A New Approach to Generate Set of Prime Numbers with Using Non-Overlapping and Non-Empty Sets

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***Abstract*—This is research review article regards one of the methods to generate prime numbers. For Set P containing Prime elements p_1 through p_n, each to power n, where $1 \leq n \leq \infty$.**

P= $\{p_1^n, p_2^n, p_3^n, \dots \dots . p_n^n\}$ where furthermore, Part A and Part B are partitions on P (i.e. Non overlapping, non-empty subsets that together account f or all elements in $A \subset P, B \subset P, A \notin B, B \notin A, A \cup B = P$) Such that any product derived by multiplying elements of part A will be co-prime to any product derived by multiplying elements of Part B. Now I have to proposed one method to generate Set of Prime numbers with using of following condition. "The absolute difference of any product obtained by multiplying each and every prime element in partition A and then adding or subtracting Any product obtained by multiplying each and every prime element in partition B must also be prime (or the number 1) as long as the result meets the condition that it is less than p_(n+1)^2. Mathematically denoted by following equation." If the following condition is met $p_x = |\prod_{n \in A} n \pm \prod_{n \in B} n| < p_{n+1}^2$ then, p_x is a prime number or the number 1.

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Main Text

The generation of prime numbers plays vital role in a computation stage appearing during various cryptographic setups. this paper shows a simple way to substantially reduce the value of hidden constants to provide much more efficient prime generation algorithms with using of following condition (Theorem).

Theorem 1: "The absolute difference of any product obtained by multiplying each and every prime element in partition A and then adding or subtracting Any product obtained by multiplying each and every prime element in partition B must also be prime (or the number 1) as long as the result meets the condition that it is less than p_(n+1)^2. Mathematically denoted by following equation." If the following condition is met $p_x = |\prod_{n \in A} n \pm \prod_{n \in B} n| < p_{n+1}^2$ then, p_x is a prime number or the number 1.

Proof: Consider that all primes through p_n are represented in either of the two partitions of the set, Part A or Part B. The rest of the primes can be represented by any prime equal or greater to p_(n+1). Hence all primes are accounted for. There are three parts to this proof. To prove that each result for p_x which falls within the defined range is prime, we will first show that p_x is prime to all elements in Part A (Partition A). In the second step, we will prove that p_x is prime to all elements in Part B. Having then shown that p_x is co-prime to all elements p_1 through p_n, in the third and final step we'll prove that p_x is also co-prime to all elements greater than p_(n+1). In combining these three parts we will have proven that any p_x in the defined range of the result set is not divisible by any prime number up through infinity and hence is a prime number.

Step1.
In this step I will show that all results for p_x within the defined range are co-prime (i.e. cannot be divided evenly) to all elements in Partition A. Take variables X,Y, and Z where the relationship can be represented as a single operation of multiplication, that is XY = Z, where X and Y are integers. We can rearrange the equation to say that Z is evenly divisible by X with result Y.

i.e. Z ÷ X = Y. From the definition of multiplication, it follows that if Z is divisible by X, then if we add or subtract from Z any number other than a multiple of X, then this resulting sum is NOT divisible by X evenly. We can represent this as follows.

If Z%X=0, and N%X != 0
Then (Z-N)%X !=0, and (Z+N)%X!=0

If XY%X=0, and N%X!=0
(We are using != to mean 'not equal to' and % to

represent the mod function. The above therefore means that X multiplied by Y is evenly divisible by X and that N is not evenly divisible by X) Then, (XY± N) % X! = 0.

A simple example to illustrate this point would be X = 2, Y=8, N=3. Hence 2*8-3 = 13. Obviously 13 is not divisible by 2. This is because Y is 2^3 while N is NOT a multiple of 2. Now consider that any product of all elements in the set of elements in Part A. We can substitute the product of elements in Part A for the expression XY in the equation above, where X can be represented by any element p_x in Part A and Y can is then represented by the resulting product of all the remaining elements in Part A. We can also substitute the product of all elements in Part B for N in the equation above because we know that any product of Part B cannot be a multiple of any element of Part A due to the very construction that we have defined, that is that Part B contains only prime elements unique from Part A. Because the two parts are 'co prime' to one another, then from the Fundamental Theorem of Arithmetic, we also know that the two products of multiplication of elements within each those sets respectively will also be co-prime to one another. That is, you can't multiply one set of prime numbers then factor it and end up with a different set of prime numbers — that would violate the Fundamental Theorem of Arithmetic. No resulting products of the set of elements in Part A will ever equal the product of the other set of elements of Part B. Moreover, the factors of those two products cannot contain any of the same prime's elements.

Let's get on with the substitute multiplying an integer by a number and then dividing by the same number always produces the same integer so we can a say that $\prod_{n\in A} n\%n_{n\in A}= 0$, and thereby substitute this for XY , and since we know by definition that Part A and Part B are

co-prime, we can express this as $\prod_{n\in B} n\%n_{n\in A}!= 0$ and substitute this for 'n' to prove that

$(\prod_{n\in A} n \pm \prod_{n\in B} n)\%n_{n\in A}!=0$

Finally, since the assignment of elements to Part A and Part B is arbitrary, we can easily switch the labels on the partitions, that is, which is Part A and

which is Part B, and we can represent this as an absolute difference so that we are only considering positive integers in the result set.

If $\prod_{n\in A} n\%n_{n\in A}= 0$ and $\prod_{n\in B} n\%n_{n\in A}!= 0$ then $|\prod n \pm \prod n|\%\, n_{n\in A}\ ! = 0$

Put simply, the result adding or subtracting the Product of all elements in Part B from the Product of all elements Part A and then taking the absolute value or difference will NOT be evenly divisible by any element in Part A. i.e. the resulting product will be co-prime to Part A. We have achieved the first step in our proof and can restate the above statement as:

$|\prod_{n\in A} n \pm \prod n|$ is co-prime to all elements of Part A

Step 2.

In this step, I will prove that all results for p_x within the defined range are co-prime to all elements of Part B.First I will prove that:
If Y%X != 0 (Y is not evenly divided by X) then (Y-X)%X !=0 (Y minus X is not evenly divided by X),

Y-X+X = Y

Rearrange using the associative property of addition/subtraction

⇒ Y=(Y-X)+X

We can substitute (Y-X)+ X for Y into the initial equation Y%X != 0

((Y-X) +X) % X !=0

We can then deduce that (Y-X) is also NOT a multiple of X because if it were then we could represent it as X*N and adding X would also be a multiple of X so that the above equation would then be written as X*N + X = X(N+1)/X !=0

The above equation is an impossible contradiction because any multiple of X must also be divisible by X. Hence, we can say that (Y-X) cannot be represented as X*N and therefore is not a multiple of X. We can write this using the mod function such that when Y-X is divided by X there will be no remainder (i.e. a remainder not equal to zero). Y – X%X ! = 0.

We can substitute ANY product derived from multiplying elements of Part A for variable Y and any product derived from multiplying elements of Part B for variable X to full fill the initial condition in this step that Y%X != 0. We can do this precisely because we have defined these sets to be co-prime and this equation represents what it means to be co-prime. i.e. Now, making the above substitution into the equation

(Y-X)%X !=0, $\prod_{n\in A} n - \prod_{n\in B} n$ is coprime to all elements of Part B.

The above equation could produce negative values when the second product is greater than the first product. In such instances we could switch the labels on the two partitions (since they are arbitrary), or we could just consider the 'absolute difference' as below, which produces the same positive value regardless of which partition is subtracted from the other: We have now completed the second step of our proof. $|\prod_{n\in A} n - \prod_{n\in B} n|$ is coprime to all elements of part B.

We can go through all the steps of this step a second time, just replacing the '-' sign at each and every step with a '+' sign to show that the equation is true regardless of whether

the sign is + or -.

$|\prod_{n\in A} n \pm \prod_{n\in B} n|$ is co prime to all elements of Part B.

Step 3.

In the third and final step, I'll prove that the all results for p_x within the defined range are co-prime to all elements greater or equal to p_(n+1.)

For any number X, it may not have any factor, Y, which greater than the square root of X. Y ≤ square root of X Conversely:
if X < Y^2, then neither Y nor any number greater than Y is a factor of X

We can substitute $|\prod_{n\in A} n \pm \prod_{n\in B} n|$ for variable X ,

and substitute P_(n+1) for Y, so that we have:

if $|\prod_{n\in A} n \pm \prod_{n\in B} n| < p_{n+1}^2$ then $|\prod_{n\in A} n \pm \prod_{n\in B} n|$ is co prime to all prime numbers ≥ p_{n+1}

Combining Conclusions of Step 1 Through 3

We have now completed our third statement and can combine all three statements as follows:

$p_x = |\prod_{n\in A} n - \prod_{n\in B} n|$ is co prime to part A, co prime to Part B, and if less than p_{n+1}^2, then

Also co-prime to all numbers ≥ p_{n+1}

Since Part A and Part B comprise all elements p_1 through p_n, we can restate the above as:

$p_x = |\prod_{n\in A} n - \prod_{n\in B} n|$ is co prime to all elements of Set P from p_1 through p_n and if less than p_{n+1}^2 , then also co prime to all elements ≥ than p_{n+1} and to refine further if $p_x < p_{n+1}^2$ then

$p_x = |\prod_{n\in A} n - \prod_{n\in B} n|$ is not divisible by any elements and is a prime number. We have known concluded our proof that: if the following condition is met $p_x = |\prod_{n\in A} n \pm \prod_{n\in B} n| < p_{n+1}^2$ then p_x is prime number, or the number 1.

A simple example to Illustrate:

Consider the Set P = {1,2,3,5,7} and P_(n+1)^2=11²= 121

There are many ways to partition this small set of prime numbers into two parts. Consider:

Table 1 : Some partitions of the Set P = {1,2,3,5,7} and P_(n+1)^2=11²= 121

PART A	PART B
{1}	{2,3,5,7}
{2}	{1,3,5,7}
{3}	{1,2,5,7}
{5}	{1,2,3,7}
{7}	{1,2,3,5}
{1,2}	{3,5,7}
{1,3}	{2,5,7}
{1,5}	{2,3,7}
{1,7}	{2,3,5}
{2,3}	{1,5,7}
{2,5}	{1,3,7}
{2,7}	{1,3,5}
{3,5}	{1,2,7}
{3,7}	{1,2,5}
{5,7}	{1,2,3}

For this example, let's consider ONLY the following partition:
Part A = {1,2,3}, Part B = {5,7}

Actually, we want to include all powers n of each prime for n is greater or equal to 1, so the correct expression of Part A and Part B is…

Part A = {1^n,2^n, 3^n}, Part B = {5^n,7^n}

Next, we can multiply each prime as many times as we want to derive our two products for each part, as long as we multiply each prime element at least once.

So, then the absolute value of the difference is a prime number if it falls in the acceptable range less than p_(n+1)² ,which in this case is 11² or 121.

Table 2: Part A = {1^n,2^n, 3^n}, Part B = {5^n,7^n}

Few possible products of Part A={1^n,2^n, 3^n}	Few possible products of Part B={5^n,7^n}	$p_x = \left\|\prod_{n\in A} n \pm \prod_{n\in B} n\right\| < p_{n+1}^2$
1*2*3 = 6	5*7 = 35	35+6 = 41 < 121 so 41 is prime 35–6 = 29 < 121 so 29 is prime
1*2*3*2=12	5*7 = 35	35+12 = 47 < 121 so 47 is prime. 35–12 = 23 < 121 so 121 is prime
1*2*3*3=18	5*7 = 35	35–18 = 17 < 121 so 17 is prime 35+18 = 53 <121 so 53 is prime
1*2*3*3*3*3 = 2*34=162	5*7*5 = 175	175–162 = 13 < 121 so 13 is prime

Corollary 1 : An important Corollary to step 1 of this theory is that the size of the gaps between consecutive prime numbers can be infinitely large. Moreover, it provides us with a formula for explicitly identifying all the consecutive non-prime numbers falling within the defined minimum gap.

If you multiply all consecutive primes p_1 through p_n where p_n approaches infinity, you get a product where adding 2 cannot possibly produce a prime number because adding 2 to a multiple of 2 i.e. (2+2n) by definition, results in a number that is also divisible by 2 precisely (n+ 1) times, i.e. 2(n+1) = 2+2n

Adding 3 to any multiple of 3 will likewise produce a result that can be divided by 3. And adding p_n to any number p_n will likewise produce a result divisible by p_n.

If instead of adding, we subtract any p_n from any multiple of p_n, we can deduce the same thing — that the sum is divisible by p_n.

To simplify what we have deduced thus far…

The product of consecutive primes starting with 2 {2,3,…p_n} is called a 'primorial'. Primorials are evenly divisible by each and all primes in the set 2 through p_n

$\prod_{n=2}^{p_n} n\%n_{n\in P}$= 0. As for whether adding any prime number greater than p_(n+1) will produce a prime, $\prod_{n=2}^{p_n} n \pm p_{n+1}$

Corollary 2: we don't know from the Another form of Prime number theorem outside the very small prime numbers. For example 2*3*5 +/- 7 = {23,37} which also fall within the defined range, less than 11² = 121, so both are prime numbers. Whereas for 2*3*5*7 +/- 11 = {199, 221} are outside of the defined range, that is they are greater than 13² = 169. Hence, we should assume in calculating the minimum gap between consecutive primes that the result of the above equation is prime.

Corollary 3: Next we can consider the product of $\prod_{n=2}^{p_n} n \pm 1$

Where the above value is prime, this is known as a 'primorial prime'. we know that 2*3 +/- 1 = {5,7} are prime, that 2*3*5 +/-1 = {29,31} are prime. But with 2*3*5*7 +/- 1= {209,211} we can't say one way or another from the Another form of Prime number theorem because the results fall outside the defined range, which for the set 1, 2,3,5,7 is up to 11²= 121. So for all p_n> 7 we do not know whether the p_n +/- 1 is or is not prime. Hence in calculating the minimum gap size, we will have to assume that that both p_n + 1 and p_n - 1 are both prime.

Corollary 4: Hence the minimum gap size before and after $\prod_{n=2}^{p_n} n$ is p_(n+1) -1

Since p_n in this equation can be any prime and since we know that prime numbers approach infinity (i e. there is no largest prime), than so too do we know that p_(n+1) approaches infinity (there is no largest prime gap).

CONCLUSION

This theorem states only that the equation given produces ONLY primes. For Set P containing Prime elements p_1 through p_n, each to power n, where $1 \leq n \leq \infty$.

P= $\{p_1^n, p_2^n, p_3^n, \dots \dots .. p_n^n\}$ where furthermore, Part A and Part B are partitions on P (i.e. Non overlapping, non-empty subsets that together account f or all elements in $A \subset P, B \subset P$,

$A \notin B,\ B \notin A,\ A \cup B = P$) Such that any product derived by multiplying elements of part A will be co-prime to any product derived by multiplying elements of Part B. Now we have to generate the Prime numbers with using of following condition. “The absolute difference of any product obtained by multiplying each and every prime element in partition A and then adding or subtracting Any product obtained by multiplying each and every prime element in partition B must also be prime (or the number 1) as long as the result meets the condition that it is less than $p\_(n+1)^2$.

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