Connected Network Dominating Set of a Circular arc graphs

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Abstract - A graph is a circular arc graph corresponding to circular arc family if it is the intersection graph of a finite set of arcs on a circle. The connected dominating sets are useful in the computation of rooting for mobile networks and circuit networks. A connected dominating set is used as a backbone for communications and vertices that are not in this set communicate by passing message through neighbors that are in the set. Resent advances in technology have made possible the creation of wireless sensor network. Although there is no physical backbone infrastructure a virtual backbone can be formed by constructing a connected dominating set. In this paper we have presented connected network dominating set of a circular arc graph using an algorithm.

Key words-Circular arc family, circular arc graph, dominating set, connected dominating set, network

I. INTRODUCTION

A minimum connected dominating set of a graph A_G is connected dominating set with the smallest possible cardinality among all connected set. Let A_G = (V, A) be a graph. The neighborhood of a vertex V in A_G is defined as V and the set of vertices that are adjacent to V (including V) as A in A_G. The neighborhood is denoted by N[v]. A set B of vertices in A_G is called a neighborhood set in A_G if

$$A_G = \bigcup_{v\in B}\langle N[v]\rangle$$

Where $< N[v] >$ Is a subgraph of A_G induced by N[v]. In this we discuss a method for finding minimal connected network dominating set for circular arc graph by using directed network and also introduce an algorithm for finding connected dominating set of a circular arc graph A_G.

II. PRELIMINARIES

Let A_G = (V, A) be a circular arc graph. A dominating set D is said to be connected dominating set of the induced subgraph $<A_d>$ is connected. The connected dominating number $\gamma_C(A_G)$ of A_G is minimum cardinality of a connected set. For each arc A_i let nbd [A_i] denote the set of arcs that intersect A_i. Let min (A_i) denotes the smallest interval and max (A_i) denotes the maximum arc in nbd [A_i]. fright (A_i) is the first right intersecting arc NIA(i) of the arc A_i as below

NIA(i)= j if $q_i<p_i$ and there do not exist an arc l such that $q_i<p_l<p_j$.

If there is no such j, then define NIA(i)= null.

Define

Next(i)= min ({nbd [NIA(i)]} \ {nbd [min(i)]})

We confine our discussion to connected graphs only.

First, we assume A with two dummy arcs say A_0 and A_{n+1}, where A_0= [p_o, q_0] and

A_{n+1} = [p_{n+1}, q_{n+1}] such that $q_0<\max\{p_l\}$ and $p_{n+2}>\max\{q_k\}$

Let A_d= A∪{A_0, A_{n+1}}. We assume that an arc A_d is indexed by increasing order of their right end point namely $q_0<q_1<q_2<\ldots<q_{n+1}$. Here we construct a directed network and show that the arc in any shortest directed path in it corresponds to a CNDS of A_G.

A directed network D= (N, A_L) is constructed as follows the nodes in N corresponds to the arcs in A_d which are not properly contained within another arc. The arcs in A_L are partitioned into two disjoint arc sets A_{L1} and A_{L2} which are defined as follows. For $A_i \in D$,there, is directed arc (A_0, A_i) between A_0 and A_i that belongs to A_{L1} if and only if no arc A_h such that $q_j<p_h<q_h<p_{n+1}$.

Similarly, there is a directed arc (A_j, A_{n+1}) between A_j and A_{n+1} that belongs to A_{L2} if and only if no arc

A_h that $q_j<p_h<q_h<p_{n+1}$. This gives the scope to join the arcs A_0 and A_{n+1} to other arcs in A and it is obvious that all such joined directed arcs belongs to A_{L1}. Next for A_i, $A_j \in D$, here is a directed arc (A_i, A_j) between A_i and A_j that belongs to A_{L2} if and only if A_j = Next (A_i).

III. AN ALGORITHM FOR FINDING MINIMAL CONNECTED NETWORK DOMINATING SET OF A CIRCULAR ARC GRAPH.

Input: circular arc family A = {A_0, A_1, ...A_n}.
Output: minimal connected network dominating set of a circular arc graph from a directed network D.

Step 1: For A_i=1,2,3, ...n
{

a = NIA(A_i)

b = min (A_i)

c = nbd [a]

d = nbd [b]

Next (A_i) = min ({c}\{d})

If

Next (A_i)= null then go to step 1

else

Join A_i to Next (A_i) and go to step 1

}

Step 2: for A_j = 2 to n

{

If A_1 is intersect to A_j or contained in A_j

Join A_0 to A_j

If A_n is intersect to A_j or contained in A_j

Join A_{n+1} to A_j

}

Step 3: find paths P_0, P_1, ...P_s for some S from A_0 to A_{n+1}

Step 4: for j = 1 to S

{

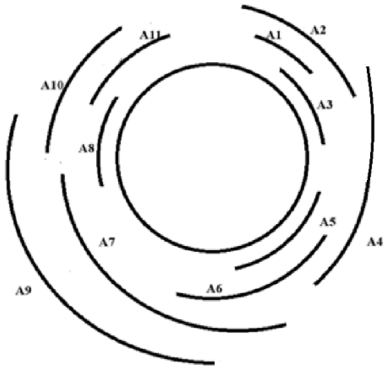
If nodes of P_j are connected in A_G

MCNDS = {Nodes of P_j}

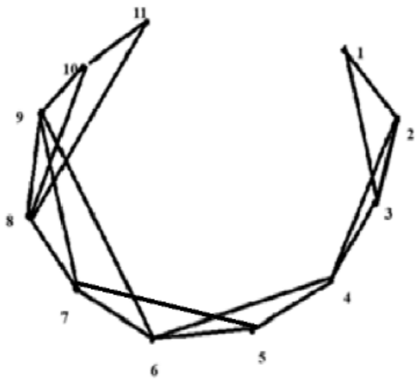
}

Step 5: End

IV. ILLUSTRATION



Circular arc family A_G



circular arc graph A_G

We find nbd [A_i], min (A_i) and NIA(A_i)

nbd [1] = {1,2,3} min (1) = 1, NIA =4

nbd [2] = {1,2,3,4}, min (2) = 1, NIA =5

nbd [3] = {1,2,3,4}, min (3) = 1, NIA = 5

nbd [4] = {2,3,4,5,6}, min (4) = 2, NIA = 7

nbd [5] = {4,5,6,7}, min (5) =4, NIA =8

nbd [6] = {4,5,6,7,9}, min (6) = 4, NIA = 8

nbd [7] = {5,6,7,8,9}, min (7) = 5, NIA = 10

nbd [8] = {7,8,9,10,11}, min (8) = 7, NIA = Null

nbd [9] = {6,7,8,9,10}, min (9) = 6, NIA = 11

nbd [10] = {8,9,10,11}, min (10) = 8, NIA = Null

nbd [11] = {8,10,11}, min (11) =8, NIA = Null

Next (A_i) = min ({nbd [NIA (A_i)]} \ {nbd [min (A_i)]})

Next (1) = min ({nbd [NIA (1)]} \ {nbd [min (1)]})

= min ({nbd [4]} \ {nbd [1]})

= min ({2,3,4,5,6} \ {1,2,3})

= min {4,5,6}

=4

Next (2) = min ({nbd [NIA (2)]} \ {nbd [min (2)]})

= min ({nbd [5]} \ {nbd [1]})

= min ({4,5,6,7} \ {1,2,3})

=min {4,5,6,7}

= 4

Next (3) = min ({nbd [NIA (3)]} \ {nbd [min (3)]})

= min ({nbd [5]} \ {nbd [1]})

= min ({4,5,6,7} \ {1,2,3})

= min {4,5,6,7}

= 4

Next (4) = min ({nbd [NIA (4)]} \ {nbd [min (4)]})

= min ({nbd [7]} \ {nbd [2]})

= min ({5,6,7,8,9} \ {1,2,3,4})

= min {5,6,7,8,9}

= 5

Next (5) = min ({nbd [NIA (5)]} \ {nbd [min (5)]})

= min ({nbd [8]} \ {nbd [4]})

= min ({7,8,9,10,11} \ {2,3,4,5,6})

= min {7,8,9,10,11}

= 7

Next (6) = min ({nbd [NIA (6)]} \ {nbd [min (6)]})

= min ({nbd [8]} \ {nbd [4]})

= min ({7,8,9,10,11} \ {2,3,4,5,6})

= min {7,8,9,10,11}

= 7

Next (7) = min ({nbd [NIA (7)]} \ {nbd [min (7)]})

= min ({nbd [10]} \ {nbd [5]})

= min ({8,9,10,11} \ {4,5,6,7})

= min {8,9,10,11}

=8

Next (8) = min ({nbd [NIA (8)]} \ {nbd [min (8)]})

= min ({nbd [null]} \ {nbd [7]}

= null

Next (9) = min ({nbd [NIA (9)]} \ {nbd [min (9)]})

= min ({nbd [11]} \ {nbd [6]})

= min ({8,10,11} \ {4,5,6,7,9})

= min {8,10,11}

= 8

Next (10) = min ({nbd [NIA (10)]} \ {nbd [min (10)]})

= min ({nbd [null]} \ {nbd[8]})

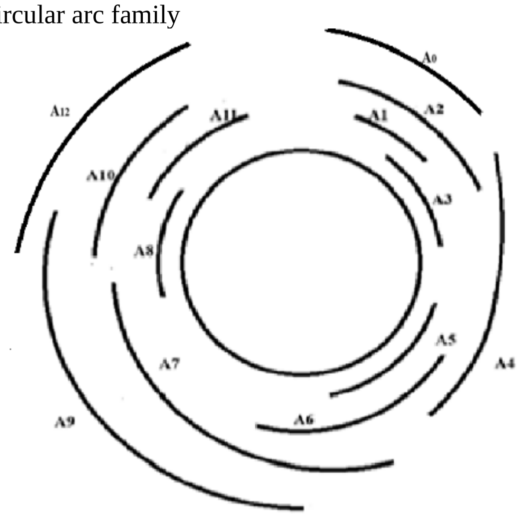
= null

Next (11) = min ({nbd [NIA (11)]} \ {nbd [min (11)]})

= min ({nbd [null]} \ {nbd [8]})

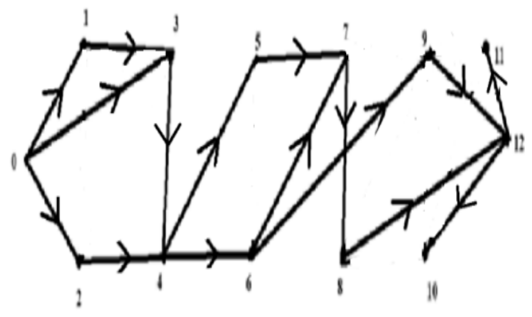
= null

Now the dummy arcs A_0 and A_{n+1} are augmented to circular arc family



Circular arc family A_G

Directed network D = (N, A) is constructed as follows



Directed network D(N, A)

From above
0-1-3-4-6-9-12

0-2-4-5-7-8-12

0-3-4-6-9-12

By deleting dummy nodes 0 and 12

We get CNDS {1,3,4,6,9} {2,4,5,7,8}

V. MAIN LEMMAS

Lemma 1:

Let A = {A_1, A_2, ...A_n} be a circular arc family. If A_i, A_k are any two arcs which are intersecting A_j is such that $A_i<A_j<A_k$ then A_j intersects A_k.

Proof:

Given that A is a circular arc family. Since the arcs labelled in increasing order of their right end

points, it is easy to see that when Ai<Aj<Ak, then qi<qj<qk. Now Ai intersects Ak implies that

$p_k<q_i$, where p_k is the left end point and q_i is the right end point. Therefore $p_k<q_i<q_j<q_k$ which implies that A_j also intersects A_k.

Lemma 2:

If the directed arc (A_0, A_j) ∈ A_{L1}. Where Aj is any arc of A_0, then the arcs between A_0 and A_j belongs to nbd [A_j].

Proof:

Let A = {A_1, A_2, ...A_n} be a circular arc family. Suppose that (A_0, A_j) ∈ A_{L1}. By the definition of arcs in A_{L1} it follows that there is no arc A_k such that $q_0<p_k<q_k<p_j$ so any arc between A_0 and A_j must intersect with A_j. Therefore, the arcs between A_0 and A_j belongs to nbd [A_j].

Lemma 3:

If the directed arcs (A_j, A_{n+1}) ∈ A_{L1} where A_n is any arc of A, then the arcs between A_j and A_{n+1} are connected by A_j.

Proof:

Let A = {A_1, A_2, A_3, ...A_n} be a circular arc family. Thus, it is clear by lemma 2 that if there is a directed arc (p_i, q_i) ∈ A_{L1} then the intervals between A_i and A_j are adjacent with A_i or A_j.

VI. MAIN THEOREMS

Theorem 1: If the directed arc (A_i, A_j) ∈ A_{L2} then arcs between A_i and A_j are dominated by A_i or A_j but not both.

Proof: Let (A_i, A_j) ∈ A_{L2}then Aj=Next (Ai). Let A_h be any arc between A_i and A_j. Suppose A_h intersects neither A_i not A_j. The lemma follows immediately if A_h intersects A_i or A_j. If A_h = NA(A_i) then by definition of A_j, clearly A_h intersects A_j. So A_h not equal to NA(A_i). Then two cases will arise

Case (i): Suppose A_h intersects NA (A_i). Again, this implies by the definition of

Next (A_i) = min {nbd [NA(A_i)]} \ {nbd [min (A_i)] that A_h intersects A_j.

Case (ii) Suppose A_h does not intersect NA(A_i). Then NA(A_i)<A_h

By our assumption $A_i<A_h<A_j$. Therefore, NA(A_i) and $A_j<A_h<A_j$. By the definition of A_j, NA(A_i) and A_j intersect. Hence by lemma 1, A_h and A_j intersect. Thus, for all cases either A_h intersects A_i (or) A_j which implies that the arcs between A_i and A_j are dominated by A_i (or) A_j. we now show that if A_h is any arc between A_i and A_j then A_h is not dominated by both A_i and A_j. since min (A_i)

Next (A_i) = min ({nbd [NA(A_i)]} \ {nbd [min (A_i)]})

Does not intersect Next (A_i) = A_j. so, the arcs between A_i and min (A_i) does not intersect A_i. Hence there does not exist an arc A_h between A_i and A_j that intersect both A_i and A_j.

Theorem 2: If the connected network dominating set (CNDS) denote that the set of vertices in the shortest path between the nodes A0 and An+1 in the directed network D (N, A), then there with not be any arc Ak in A such that it intersect all the arcs in CNDS.

Proof: Suppose there is an arc Ak in A such that Ak is dominated by all the arcs in connected network dominating set.

Let CNDS = {A_{i1}, A_{i2}, ...A_{in}}

Then we have either $A_{i1}<A_{i2}<........<A_{in}<A_k$ (or) $A_{i1}<A_{i2}<A_{i3}<.........<A_k<A_{in}$

Suppose $A_{i1}<A_{i2}<A_{i3}<$ ……………$<A_{in}<A_k$

Then min (A_{i1}) = A_{i1} and nbd [min (A_{i1})] = nbd [A_{i1}]

= {A_{i1}, A_{i2} ………. A_{im}} for A_{im}<NA(A_{i1})

Now

nbd [NA(A_{i1})] = {NA (A_{i1} ……… A)} and hence {nbd [NA(A_{i1})]} \ {nbd [min (A_{i1})]} = null

That is Next (Ai1) = A_{i2} = null, a contradiction.

Similarly in the case if Ai1<Ai2<....

Thus, there is no arc A_k in A such that A_k intersects all the arcs in CNDS.

Theorem3: The vertices in the path between nodes A_0 and A_{n+1} in D (N, A) corresponding to a connected dominating set of A_G having arcs properly contained within any arc of the given circular arc graph A_G of A.

Proof: Let P be a path from A_0 to A_{n+1} in D (N, A)

Define

B = {A_i: node i appears in P, i ≠ A_0, i ≠A_{n+1}}

For each directed arc (A_i, A_j) in P by lemma 2,3 and 4 it follows that all intermediate arc A_{i+1}, A_{i+2}, A_{i+3}, ...A_{j-1} between A_i and A_j are belongs to N[A_i] ∪ N[A_j]. Hence all the intermediate arcs between the arcs in B belongs to

$\bigcup_{iJ\dot{\epsilon}B} A_G$ [N [A_i] ∪ N [A_j]]

Since the arcs in B corresponding to the nodes in the path P between A_0 and A_{n+1}, the arcs between A_0 and the first arc in B as well as the arcs between the last arc in B and A_{n+1} also belongs to $\bigcup_{iJ\dot{\epsilon}B} A_G$ [N [A_i]. Thus, all nodes in A_G are adjacent to nodes in B.

That is, V(A_G) = $\bigcup_{iJ\dot{\epsilon}B} A_G$ [N [A_i]

But the subgraph AH by the arc set {A_i, A_{i+1}, A_{i+2}, ...A_j} is a subgraph of the induced subgraph A_G [N [A_i] ∪ N [A_j]], where (A_i, A_j) is any arc in CNDS.

Therefore A_G (A_i, A_{i+1}, A_{i+2}, ...Aj) ⊆ A_G[N[Ai] ∪ N[Aj]] where A_i, A_j ∈ B.

Therefore

$\bigcup_{iJ\dot{\epsilon}B} A_G$ [{A_i, A_{i+1}, …… A_j}] ⊆ $\bigcup_{iJ\dot{\epsilon}B} A_G$ [N [A_i] ∪ N [A_j]]

Since V (A_G) = $\bigcup_{ij\in B} N[Ai]$, it follows that A_G = $\bigcup_{A_i\in B} A_G$[N[A_i]]

Thus, B is neighborhood set of A_G [A]. Therefore, the nodes in B are nonadjacent. Therefore, B forms a connected network dominating set of A_G [A]. Since P is the path, it follows that B is minimum connected network dominating set of a circular arc graph A_G [A].

VII. AN ALGORITHM FOR FINDING A CONNECTED DOMINATING SET OF A CIRCULAR ARC FAMILY

Input: Circular arc family A= {1,2,3…………n}

Output: connected dominating set of a circular arc family of given circular arc family.

Step1: CD = max (1)

Step2: LA = The largest arc in CD

Step3: Find max (LA)

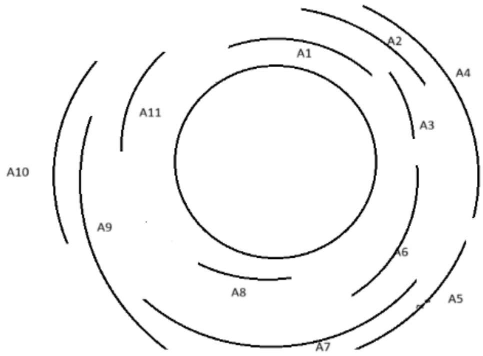
Step4: If max (LA)=LA (or) max (LA)=n then go to step 7

Step5: If max (LA)=n and max (LA)=fright (LA) and there is no arc which contained in max (LA) then go to Step 7

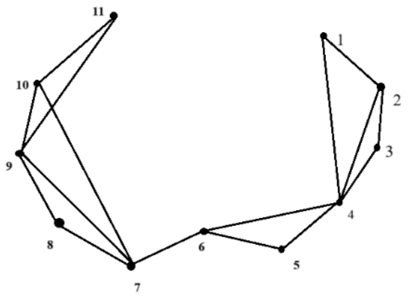
Step6: CD = CD ∪ {max (LA)} then go to step 2

Step7: End

VIII. ILLUSTRATION FOR THEOREM 3



Circular arc family A



Circular arc graph A_G

Will find nbd [A_i] and max (A_i) where i = 1,2,3……….11

nbd [A_1] ={1,2}, max (A_1) = 2

nbd [A_2] = {1,2,3,4}, max (A_2) =4

nbd [A_3] = {2,3,4}, max (A_3) = 4

nbd [A_4] = {1,2,3,4,5,6}, max (A_4) = 6

nbd [A_5] = {4,5,6}, max (A_5) = 6

nbd [A_6] = {4,5,6,7}, max (A_6) = 7

nbd [A_7] = {6,7,8,9,10}, max (A_7) =10

nbd [A_8] = {7,8,9}, max (A_8) = 9

nbd [A_9] = {7,8,9,10,11}, max (A_9) = 11

nbd [A_{10}] = {7,9,10,11}, max (A_{10}) = 11

nbd [A_{11}] = {9,10,11}, max (A_{11}) = 11

Procedure to find a connected dominating set

Input: Circular arc family A = {1, 2, ………11}

Step1: CD = 4

Step2: LA = The largest arc in the CD = 4

Step3: max (LA) = max (4) = 6

Step4: CD = {4}∪ {6} = {4,6} go to step 2

Step5: LA = 6

Step6: max (6) = 7

Step7: CD = {4,6} ∪ {7} = {4,6,7} go to step 2

Step8: LA = 7

Step9: max (7) = 10

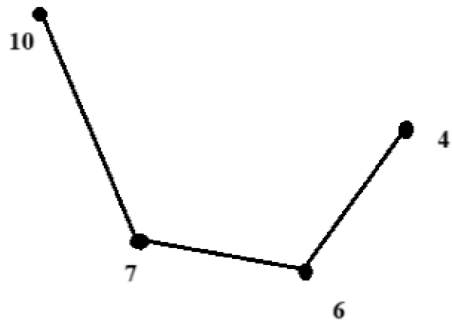
Step10: CD = {4,6,7} ∪ {10} = {4,6,7,10} go to step 2

Step11: LA = 10

Step12: max (10) = 11

Step13: end

Output: CD = {4,6,7,10} is a connected dominating set of a circular arc graph A_G as



The induced subgraph <A_D>

IX. CONCLUSION

Let G be a circular arc graph corresponding to circular arc family A.The circular graphs are rich in combinatorial structures and have found applications in several disciplines such as electrical networks, traffic control, Ecology, Genetics and Computer science and particularly useful in cyclic scheduling and computer storage allocation problems. In this paper we have presented individualized circular arc graphs as various circular arc graphs. We then extended the results to trace out a specific type of circular arc graphs having every pair of vertices or nodes as a minimal connected dominating set. Moreover, we presented an algorithm to identify circular arc graphs having every pair of vertices as a minimal network connected dominating set.

X. AKNOWLEDGEMENT

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