A Mathematical Approach to Language Learning Skill and Acquisition within a Population

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Abstract*— Language is the pillar of a society on which its members rely to communicate and connect among themselves. Growth and development of languages have given humans the freedom to choose and share in any language of their choice. There are many languages that we find ourselves surrounded with. To be able to communicate our messages properly, there is a constant need to upgrade our language skills and keep learning. With several languages out there, it is the need of the hour for a society to start learning a global language to be able to participate globally in an active manner. The BPI model has been introduced here to understand the learning process of a language within a population along with a mathematical model of the same. In the end, Numerical analysis has been done for validation of the assumptions.

Index Terms- Basic Influence Factor, Language Learning Skill, Mathematical Model, Numerical Simulation.

I. INTRODUCTION

Language is a medium which gives humans the power to share their ideas, emotions, thoughts, information. The development of a common language among a community played a pivotal role as it helped the members to easily connect with each other. With the progress of humanity, more languages and society came into existence. When we talk of a modern and global world, we see the need to connect globally and that leads to sharing and learning of a new language whenever and wherever required. To learn a language, there are four basic skills known as LSRW that is Listening, Speaking, Reading and Writing. When a child grows, LSRW becomes a process to learn a language. By listening to a particular language which includes words, phrases, tone, modularity, fluency, sentences, a child starts to speak. Speaking in the initial phase includes broken words, signaling, and body language. After that when it comes to Reading, a child gets aware of the letters, simple words with their meanings attached, and later on begins with simple sentences. Writing is the next skill for a child where comprehension begins. Among the four basic skills for communication in a language, two are receptive skills that is Listening and Reading whereas two are expressive skills that is Speaking and Writing. Learning a new language for a population which has a language of its own is an arduous process if not getting proper guidance. Learning there becomes a complicated task and through this paper, we are going to discuss the population which takes that task. To reduce the complication of learning a new language among a well known population of another language, BPI model has been introduced in this paper. The study is focused on helping the society to ease the process of learning a new language skill.

II. MODEL FORMULATION

We consider BPI (Beginner, Proficient, and Influencer) Model for language learning skill as learning is considered as an epidemic process here. The idea came from the basic SIR model [1]. We divided here the population into three classes that is Beginner, Proficient, and Influencer according to their skill. We subdivide the population into Beginner (B), Proficient (P) and Influencer (I) class such that $B(t) + P(t) + I(t) = 1$. Beginner individuals are recruited into the population by input rate μ. The Beginner population moves to the Proficient class P(t) at the rate β while they are trained by Influencer. After proper training the Beginner population becomes Proficient. The Proficient class contains those one who has perfect language skill but do not act as Influencer in the society. When Proficient actively participates in training Beginners they become an Influencer. Here the rate with which Proficient class becomes influencer is γ. Here natural emigration rate is μ of each class. The compartmental model which shows the mode of transmission knowledge is depicted in the figure1.

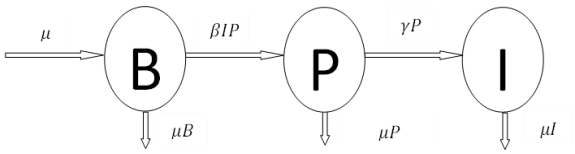

Fig.1

II. BASIC INFLUENCE FACTOR (F_O)

Basic Influence Factor is defined as number of Beginner influenced by an Influencer in their service period. The idea of defining the Basic Influence Factor here came from idea of Basic Reproduction Number for epidemic models [2]. If the value of F_O is less than and equal to one then population has no Influencer and if F_O is greater than one then population become

$$\frac{dB}{dt} = \mu - \beta PI - \mu B \quad (2.1)$$

$$\frac{dP}{dt} = \beta PI - (\mu + \gamma)P \quad (2.2)$$

$$\frac{dE}{dt} = \gamma P - \mu I \quad (2.3)$$

trained for a specific language skill [3]. The value of the Basic Influence Factor for the model is given by

$$F_0 = \frac{\beta\gamma}{\mu(\mu+\gamma)} \quad (3.1)$$

III. NUMERICAL ANALYSIS AND RELEVANCE TO REAL WORLD

From table 1, we can see that the initial Beginners population is very high in comparison to the Proficients and Influencers. Here, we considered a situation where Influencer Factor is very low that is less than one. The situation results in the reduction rate of Beginners and Influencers where as the number of Proficients increases with time. At t_0 1.8889, we see that there is a shift in Proficient number, it starts to decrease here which shows that the interest in a language becomes less for the population. The experiment shows that Influencer Factor plays a significant role in a population to train it for learning a new language. The above situation can clearly be made from the first graph of figure 2.

In table 2 we considered another situation in which the value of Influencer Factor is very high that is more than one. We considered the Beginners population as maximum with no Proficient available, but a few Influencers are there to train the Beginners. From the situation we find that with the time, the number of Beginners decrease and they become Proficient and at the same time, the number of Influencers also increases. For a certain time, the Proficient and number of Influencer increases which is the normal situation in the case but after that it starts to decrease which makes the model unique from the existing model in the field of Languages. We see that the number of Beginners again starts to increase which signifies that the interest in a particular language among the population has increased as the population starts moving towards learning the new language.

With time at the end, we have a certain population who has interest towards learning the language and a number of Proficients who are trained in the language with some Influencers in the population to continue to influence the Beginners towards learning a new language. This creates a healthy situation in the population from learning a new language perspective where we have Beginners who are eager to learn. The above situation can also be understood with the help of the second graph of figure 2.

S. No	t_0	B	P	I	
1	0	90.0000	0	1.0000	F_0=0.0048<1
2	10.0163	89.9986	0.0002	1.0000	
3	12.2596	89.9935	0.0007	0.9999	
4	15.5800	89.7687	0.0250	0.9977	
5	17.0672	88.8481	0.1232	0.9885	
6	19.3274	77.7813	1.1602	0.8869	
7	21.2431	54.2091	2.4942	0.7145	
8	23.8240	30.7578	2.5824	0.5717	
9	26.5598	16.6331	1.8889	0.4533	
10	29.2041	9.3597	1.2100	0.3444	
11	30.7207	6.8501	0.9042	0.2869	
12	37.5946	2.4682	0.2531	0.1217	
13	39.9854	1.9798	0.1701	0.0913	
14	43.2086	1.6554	0.1139	0.0679	
15	46.4318	1.4391	0.0762	0.0500	
16	50.7844	1.2996	0.0518	0.0371	
17	52.9608	1.2476	0.0428	0.0319	
18	55.1371	1.2047	0.0353	0.0274	
19	57.3743	1.1404	0.0242	0.0202	
20	60.7301	1.0799	0.0137	0.0127	
21	62.9673	1.0548	0.0094	0.0093	
22	68.6733	1.0258	0.0045	0.0049	
23	73.8100	1.0145	0.0026	0.0030	
24	75.6139	1.0120	0.0021	0.0025	
25	77.4177	1.0099	0.0018	0.0022	
26	82.8861	1.0037	0.0007	0.0009	
27	89.0848	1.0014	0.0003	0.0004	
28	93.7633	1.0007	0.0001	0.0002	
29	97.2850	1.0004	0.0001	0.0001	
30	100	1.0001	0.0000	0.0000	

Table 2

S.No.	t_0	B	P	I	
1	0	99.0000	0	1.0000	F_0= 6.3000>1
2	0.0529	95.6050	3.0792	1.0545	
3	0.3063	71.6344	23.6894	3.1717	
4	0.4236	52.4807	39.7222	5.7225	
5	0.5426	30.2848	57.3340	9.7315	
6	0.6225	17.4877	66.3132	13.1651	
7	0.6865	9.9332	70.5348	16.1916	
8	0.7717	3.8843	71.9806	20.3878	
9	0.8641	1.1076	69.8237	24.8825	
10	1.3268	0.0020	50.1861	43.4573	
11	2.2976	0.0013	24.2663	64.9883	
12	5.1041	0.0011	3.0157	74.6844	
13	7.0004	0.0012	0.7778	69.9837	
14	9.0577	0.0013	0.2185	63.7235	
15	11.3373	0.0015	0.0940	57.0674	
16	13.8946	0.0016	0.0705	50.3495	
17	16.8295	0.0019	0.0669	43.6076	
18	18.4585	0.0021	0.0665	40.2695	
19	20.2372	0.0022	0.0664	36.9220	
20	24.3260	0.0027	0.0663	30.2671	
21	26.7129	0.0031	0.0663	26.9665	
22	29.4188	0.0035	0.0662	23.6715	
23	32.4966	0.0040	0.0662	20.4273	
24	36.0959	0.0048	0.0661	17.2154	
25	40.4057	0.0059	0.0660	14.0573	
26	45.7218	0.0075	0.0658	10.9914	
27	52.5864	0.0101	0.0655	8.0650	
28	61.9945	0.0151	0.649	5.3811	
29	76.0388	0.0255	0.0639	3.1210	
30	100	0.0498	0.0620	1.5552	

Table 1

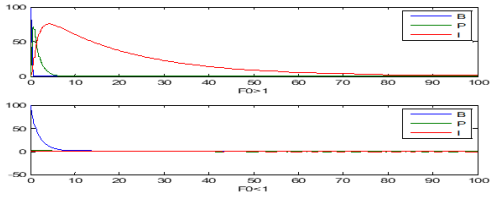

Fig.2

CONCLUSION

The BPI model can prove to be beneficial for a society where a new language is introduced in the population.

The Mathematical Model for learning a new language gives an idea to maximize the Proficients and Influencers, to raise awareness towards a new language in a society which will help the society to grow more on a global level. The learning process discussed here will ease the medium through which language gets transmitted in a population and hence, the population will be able to participate actively to communicate their message. The Numerical Analysis and Graphical Representation validates our results.

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