + 5355^4 = 6001^4$
5	$437^5 + 989^5 + 1058^5 + 1081^5 + 1541^5 = 1656^5$

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