The Sum of Roots of n^{th} Degree of Polynomial Equations is Equal to the n×(n-1) Derivation of n^{th} Degree of Polynomial Equations

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Abstract: An Ordinary differential equation applications in a real life are used to calculate the momentum are flow of electricity, motion of an object. The differential equation applications have significance in both academic and real life. An equation denotes the relation between two quantity or two functions or two variables or set of variables or between two functions and roots. And also the Quadratic equation is usually not any better than using the quadratic formula, but very useful if we need to we write a quadratic function in vertex from. It also used to rewrite the equations of circles, ellipse and hyperbola's in standard form. In the chapter we have presented to found the number of roots of given quadratic equation in this we have discredited "The sum of roots of n^{th} degree of polynomial equation is equal to the n×(n-1) derivative of n^{th} degree of polynomial."

Key words: Quadratic equation, Derivative, Cubic equation, Double Derivative and n^{th} degree of polynomial.

I. INTRODUCTION

The derivative of a function describes the function instantaneous rate of change at a certain point. Another common interpretation is that the derivative gives us the slope of the line tangent to the functions graph at that point. Limits and Derivatives are the foundation stones of calculus. Just as limits help in measuring exponential functions, derivatives help study the nature of their change. Derivatives have addition, subtraction, multiplication, and division rules, which allow us to bypass the variable's value and yet find desired measurements. These applications are indirectly used in various fields such as Engineering, Astronomy, Physics, Information Technology, Biotechnology, Agro sciences, etc. A function is continuous in its motion. The meaning of derivative can be defined as the momentary change in the constant motion of an exponent. The derivative of a function is a small unit of it. It is denoted as dx. The unit dx can be an integer but never zero. A derivative is a fundamental part of calculus. It can be explained as the instant by the instant varying rate of change of the function of a variable to an independent variable.

When scientists want to study a dynamic system, i.e., a system whose components are constantly changing, they use calculus. For example, if they wish to study a particular molecule of water in the ocean, they will obtain its rate of change compared to another water molecule. This is the meaning of derivatives in calculus. This information can be further used by the scientists to apply in a differential equation and then use integration to study the behavior of the above system in different conditions. A quadratic equation is the equation of the second degree this means that it comprises at least one term that is squared. One of the standard formula for solving quadratic equation is $ax^2+bx+c=0$ by Harripersaud [1]. Here a, b and c are constants are numerical coefficients, Here 'x' unknown variable. Quadratic equation can be defined as a polynomial equation of a second degree, which implies that it comprises a minimum of one term it is squared. It is also called quadratic.

The solutions to the quadratic equation are the values of the unknown variable x, which satisfy the equation. These solutions are called roots or zeros of quadratic equations. İnce, M. [3]. The roots of any polynomial are the solutions for the given equation. Generally they are four methods of solving quadratic equations. They are factoring, completing the square, using quadratic formula, and taking the square root.

In this connection we are using different method to find the roots of quadratic equation by López, J. Robles, I. [4]. which is the first derivative of calculus. But there is no any difference between them. The roots of derivative is faster than the roots of quadratic equation. This is only new concept in this paper. This is the best verification to find the roots of the quadratic equation.

II. PRILIMINARIES

The quadratic equations are second-degree algebraic expressions and are of the form $ax^2+bx+c= 0$. In this connection they found sum of the roots and product of the roots in fast days. In other words a quadratic equation is an equation of degree '2'. They are many scenarios. Where a quadratic equation is used. Like the sum of the roots is equal to -b/a and the product of roots is equal to c/a. And generally we can find the roots towards $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, since $\Delta=0$, $\Delta<0$, $\Delta>0$. This called the opinion of roots like distinct, repeated, complex roots.
Now in this chapter we are going to found the number of roots towards the first derivative of differential equations. In our contribution to find the new formula is $2\frac{d}{dx}$ of quadratic equation is $ax^2+bx+c=0$ by Irianti, O. F., & Qohar [2]. The sum of roots $=-\frac{b}{a} = 2f'(x)$. And also we find upto cubic equations, as well as n^{th} degree of polynomials.

III. MAIN THEOREMS

Theorem: 1. If f(x) be a quadratic equation. Let a, b and c are real numbers. Such that the sum of roots of quadratic equation is equal to the twice first derivative of f(x) and also $\Delta=0$, $\Delta>0$, $\Delta<0$.

Proof: Let x_1 and x_2 are the roots of the quadratic equation is $ax^2+bx+c=0$.

Let the roots be $x=x_1$, $x=x_2$.

Here $x-x_1=0$ ----- (1)

And $x-x_2=0$ ----- (2)

The product of equation (1) and (2) is

$(x-x_1)(x-x_2)=0$

$x^2-xx_1-xx_2+x_1x_2=0$

$x^2-(x_1+x_2)x+x_1x_2=0$ ------(3)

On the other hand, the quadratic equation

$ax^2+bx+c=0$ ------- (4)

Which implies that, dividing on both sides by 'a'

$\frac{ax^2+bx+c}{a}=\frac{0}{a}$

$\frac{a}{a}x^2+\frac{b}{a}x+\frac{c}{a}=0$

$X^2-(-\frac{b}{a})x+(\frac{c}{a})=0$. (from equation 3)

Therefore $x_1+x_2=(-\frac{b}{a})$ and $x_1x_2=(\frac{c}{a})$

Sum of the roots $= (-\frac{b}{a})$

The derivative of quadratic equation is $f(x) = ax^2+bx+c$

$f'(x) = \lim_{h\to0}\frac{[f(x+h)-f(x)]}{h}$

$= \lim_{h\to0}\frac{[a(x+h)^2+b(x+h)+c-(ax^2+bx+c)]}{h}$

$= \lim_{h\to0}\frac{[ax^2+ah^2+2axh+bx+bh+c-ax^2-bx-c]}{h}$

$= \lim_{h\to0}\frac{[ah^2+2axh+bh]}{h}$

$= \lim_{h\to0}\frac{[h(ah+2ax+b)]}{h}$

$= \lim_{h\to0}[ah+2ax+b]$

$= a(0)+2ax+b$

$f'(x)= 2ax+b$

Since $f'(x) = 0$

$2ax+b=0$

Therefore, $x=-\frac{b}{a}$

$-\frac{b}{a}=2\left(-\frac{b}{2a}\right)$

Therefore, The sum of roots of quadratic equation is equal to the twice first derivative of f(x).

Examples:

Q) $x^2+4x+4=0$

Sol). a=1,b=4,c=4

Formula: $\frac{[-b\pm\sqrt{b^2-4ac}]}{2a}$

$\Delta= b^2-4ac = 4^2-4.1.4 = 16-16 = 0$

This quadratic equation is equal roots

$x=\frac{[-4\pm\sqrt{4^2-4.1.4}]}{2.1}=\frac{[-4\pm\sqrt{16-16}]}{2}=\frac{[-4\pm\sqrt{0}]}{2}=-\frac{4}{2}=-2$

$x^2+4x+4=0$

$x^2+2x+2x+4=0$

$x(x+2)+2(x+2)=0$

$(x+2)(x+2)=0$

$x+2=0$ (OR) $x+2=0$

$x=-2$ (OR) $x=-2$

Therefore, $x=-2,-2$

The first derivative of given equation with respect to 'x', we get

$f'^{(x)}=\left(\frac{d}{dx}\right)(x^2+4x+4)$

$f'(x)=2x+4$

Now $f'(x)=0$

$$2x + 4 = 0 \Rightarrow 2x = -4 \Rightarrow x = -\left(\frac{4}{2}\right) \Rightarrow x = -2$$

Therefore, The sum of roots is equal to the twice first derivative of f(x), we get

$$(-2) + (-2) = 2(-2) \Rightarrow -4 = -4$$

Q) $x^2 - 2x - 1 = 0$

Sol) a=1,b= -2,c= -1

Formula: $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(1)(-1) = 4 + 4 = 8$$

$$\Delta > 0$$

This quadratic equation is different roots.

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-2)\pm\sqrt{8}}{2} = 1 \pm \sqrt{2}$$

Therefore, $x = 1 + \sqrt{2}, 1 - \sqrt{2}$

The first derivative of given equation with respect to 'x', we get

$$f'(x) = \left(\frac{d}{dx}\right)(x^2 - 2x - 1)$$

$$f'(x) = 2x - 2$$

Now, $f'(x) = 0 \Rightarrow 2x - 2 = 0 \Rightarrow 2x = 2 \Rightarrow x = 1$

Therefore, the sum of roots is equal to the twice first derivative of f(x),we get

$$1 + \sqrt{2} + 1 - \sqrt{2} = 2(1) \Rightarrow 2 = 2$$

Q) $x^2 - 2x + 7 = 0$

Sol). a=1, b= -2, c=7

Formula: $\frac{(-b)\pm\sqrt{b^2-4ac}}{2a}$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(1)(7) = 4 - 28 = -24$$

$$\Delta < 0$$

The quadratic equation is imaginary roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{-24}}{2} = \frac{2 \pm i\sqrt{24}}{2}$$

$$x = 1 \pm i\sqrt{6}$$

Therefore, $x = 1 + i\sqrt{6}, 1 - \sqrt{6}$

The first derivative of given equation with respect to 'x', we get

$$f'(x) = \left(\frac{d}{dx}\right)(x^2 - 2x + 7)$$

$$F'(x) = 2x - 2$$

Now, $f'(x) = 0$

$$2x - 2 = 0$$

$$2x = 2$$

$$x = 1$$

Therefore, the sum of roots is equal to the twice first derivative of f(x), we get

$$2(1) = 1 + i\sqrt{6} + 1 - \sqrt{6}$$

$$2 = 2$$

Theorem:2. If f(x) be a cubic equation. Let a, b, c and d are real numbers. Such that the sum of roots of cubic equation is equal to the thrice double derivative of f(x) and also Δ=0, Δ<0, Δ>0.

Proof: Let x_1, x_2, x_3 are the roots of the cubic equation $ax^3 + bx^2 + cx + d = 0$

Let the roots be $x = x_1, x = x_2, x = x_3$

Here $x - x_1 = 0 \rightarrow (1)$

$x - x_2 = 0 \rightarrow (2)$

$x - x_3 = 0 \rightarrow (3)$

The product of equation (1),(2) and(3), we get

$$(x - x_1)(x - x_2)(x - x_3) = 0$$

$$(x^2 - xx_2 - xx_1 + x_1x_2)(x - x_3) = 0$$

$$x^3 - x_2x^2 - x_1x^2 + x_1x_2x - x_3x^2 + x_2x_3x + x_1x_3x - x_1x_2x_3 = 0$$

$$x^3 - (x_1 + x_2 + x_3)x^2 + (x_1x_2 + x_2x_3 + x_3x_1)x - x_1x_2x_3 = 0 \rightarrow (4)$$

On the other hand, the cubic equation

$$ax^3 + bx^2 + cx + d = 0 \rightarrow (5)$$

Which implies that diving on both sides by 'a', we get

$$\frac{ax^3 + bx^2 + cx + d}{a} = \frac{0}{a}$$

$$\frac{ax^3}{a} + \frac{bx^2}{a} + \frac{cx}{a} + \frac{d}{a} = 0$$

$$x^3 - \left(-\frac{b}{a}\right)x^2 + \frac{c}{a}x - \left(-\frac{d}{a}\right) = 0 \rightarrow (6)$$

Comparing equation (4) and (6), we get

$$x_1 + x_2 + x_3 = -\frac{b}{a}$$

$$x_1x_2 + x_2x_3 + x_3x_1 = \frac{c}{a}$$

$$x_1x_2x_3 = -\frac{d}{a}$$

Sum of the roots $= -\frac{b}{a}$

The double derivative of cubic equation is $ax^3 + bx^2 + cx + d = 0$

$$f'(x) = \lim_{h\to 0} \frac{f(x + h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{a(x + h)^3 + b(x + h)^2 + c(x + h) + d - (ax^3 + bx^2 + cx + d)}{h}$$

$$= \lim_{h\to 0} \frac{h(ah^2 + 3ax^2 + 3axh + bh + 2bx + c)}{h}$$
$$= \lim_{h\to 0} ah^2 + 3ax^2 + 3axh + bh + 2bx + c$$
$$= a(0)^2 + 3ax^2 + 3ax(0) + b(0) + 2bx + c$$
$$f'(x) = 3ax^2 + 2bx + c$$
$$f''(x) = \lim_{h\to 0} \left[\frac{f'(x+h) - f'(x)}{h}\right]$$
$$= \lim_{h\to 0} \left[\frac{3a(x+h)^2 + 2b(x+h) + c - (3ax^2 + 2bx + c)}{h}\right]$$
$$= \lim_{h\to 0} \left[\frac{3ah^2 + 6axh + 2bh}{h}\right]$$
$$= \lim_{h\to 0} \left[\frac{h(3ah + 6ax + 2b)}{h}\right]$$
$$= \lim_{h\to 0} 3ah + 6ax + 2b$$
$$= 3a(0) + 6ax + 2b$$
$$f''(x) = 6ax + 2b$$

Let $f''(x) = 0$

$$6ax + 2b = 0$$
$$2(3ax + b) = 0$$
$$3ax + b = 0$$
$$3ax = -b$$
$$x = -\frac{b}{3a}$$

Therefore, $-\frac{b}{a} = 3\left(-\frac{b}{3a}\right)$

$$-\frac{b}{a} = -\frac{b}{a}$$

Therefore, the sum of roots of cubic equation is equal to the thrice double derivative of f(x).

Examples:

Q) $x^3 - 6x^2 + 12x - 8 = 0$

Sol: Given that $x^3 - 6x^2 + 12x - 8 = 0$

$(x-2)(x^2 - 4x + 4) = 0$

$(x-2)(x-2)(x-2) = 0$

$(x-2)^3 = 0$

$x = 2,2$ and 2

This cubic equation is equal roots.

The first derivative of given equation with respect to 'x', we get

$$f'(x) = \frac{d}{dx}(x^3 - 6x^2 + 12x - 8)$$
$$f'(x) = 3x^2 - 12x + 12$$

Again derivative of given equation with respect to 'x', we get

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$f''(x) = \frac{d}{dx}(3x^2 - 12x + 12)$$
$$f''(x) = 3(2x) - 12$$
$$f''(x) = 6x - 12$$

Now f'(x) = 0

$6x - 12 = 0 \Rightarrow 6x = 12 \Rightarrow x = 2$

Therefore, the sum of roots of cubic equation is equal to the thrice double derivative of f(x)

$$2 + 2 + 2 = 3(2)$$
$$6 = 6$$

Q) $x^3 - 3x^2 + x + 1 = 0$

Sol: Given that $x^3 - 3x^2 + x + 1 = 0$

$(x-1)(x^2 - 2x - 1) = 0$

$x^2 - 2x - 1 = 0$

$a = 1, b = -2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4+4}}{2} = \frac{2}{2} \pm \frac{\sqrt{8}}{2} = 1 \pm \sqrt{2}$$

Therefore, $x = 1, 1+\sqrt{2}, 1-\sqrt{2}$

This cubic equation is different roots.

The first derivative of given equation with respect to 'x', we get

$$f'(x) = \frac{d}{dx}(x^3 - 3x^2 + x + 1)$$
$$f'(x) = 3x^2 - 6x + 1$$

Again derivative of given equation with respect to 'x', we get

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$f''(x) = \frac{d}{dx}(3x^2 - 6x + 1)$$
$$f''(x) = 3(2x) - 6(1)$$
$$f''(x) = 6x - 6$$

Now $f''(x) = 0$

$6x - 6 = 0$

$x = 1$

Therefore, the sum of roots of cubic equation is equal to the thrice double derivative of f(x)

$$1 + 1 + \sqrt{2} + 1 - \sqrt{2} = 3(1)$$
$$3 = 3$$

Q) $x^3 + 3x^2 + 2x + 6 = 0$

Sol: Given that

$x^3 + 3x^2 + 2x + 6 = 0$

$x^2(x+3) + 2(x+3) = 0$

$(x+3)(x^2+2) = 0$

$x = -3$ and $x^2 = -2$

$$x = \pm\sqrt{-2}$$
$$x = \pm i\sqrt{2}$$

$x = -3, i\sqrt{2}, -i\sqrt{2}$

Therefore, the first derivative of given equation with respect to 'x', we get

$$f'(x) = \frac{d}{dx}(x^3 + 3x^2 + 2x + 6)$$
$$f'(x) = 3x^2 + 6x + 2$$

Again derivative of given equation with respect to 'x', we get

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$f''(x) = \frac{d}{dx}(3x^2 + 6x + 2)$$
$$f''(x) = 3(2x) + 6(1)$$
$$f''(x) = 6x + 6$$

Now $f''(x) = 0 \Rightarrow 6x + 6 = 0 \Rightarrow x = -1$

Therefore, the sum of roots of cubic equation is equal to the thrice double derivative of f(x)

$$-3 + i\sqrt{2} - i\sqrt{2} = 3(-1) \Rightarrow -3 = -3$$

Theorem: 3. If $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$ (n^{th} degree of polynamial).Then the sum of roots is n^{th} degree of polynomial is equal to n*(the n-1 derivative of n^{th} degree of polynomial). [i.e., $f^{n-1}(x)$] of Δ=0, Δ<0,Δ>0

Proof: Let $x_1, x_2, x_3, \ldots\ldots, x_n$ are the roots of the n^{th} degree of polynomial

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

Let the roots $x = x_1, x_2, \ldots, x_n$

Here $x - x_1 = 0$

$x - x_2 = 0$

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$x - x_n = 0$

The product of above equations, we get

$$(x - x_1)(x - x_2) \ldots (x - x_n) = 0$$
$$x^n - (x_1 + x_2 + \cdots + x_n)x^{n-1} + \cdots + (x_1 x_2 \ldots x_n) = 0 \rightarrow (1)$$

On the other hand, the n^{th} degree of polynomial. Which implies that, diving on both sides 'a_n', we get

$$\frac{[a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0]}{a_n} = \frac{0}{a_n}$$
$$x^n + \frac{a_{n-1}}{a_n}x^{n-1} + \cdots + \frac{a_1}{a_n}x + \frac{a_0}{a_n} = 0 \rightarrow (2)$$

Comparing equation (1) and (2),we get

$$x_1 + x_2 + \cdots + x_n = -\frac{a_{n-1}}{a_n}$$
$$x_1 x_2 \ldots x_n = \frac{a_0}{a_n}$$

Sum of roots $= -\frac{a_{n-1}}{a_n}$

The (n-1) derivative of n^{th} degree of polynomial, we get

$$f^{n-1}(x) = \lim_{h \to 0} \frac{f^{n-2}(x + h) - f^{n-2}(x)}{h}$$
$$= n(n-1)\ldots(n-(n-1))a_n x^{n-(n-1)} + (n-1)(n-2)\ldots(n-(n-1))a_{n-1}x^{n-1-(n-1)}$$
$$= n(n-1)(n-2)\ldots 2.1.\, a_n x + (n-1)(n-2)\ldots 2.1.\, a_{n-1}x^0$$
$$f^{n-1}(x) = (n-1)(n-2)\ldots 2.1.\,[n a_n x + a_{n-1}]$$

Now $f^{n-1}(x) = 0$

$$(n-1)(n-2)\ldots 2.1.\,[n a_n x + a_{n-1}] = 0$$
$$n a_n x + a_{n-1} = 0$$
$$n a_n x = -a_{n-1}$$
$$x = -\frac{a_{n-1}}{n a_n}$$

Therefore, the sum of roots is n^{th} degree of polynomial is equal to n*(the n-1 derivative of n^{th} degree of polynomial).

$$-\frac{a_{n-1}}{a_n} = n\left(-\frac{a_{n-1}}{n a_n}\right)$$
$$-\frac{a_{n-1}}{a_n} = -\frac{a_{n-1}}{a_n}$$

Examples:

Q) $x^4 - 20x^2 + 64 = 0$

Sol : Given that $x^4 - 20x^2 + 64 = 0$

$$(x - 2)(x^3 + 2x^2 - 16x - 32) = 0$$
$$(x - 2)(x + 2)(x^2 - 16) = 0$$
$$(x - 2)(x + 2)(x - 4)(x + 4) = 0$$
$$x = 2, -2, 4, -4$$

formula :

sum of roots = n[(n-1) derivative of given equation]

Where n=4

Sum of roots =4[f'''(x)]

First derivative of given equation, we get

$$f'(x) = \frac{d}{dx}(x^4 - 20x^2 + 64)$$
$$f'(x) = 4x^3 - 20(2x)$$
$$f'(x) = 4x^3 - 40x$$

Double derivative of given equation, we get

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$f''(x) = \frac{d}{dx}(4x^3 - 40x)$$
$$f''(x) = 4(3x^2) - 40$$
$$f''(x) = 12x^2 - 40$$

Triple derivative of given equation, we get

$$f'''(x) = \frac{d}{dx}(f''(x))$$
$$f'''(x) = \frac{d}{dx}(12x^2 - 40)$$
$$f'''(x) = 12(2x)$$
$$f'''(x) = 24x$$

Now, f'''(x)= 0

$$24x = 0 \Rightarrow x = 0$$

sum of roots = 4[f'''(x)]

$$2 + (-2) + 4 + (-4) = 4(0)$$
$$0 = 0$$

IV CONCLUSION

In the chapter we have found the difference between the roots of quadratic equation and the roots of first derivative of differential equation. In this connection the quadratic equations are used in many real-life situations such as calculating the areas of an enclosed space, the speed of an object, the profit and loss of a product, or curving a piece of equipment for designing. Also the applications of derivatives are used to determine the rate of changes of a quantity with respect to the other quantity to words the number of roots of first order differential equations.

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