Construction and analysis of mechanical energy-storage incremental capacitor cum virtual resistor

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***Abstract*—We have prescribed a method of storage of energy. Electrical energy of capacitor is converted into mechanical energy of springs as a result of our analysis and stored for future use. In the process we designed a charge dependent polynomial capacitance. We use one or more non-conducting identical springs in each capacitor to store energy. We choose circular or elliptical plates for the capacitors to avoid sharp edge discharging. We have shown that this device may also be used as an incremental capacitor and under some particular limit as special purpose virtual resistor.**

***Index Terms*—non rigid plate separation; polynomial capacitance; electrical to mechanical energy; dielectric spring; incremental capacitor; virtual linear resistor;**

Alternative energy source or conversion of energy from one form to another is of utmost importance in present era. Lots of research is going on energy storage systems. One example of storing electrical energy is in the form of capacitor (depending upon the shape and insulation material of the capacitor). *Kirk et.al.* [1] prepared a mechanical capacitor using a spoke-less magnetically levitated composite ring rotor. In a different work *Michaelis et.al.* [2] designed and analyzed a 10 kW-hr, 15 kW mechanical capacitor system. It was determined that magnetically supported wheels constructed of advanced composites, have the potential for high energy density and high-power density. They analyzed the structural concepts yielding the highest energy density for any kind of design. *Kirk* in a different work [3] discussed about the energy storage capabilities of fly-wheels.

In this manuscript we consider conversion of electrical energy to usable mechanical energy. Electricity can be stored as the elastic potential energy of a spring, however the volume of the equivalent spring that will supply energy equal to a typical electrical cell will be huge in comparison to the volume of the corresponding electrical cell. Difference will be clear once we think about the clockwork spring. If we compress the spring up to maximum, it would drive the clock only a few days; by contrast, two pieces of small 1.5 V batteries would drive it for years. So clearly there will be no chance of volume comparison in this case. On the contrary if we emphasize into the cost effectiveness, then definitely the spring will be more advantageous, whatever may be the number of spring combination required to generate the same power as battery, does not matter. Though we know that springs are characterized by relatively low energy density (about *0.1 Wh/kg* for steel), being therefore a relatively bad choice for large scale application in energy storage devices. But still they possess (in general) high power density (around *10 Wh/kg* for steel), resulting in capacity to generate high forces from relatively small compression or expansion displacements. This typical property can help springs to be chosen as power storage devices. Storage of spring energy also requires continuous application of force to keep spring in compressed form. In a recent research (2015) *Rossi et.al.* [4] discussed about the benefits and challenges of using mechanical spring like systems when they are used as energy storage. In his/her work [4] he/she mentioned the importance of storing energy in elastic format and indicates the advantage of this format in comparison to the electrical, electrochemical, chemical, and thermal energy storages because of its ability to discharge quickly that enables high power densities. He/she mentioned that this stored energy may be delivered not only to mechanical loads, but also to systems those convert it to drive an electrical load. He/she worked on the energy storage systems with conventional torsional springs and presented a data showing the relative energy densities of different spring materials. In another patented work of *Cripps* [5], a self-sustaining electrical power generating system including a spring system with stored energy was discussed and invented. The spring system used has an input and an output for recharging and releasing the stored energy respectively. With a view of such background idea we plan to design parallel plate capacitors whose plates are connected by non-conducting spring or identical parallel spring combinations. We may call them non-rigid capacitors whose plate separations are variable depending upon the compression or expansion of the spring(s). With the charging of the capacitor the springs start compressing from their natural length. However with discharging, springs will start expanding from the compressed state. Because of the non-conducting natures of the springs, no leakage current will flow through them. Also to prevent charge leakage the geometrical shapes of the capacitor plates are chosen not to have sharp edges as far as practicable. One may choose circular or elliptical type plates for better charge storage.

We consider two parallel plates each of area A separated by a single non-conducting spring of spring constant K or symmetrically placed non-conducting parallel spring combinations with equivalent spring constant K. Initial plate separation or original length of the spring is d. Once we start charging the capacitor through a resistance R and battery of e.m.f. E, the plates accumulate charge continuously. As the plates accumulate charge, the attractive force between the plates increases and the spring starts compressing. Let at any instant of time t during charging q charge accumulates in each plate and the corresponding spring compression is x, which leads to a decreased plate separation d-x. Immediately one may arrive at the electro-mechanical balanced force equation

$$\frac{q^2}{A\varepsilon_0} = Kx\,, \tag{1}$$

and the capacitance will become

$$C = \frac{A\varepsilon_0}{d - \dfrac{q^2}{AK\varepsilon_0}}\,. \tag{2}$$

Binomial expansion of (2) leads to a polynomial function

$$C = \frac{A\varepsilon_0}{d}\left[1 + \left(\frac{q^2}{AK\varepsilon_0 d}\right) + \left(\frac{q^2}{AK\varepsilon_0 d}\right)^2 + \left(\frac{q^2}{AK\varepsilon_0 d}\right)^3 + \ldots\right], \tag{3}$$

leading to a polynomial capacitance. Growth equation of CR circuit becomes (following (1) and (2))

$$R\frac{dq}{dt} + \frac{q}{A\varepsilon_0}\left(d - \frac{q^2}{AK\varepsilon_0}\right) = E. \tag{4}$$

During the charge growth process, at some finite instant of time the capacitor will reach its maximum compression and that will lead to the endpoint of conversion from electrical to mechanical energy using this particular device.

The most important application of this device lies in its energy storing mechanism. Part of the energy is stored in electrical format in the capacitor and the rest part in mechanical format as the elastic potential energy of the spring. At any instant of time t the energy stored inside the device will be

$$E' = \frac{q^2}{2C} + \frac{1}{2}Kx^2 = \frac{q^2}{2A\varepsilon_0}\left(d - \frac{q^2}{AK\varepsilon_0}\right) + \frac{q^4}{2A^2\varepsilon_0^2 K} = \frac{q^2 d}{2A\varepsilon_0}\,, \tag{5}$$

which is identical with the normal parallel plate capacitor without spring, as expected, i.e. energy lost by the capacitor due to spring compression (plate separation reduction) was exactly transferred to the spring in the form of elastic potential energy. Once we charge the capacitor, we then isolate it (may be with some insulating handle) from any electrical device to prevent any charge loss, also the non-sharp edge shape of the plates will help in this regard as mentioned earlier. In this way ready to use electro-mechanical energy storages may be manufactured.

We now move into the design and constructional part of the capacitor. Earlier we assumed *d* as the initial (before charging) plate separation (natural length of the spring) of the capacitor. Here we have to keep in mind the fact that during charging phase the plates of the capacitor may be horizontal or vertical to the ground level. If the plates are vertical then gravity will not be an issue but if the plates are placed horizontally then the weight of the upper plate may create some initial compression in the spring and may decrease *d*. So, to avoid this unwanted scenario both the plates are made sufficiently light (as we do not know which one will be set as upper plate) in compare to the rigidity of the internal spring, so as to have negligible initial gravitational compression of the spring. Also, there will be a restriction on the maximum compression (or minimum length d_{min}) of the spring (already mentioned) as it cannot be compressed less than the product of number of turns (*n*) and thickness of spring wire (θ). Immediately there will be a restriction on the maximum charge (q_{max}) stored on each capacitor plate and consequently the potential difference V_{max} across the plates (with $q_{max} = CV_{max}$). Capacitor will not be fully charged because of the minimum length of the spring, consequently the charging phase will end within a finite laboratory time interval even theoretically, unlike the infinite time requirement for normal capacitors. We mention below the relevant constrained equation

$$d'_{min} = n\theta = d - \frac{q_{max}^2}{AK\varepsilon_0}. \qquad (6)$$

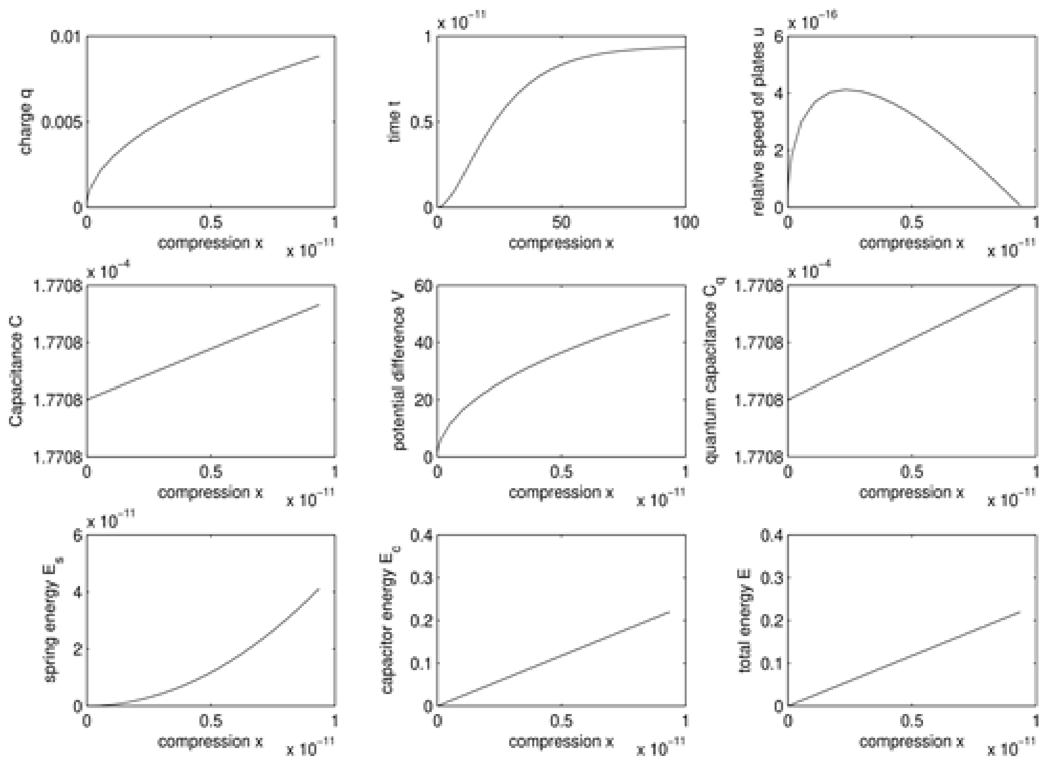

Figure 1: Parameters of the model are all varied with the compression of the spring. Starting from top and extreme left clockwise: a)charge stored on capacitor plates increase at a decreasing rate; b)current decreases and reaches a definite minimum and remains almost constant afterwards; c)relative speed of plates decreases; d)quantum

capacitance changes sign and becomes negative at a certain stage close to the maximum voltage and minimum current; e) total energy of the capacitor-spring system increases at a constant rate; f) capacitor energy increases at a decreasing rate; g)spring energy increases at a increasing rate; h)capacitance increases at a fast rate; i)potential difference increases and reaches a definite maximum and remains almost constant afterwards.

This again leads to a final energy that may be stored (at maximum possible compression of the spring) partly inside the capacitor (E_{max}^{C}) and partly inside the spring (E_{max}^{S}) (following (4))

$$E'_{max} = E_{max}^{C} + E_{max}^{S} = \frac{q_{max}^{2}}{2A\varepsilon_0}\left(d - \frac{q_{max}^{2}}{2A\varepsilon_0}\right) + \frac{q_{max}^{4}}{2A^2\varepsilon_0^2 K} \tag{7}$$

While designing the capacitor-spring system we have to maximize the spring energy and minimize the capacitor energy. Using (5), (6) and (7) one obtains the ratio (*r*) of spring energy to capacitor energy corresponding to maximum spring compression

$$r = \frac{q_{max}^{2}}{A\varepsilon_0 K n\theta} = \frac{d}{n\theta} - 1. \tag{8}$$

In the above equation we have used the relation (following (1))

$$q_{max} = \sqrt{A\varepsilon_0 K(d - n\theta)} \; . \tag{9}$$

Clearly (8) indicates that capacitor retains smaller part of energy, it transfer greater part of energy to spring (since $d > n\theta$). If one uses more parallel springs instead of one, then spring constant has to be modified taking into account all the spring constants. (8) suggests that to obtain more spring energy in compressed form, we need to construct the spring in a way so that it has more spaces in its loops in normal state. The energy flow direction will be like this: Source to capacitor (with some dissipation at the resistor), then to spring.

Since the life-span of the maximum compressed state of the spring depends entirely upon the duration of energy storage by the capacitor, hence we must choose proper insulating material to construct the spring(s). We have to use some insulating coating to cover the entire system also. There is another very important factor that has to be taken into consideration while designing the same. The charging time period for maximum compression must neither be too large nor too small. If it is too large then for practical purposes the device is of no use. On the contrary if the charging time period is too small then there will be a possibility of overcharging the device that may damage the spring. From (4) we derive the integral equation corresponding to charging time period

$$t = \int_0^{\sqrt{A\varepsilon_0 K}(d - n\theta)} \frac{R\,dq}{E - \frac{q}{A\varepsilon_0}\left(d - \frac{q^2}{AK\varepsilon_0}\right)}. \tag{10}$$

One may note the fact that, $K = Y\alpha / d$, Where Y is elastic coefficient of the material of the spring, α being the spring cross section. Further simplification of (10) yields

$$t = \frac{R}{2}\sqrt{\frac{AY\alpha\varepsilon_0}{d}} \cdot \int_0^{d-n\theta} \frac{dx}{\sqrt{x}\left[E - \sqrt{\frac{Y\alpha}{Ad\varepsilon_0}}\sqrt{x}(d - x)\right]}. \tag{11}$$

For parallel combination of many springs, modification of K needed accordingly. The relative velocity of approach of the capacitor plates during charging phase may be written with the aids of (1) and (4)

$$u = \frac{dx}{dt} = \frac{2}{R}\sqrt{\frac{x}{AK\varepsilon_0}}\left[E - \sqrt{\frac{K}{A\varepsilon_0}}\sqrt{x}(d - x)\right]. \tag{12}$$

This typical device is not only an electrical to mechanical energy converter and storage, it can also be used as an incremental (quantum) capacitor [6, 7]. Incremental capacitor devices are those which give incremental capacitance $C_q = dq / dv$ as a function of charge stored inside the capacitor or the potential difference between the plates. Usually for classical capacitors this element remains constant and is equal to the capacitance itself. Because of the polynomial nature of our special capacitance, here one may compute the incremental capacitance (C_q) from (2)

$$C_q = \frac{A\varepsilon_0}{d\left(1 - \frac{3q^2}{A\varepsilon_0 Y\alpha}\right)}. \tag{13}$$

Below we choose some typical sets of the parameters and measure time from (11) performing numerical integration using MATLAB. We choose ceramic (insulator) as the material of the spring with $Y = 470\times 10^9 Nm^{-2}$, $E = 50V$, $R = 1KOhm$, $\alpha = 0.1m^2$, $n = 30$, $\theta = 0.001m$, $A = 0.1m^2$ and $d = 0.05m$. We evaluate $t_{max} = 0.0119\,s$, $q_{max} = 0.129C$, $I = 4.3715\,MA$, $u = \;\; 6.6439\times 10^6 ms^{\;\;1}$, $C_q = \;\; 8.8540\times 10^{\;\;11} F$, $E' = 4.7000\times 10^8 J$, $E^C = 2.8200\times 10^8 J$, $E^S = 1.8800\times 10^8 J$, $C = 2.951\times 10^{\;\;11} F$ and finally $V = 4.371\times 10^9 volt$. We choose $\alpha = A$ because as mentioned earlier, that to avoid loss of charge we avoid sharp edges of plate and may choose it circular. Springs are also of circular cross section in general hence this equality arises. Figure 1 gives the detailed plot of different parameters of the model against the spring compression *x*. The entire figure is illustrated in the caption with all details. Numerical integration shows that for standard values of the supply voltage, capacitance and spring parameters, the time required to charge the capacitor up to V_{max} will be too small provided we use standard resistance of 1K. Now it is unphysical to operate a circuit manually for a very short period of time. We need a programmable Arduino to achieve it. Contrarily one may use a very high resistance to the circuit to achieve a moderate charging time span. But that is also unsuitable and leads to energy loss due to unnecessary Jule heating. So it depends up to our optimization technique and necessity.

Before we conclude we like to analyze the optimization/ saturation natures of the plots (figure 1) showing variations of potential difference and current with *x*. Those plots and the fact that the capacitance of this device increases with charge, one clearly leads to the following equations at that limit

$$V = \frac{q}{C}, \tag{14 a, b, c}$$

$$V + dV = \frac{q + dq}{C + dC},$$

$$dV = 0.$$

The above equations in turn reduces to

$$\frac{dC}{dq} = \frac{C}{q}. \tag{15}$$

After a few steps of algebraic simplification, one may evaluate the stable charge stored in the capacitor plate under this circumstance

$$q_{stable} = \sqrt{\frac{Y\alpha\alpha\varepsilon_0}{3}}. \tag{16}$$

The corresponding capacitor voltage evaluates to be

$$V_{stable} = \frac{2d}{\sqrt{3}}\sqrt{\frac{Y\alpha}{A\varepsilon_0}}. \tag{17}$$

Immediately applying KVL to the CR circuit the stable current through the circuit and 'virtual stable resistance' of the capacitor are evaluated as

$$I_{stable} = \frac{E}{R} - \frac{2d}{\sqrt{3}R}\sqrt{\frac{Y\alpha}{A\varepsilon_0}}.$$

$$RC_{stable} = \left(\frac{\frac{2dR}{\sqrt{3}}\sqrt{\frac{Y\alpha}{A\varepsilon_0}}}{E - \frac{2d}{\sqrt{3}}\sqrt{\frac{Y\alpha}{A\varepsilon_0}}}\right). \tag{18a, b}$$

To check the stability of current we start with (4) and compute 2nd derivative of charge with respect to time

$$\frac{d^2q}{dt^2} = \left(- \frac{d}{RA\varepsilon_0} + \frac{3dq^2}{RA^2\varepsilon_0^2 Y\alpha} \right)\left(\frac{E}{R} - \frac{q}{A\varepsilon_0 R} + \frac{dq^3}{A^2\varepsilon_0^2 RY\alpha} \right). \qquad (19)$$

At $q = q_{stable}$ the first term on the right side of the above equation will become zero, confirming the stability of current. To add a flavour into it if we compare q_{stable} with q_{max} ((16) and (9)), we find

$$n\theta = \frac{2}{3}d, \qquad (20)$$

which we are close to, by chance, in our choice $d = 0.05m$ and $n\theta = 0.03m$, during the numerical computation. Once we set this choice, simultaneously we achieve

(i) maximum stable capacitor voltage,
(ii) minimum stable current,
(iii) highest compression of the spring i.e. no further possibility of compression. One can then either detach the capacitor from the circuit for further use as a source of electromechanical energy or can keep it attached with the circuit. In the latter case immediately, the device turns from an incremental capacitor to a constant resistor. But remarkably no current passes through it. Any additional force between the plates will be absorbed by the spring (depending upon its rigidity of course). We can thus use this device as a special type of linear resistance obeying Ohm's law with a constant current passing through it virtually. The circuit will function as if it is continuous, though the capacitor plates are connected by a non-conducting spring not carrying any charge at all.

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