Analytical Study on Paraboloid Shell Roof Structure using Finite Element Analysis Software

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***Abstract* - Shells belong to the class of stressed skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry the loads primarily by direct stresses acting in their plane. In the design of new forms of concrete shell structures, the conventional practice is to select the geometry of shell first and then making the stress analysis. In this process no deliberated effort is taken to ensure the desirable state of stress in the material. Perhaps it is more logical to reverse this process. Ideally a concrete shell in its membrane state carries the external loads by pure compression, unaccompanied by shear stresses so that no tensile stresses develop and hence the reinforcement becomes necessary excepting for secondary effects like bending, shrinkage. In most of the shell roof is the predominant load is the dead weight. Hence it is advantageous to select the shape of shell in such a way that, under this condition of loading, the shell is subjected to pure compression without bending. This can be achieved by shaping the shell in the form of a which the parabolic shape is corresponding to the dead weight. An attempt is made to study the influence of thickness on the ultimate load of the Paraboloid Shells of circular Ground Plan. The Paraboloid Shells are analyzed to determine the influence of thickness in deflections (y), maximum absolute stress, membrane stresses (Sx), (Sy), (Sxy), bending stresses (Mx), (My), (Mxy) and shear stresses (SQx), (SQy) using the Software STAAD.Pro. Conclusions are made from the analysis results that the deflection, maximum absolute stress, membrane stresses, shear stresses of paraboloid concrete shells decrease with increase in thickness.**

***Index Terms* – Deflection, FEA, Paraboloid Shell, Stresses, Thickness**

I. INTRODUCTION

A Shell structure can be defined as a solid continuum lying in between two closely spaced covered surfaces. This definition suits well so far as the geometrical shape of a shell form is concerned. The distance between the two boundary surfaces is obviously the thickness “t”. Shells and folded plates belong to the class of stressed skin structures which because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by directs stresses acting in their plane. Whenever is referred to in the standard it refers to thin shell. In building construction, a thin, curved plate structure shaped to transmit applied forces by compressive, tensile and shear stresses that acts in the plane of the surface. They are usually constructed of concrete reinforced with steel mesh. Thin shells are an example of strength though forms as opposed to strength through mass. The effort in design is to make the shell as thin as practical requirements will permits so that the dead weight is reduced and the structure functions as a membrane free from large bending stresses. By this means, a minimum of materials is used to the maximum structural forms. Most shells occurring in nature are doubly curved. Shells of eggs, nuts and the human skull are common place examples. These naturally occurring shells are hard to crack or break.

The following are the characteristics of a true membrane:

1. It must be very thin and uniform in thickness. Any abrupt change of the thickness is not permitted.
2. The acting load must be continuously distributed over the membrane surfaces and the intensity of the loading must be constant. No change in the intensity of the loading is allowed.
3. It must be perfectly flexible. That is, any point on the membrane surface can have full freedom of displacement along the normal to the membrane surface through the point.
4. The internal forces at any point (or if any support is provided at that point) must be directed along the tangent at that point of the membrane surfaces.

Shells are used for many structural purposes due to its light weight, graceful form and maximum load resisting capacity. Paraboloid shells usually offer a higher ultimate strength than the shells of single curvature such as cylindrical shells. Paraboloid shells can be employed for mass construction of roofs subjected to maximum loads. Sustainability is achieved due to the minimum constituent materials used when compared to conventional methods.

The main objectives of this project is to study the effect of various thicknesses on paraboloid concrete shells over circular ground plan. The paraboloid shells with five different thicknesses are considered for this study and are designated as follows,

- PS - I - Paraboloid shell over circular ground plan with thickness, t1 (50mm)
- PS - II - Paraboloid shell over circular ground plan with thickness, t2 (60mm)
- PS - III - Paraboloid shell over circular ground plan with thickness, t3 (70mm)
- PS - IV - Paraboloid shell over circular ground plan with thickness, t4 (80mm)
- PS - V - Paraboloid shell over circular ground plan with thickness, t5 (90mm)
- PS - VI - Paraboloid shell over circular ground plan with thickness, t6 (100mm)

Paraboloid Shells over Circular Ground Plan of various thicknesses - t_1, t_2, t_3, t_4, t_5 and t_6 are analyzed to determine the deflection (y), maximum absolute stress, membrane stresses (Sx, Sy & Sxy), bending stresses (Mx, My and Mxy) and shear stresses (SQx, SQy) using the Finite Element Software STAAD.Pro.

II. LITERATURE REVIEW

Vincent , John F (2006) gives the Evaluation of Distress in Supports of Hyperbolic Paraboloid Shell which composed of a saddle-type hyperbolic paraboloid concrete shell roof supported by buttresses and brick masonry walls, was constructed in 1963. At one corner of the structure, the perimeter of the roof shell projects beyond the exterior building walls to form a canopy over the main entrance. This canopy is partially supported by two brick masonry fin walls that project outward from the main building walls. However, the configuration of the roof shell was not consistent with detailing requirements of the closed-form methods.

Josip Kacmarcik (2011) comprehensively compared three design methods for openings in cylindrical shells under internal pressure (pressure vessels).The calculation procedures used in the three methods are significantly different, so the comparison has been based on the maximum permissible design pressures for different geometries.

Thomas Simoni, Peter Madsen Nordestgaard (2017) design the hyperbolic paraboloid shape shell structure used as a roof for an outdoor music stage. The structure was calculated using FEM analysis and constructed using standard steel trusses and trapezoidal steel plates. The steel structures is lined with wood and covered with roofing felt.

Amrutha Joseph & P. E. Kavitha (2021) Shell foundations are proved to be performing much better than conventional flat footings especially when transmitting heavy super-structural loads to poor bearing soil. Shells have larger stiffness and strength compared with plane surface structural elements which ensure minimum material and maximum structural performance. So, it is more economical where the materials are scarce and overpriced. Different types of shells are used in foundations such as conical shell, triangular shell, pyramidal shell, hyperbolic paraboloid shell, cylindrical shell inverted dome, elliptic paraboloid, inverted spherical shell etc. In this research the structural performance of hyperbolic paraboloid (hypar) shell foundation is investigated using finite element software. The thickness of the shell is optimised based on the stress distribution behaviour of hypar foundation.

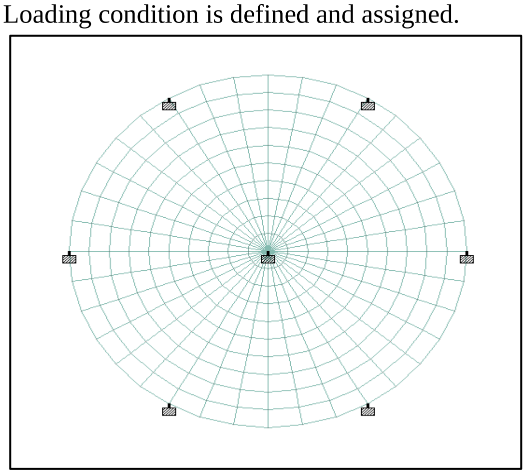
Vidya Vijay K. et. al., (2023) It is characterized by a double-curved shape of the shell which enhances the load distribution and contributes to the stability of the structure. The study explains the use of lightweight material to create lightweight structures with larger spaces by optimizing the structural frame. The paper also discusses the structural awareness required in the alteration of design in the conceptual design stage and to have the software tools required for decision support and interact concurrently with the architect in the optimization process. The analysis of shell structures reveals a fascinating intersection of architectural in Genuity and engineering precision.

III. NUMERICAL ANALYSIS USING STAAD.PRO

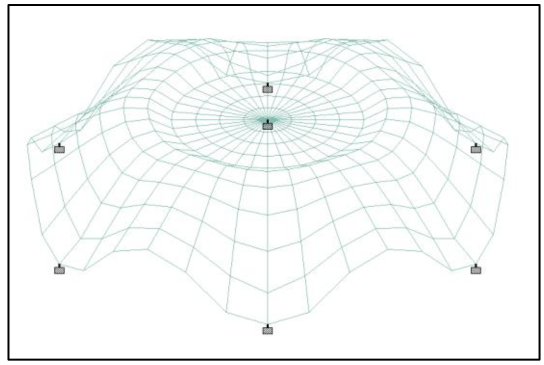
The theoretical Finite Element Analysis and the computation of membrane stresses and deflections corresponding to the applied loads on shallow funicular shells are carried using the standard program STAAD. Pro Version 8i (Select Series 6).

Shell elements are a special class of elements that are designed to efficiently model thin structures. They take advantage of the fact that the only shear on the free surfaces is in-plane. The element coordinate system for all the shell elements has the z-axis normal to the plane. Nodes are normally located on the center plane of the element. Shell has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included. The thickness is assumed to vary smoothly over the area of the element, with the thickness input at the four nodes. If the element has a constant thickness, only need be input. The analysis and the computation of shallow funicular shells are done using the standard program, by executing in the following steps:

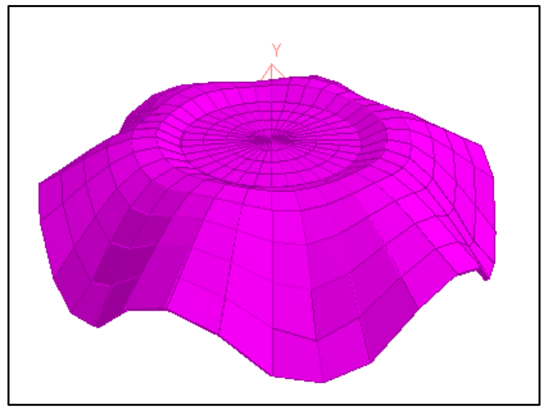
1. Choice of element - Four noded quadrilateral shell elements (Shell)
2. Discretization of shell - The shell is discretized into small.
3. Joint coordinates are taken from the shells samples cast experimentally.
4. Material properties are defined and assigned.
5. Restraints at supports are specified.
6. Loading condition is defined and assigned.



(a)



(b)



(c)

Fig. 1 Model Initialization of Structure - (a), (b) & (c)

Material properties of the Paraboloid shells are defined and the values are assigned to the elements. The values of the material properties physical properties assigned to the elements are shown in table.1

Table.1 Material and Physical Properties of Paraboloid Shell

Property	Value
Modulus of Elasticity, **E**	2 .17 x 10^4 N/mm^2
Poisson's ratio, **μ**	0.17
Density, **ρ**	2500 kg/m^3
Diameter of the Shell, **d**	5000 mm
Rise, **r**	1000 mm
Shell thickness, **t**	50 mm

Paraboloid Shell with various thicknesses is modeled with quadrilateral and triangular shell elements. The shell is discretized into small elements and the supports are restrained at the certain boundaries. To determine the deflections, membrane stresses and bending stresses and shear stresses, an arbitary plate load of 1 kN/m2 is applied all over the shell elements. From the results of the analysis, deflection and stress contour are obtained.

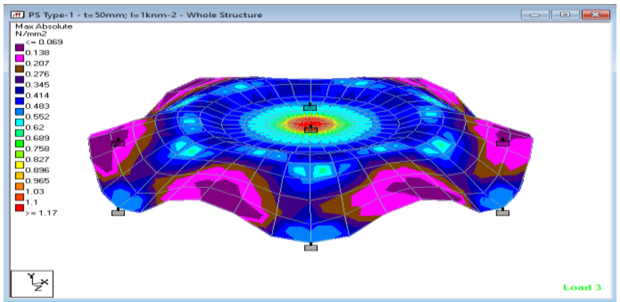


Fig.2 Maximum Absolute Stress for shell thickness t_1

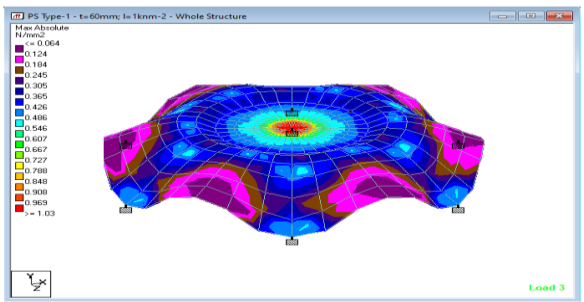


Fig.3 Maximum Absolute Stress for shell thickness t_2

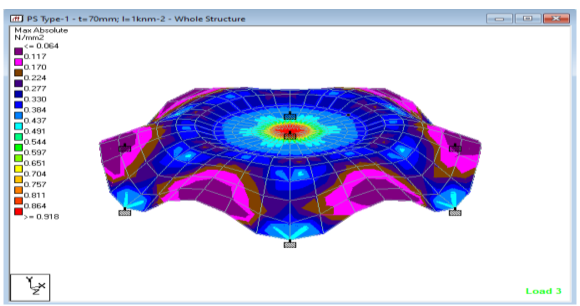


Fig.4 Maximum Absolute Stress for shell thickness t_3

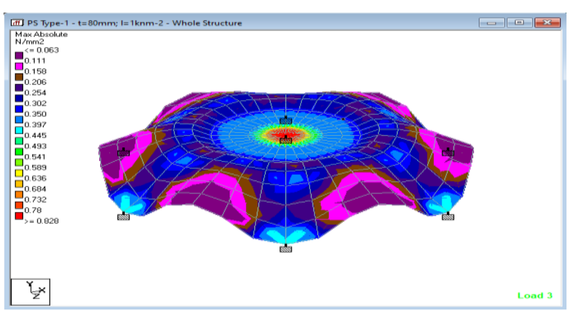


Fig.5 Maximum Absolute Stress for shell thickness t_4

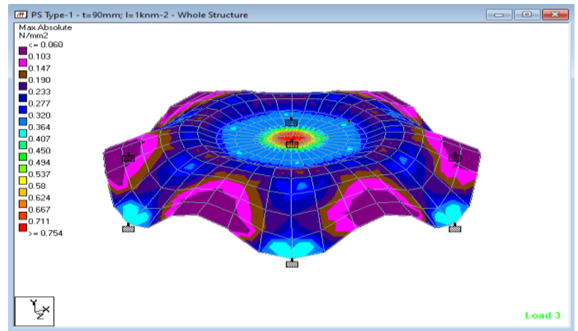


Fig.6 Maximum Absolute Stress for shell thickness t_5

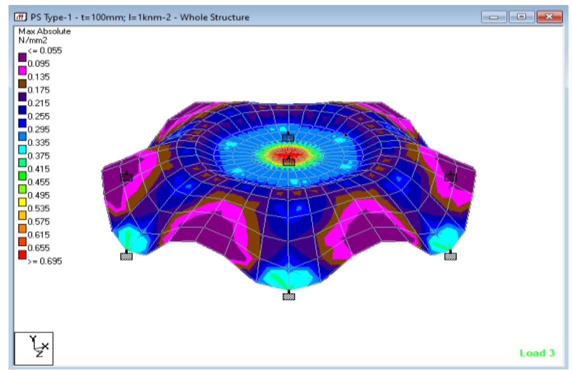


Fig.7 Maximum Absolute Stress for shell thickness t_6

IV. RESULTS AND DISCUSSIONS

Table.2 Resultant Stresses - Paraboloid Shell

Stresses (N/mm^2)	PS-I (50mm)	PS-II (60mm)	PS-III (70mm)	PS-IV (80mm)	PS-V (90mm)	PS-VI (100mm)
Max. Absolute	1.172	1.029	0.918	0.828	0.754	0.695
Sx	0.091	0.070	0.054	0.041	0.030	0.021
Sy	0.084	0.065	0.053	0.044	0.038	0.033
Sxy	0.215	0.194	0.179	0.169	0.160	0.154
SQx	0.029	0.025	0.022	0.021	0.021	0.021
SQy	0.026	0.020	0.016	0.012	0.009	0.009
Mx	0.459	0.589	0.725	0.865	1.008	1.154
My	0.459	0.589	0.725	0.865	1.008	1.154
Mxy	0.029	0.034	0.038	0.042	0.048	0.054
Deflection, y (mm)	0.230	0.176	0.139	0.114	0.095	0.081

From Table.2, a plot is made as shown in below figures to determine the deflection, Max. Absolute Stress, Membrane Stresses, Bending Stresses and Shear Stresses in Paraboloid Shells for various thickness.

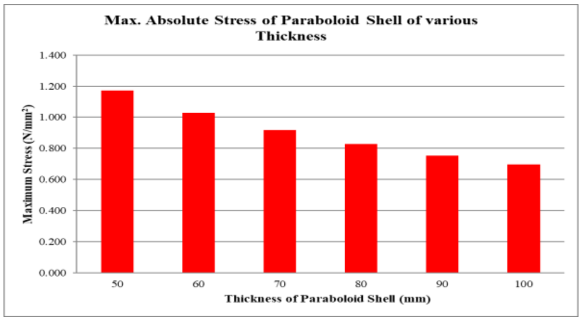


Fig.8 Paraboloid Shell - Max. Absolute Stress

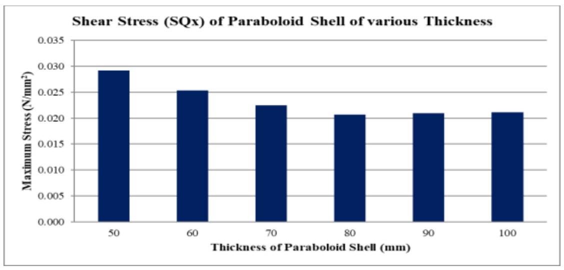


Fig.9 Paraboloid Shell - Shear Stress (SQx)

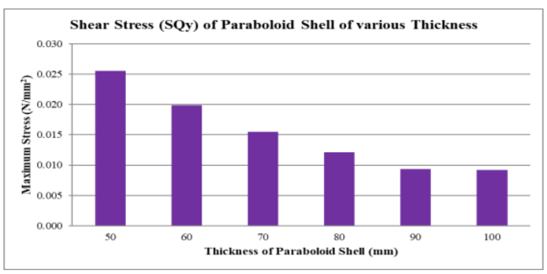


Fig.10 Paraboloid Shell - Shear Stress (SQy)

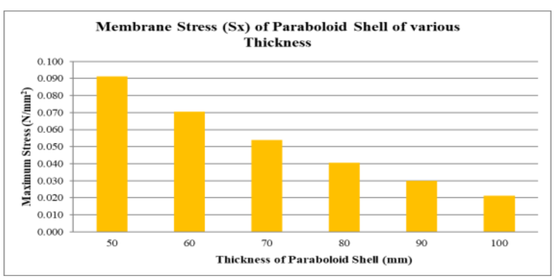


Fig.11 Paraboloid Shell - Membrane Stress (Sx)

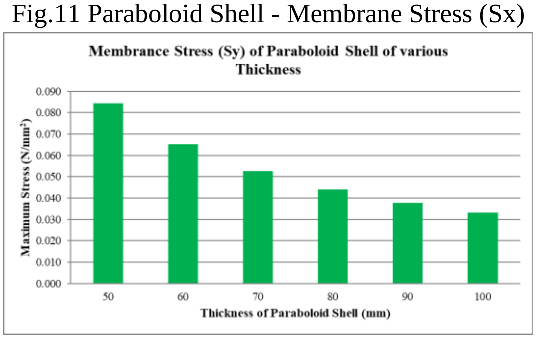


Fig.12 Paraboloid Shell - Membrane Stress (Sy)

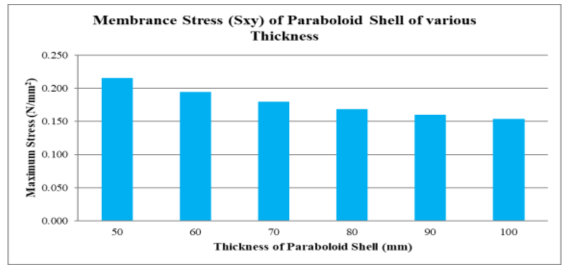


Fig.13 Paraboloid Shell - Membrane Stress (Sxy)

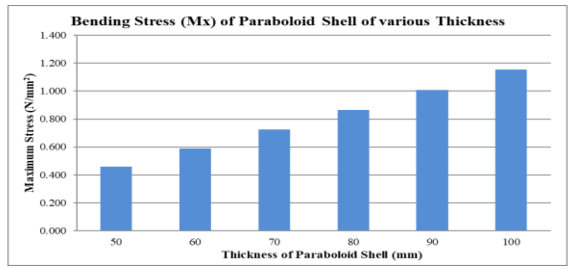


Fig.14 Paraboloid Shell - Bending Stress (Mx)

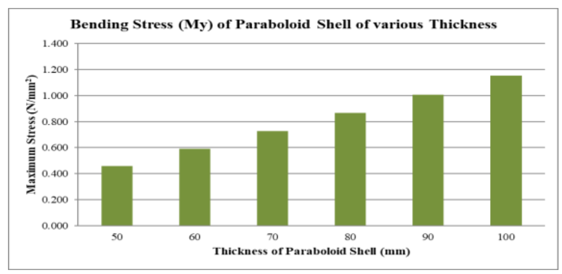


Fig.15 Paraboloid Shell - Bending Stress (My)

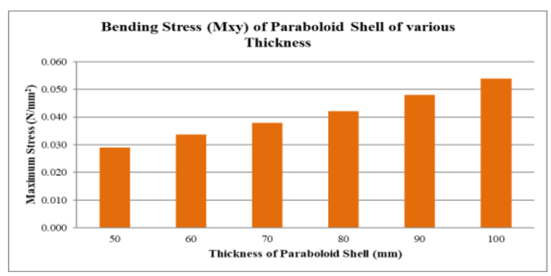


Fig.16 Paraboloid Shell - Bending Stress (Mxy)

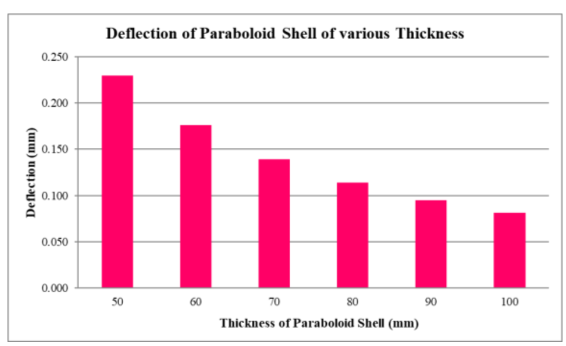


Fig.17 Paraboloid Shell - Deflection (y)

V. CONCLUSIONS

From the analysis of Paraboloid Shell with various thicknesses, the following conclusions are made:

- The deflection of Paraboloid Shell decreases with the increase in thickness of plate element.
- The maximum absolute stress of Paraboloid Shell decreases with the increase in thickness of plate element.
- The membrane stresses (Sx, Sy and Sxy) of Paraboloid Shell decreases with the increase in thickness of plate element.
- The shear stresses (SQx and SQy) of Paraboloid Shell decreases with the increase in thickness of plate element.
- The bending stresses (Mx, My and Mxy) of Paraboloid Shell increases with the increase in thickness of plate element.

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