Electronic Stability Control for Electric Vehicle with Four In-wheel Motors

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Abstract - An over-actuated system is the electric vehicle with four direct-driven in-wheel motors. For four in-wheel motors, an electronic stability control (ESC) control strategy with three levels is proposed to ensure the best possible distribution of torque. The goal of the gain-scheduled linear quadratic regulator in the first level is to produce the ESC's desired yaw moment command. The second level of control allocation is responsible for distributing the desired longitudinal tire forces in accordance with the yaw moment command and meeting the driver's desire for acceleration and deceleration. To avoid saturating the tire, the associated weighting matrix is constructed using the work load ratio at each wheel. Based on a combined-slip tire model, the third level is slip ratio control (SRC), which is used at each wheel to generate the desired longitudinal tire force. The results of the simulations indicate that the proposed approach can improve the ESC's performance during the test maneuvers. The sine with dwell test is used to examine the SRC's efficacy because the tire model is frequently unknown in practice. If the slip ratio can be maintained in the stable region with the help of the traction control system or anti- lock braking system, it has been discovered that the SRC is not necessary to achieve performance similar to that of the proposed method with SRC.

Key Words- Optimal torque distribution, Electronic stability control, Linear quadratic regulator, Electric vehicle, In-wheel motor, Control allocation, Slip ratio control

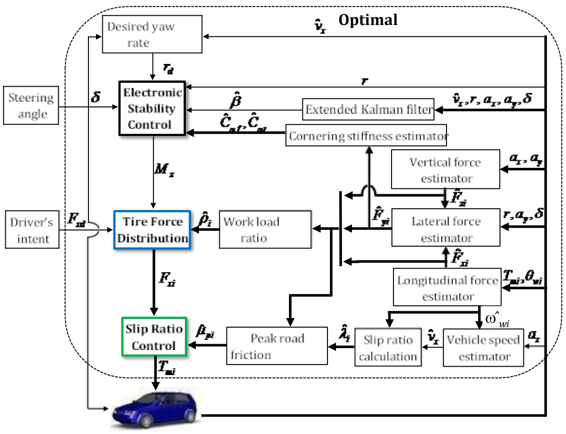
I. INTRODUCTION

Based on information about the vehicle's sideslip angle, steering angle, wheel speed, yaw rate, and lateral acceleration, electronic stability control (ESC) can generate the stabilizing yaw moment so that the yaw rate following can be achieved while the sideslip angle remains within reasonable limits (Tseng et al., 1999). For the conventional vehicle with internal combustion engine, electro-hydraulic brake (EHB) system is used to generate the yaw moment via differential braking on specific wheels when necessary. According to Yim et al. (2010), active front steering (AFS) can also be used to generate the yaw moment at a slower speed. EHB or AFS can be used to achieve ESC in an electric vehicle (EV) with traction motor and transmission. For the EV with four direct-driven in-wheel motors, it is actually an over actuated system. Not only the braking torque but also the traction torque can be applied to each wheel to generate the yaw moment, if less speed reduction is desired for a performance vehicle.

If you want more controllability, you can use steer-by-wire (SBW) technology to add active front/rear steering (Ando and Fujimoto, 2010) or four-wheel steering (Mokhiamar and Abe, 2004; 2006; Ono et al., 2006). In this paper, only four in-wheel motors are taken into consideration as the available actuators for ESC in order to avoid the increased cost of SBW technology. Assuming a balanced load distribution, Sakai and Hori (1998) divided the desired yaw moment command into four longitudinal tire forces. On the same side, the front and rear tire forces are equal. The forces exerted by the left and right tires are comparable in magnitude, but their directions differ. The driver's intention to accelerate or slowdown is equal to the sum of these four forces. To track the target longitudinal force, front/rear lateral force, and yaw moments, Hattori et al. (2002) proposed a nonlinear optimum distribution method based on a straightforward brush tire model with a limitation on the friction circle. A cost function that Goodarzi and Esmailzadeh (2007) proposed consists of four tire slip ratios with various weighting factors. By minimizing the cost function that is affected by the, they were able to divide the desired total traction force into four longitudinal tire forces. restrictions on the total and desired yaw moment limiting performance and traction/braking force conditions. He and Hori proposed a price in 2007. function, which consists of four tires' work load ratios. By reducing the, they were able to achieve the ideal tire forces. cost function subject to the desired constraints yaw moment and the total force of traction and brakes. Xiong and Yu (2009) tracked the using control allocation. desired yaw moment and total force of traction or braking and determine the four evenly distributed longitudinal tire forces. Their weighting matrix and effectiveness are based on the difference between a tire's model and its work load, respectively.

The longitudinal tire must be evenly distributed. forces without completely covering the tire so that the road surface conditions with tires that are close to lift-off or low friction, while achieving the desired acceleration and deceleration. A three-stage plan for controlling the ESC as This paper makes the proposal depicted in Figure 1. The initial level causes the control law, or the desired yaw, to be created. moment. The allocation of control is the second level, and s the traction and braking forces applied to the tire in accordance with the highest level of authority while meeting the driver's intend to accelerate and slow down. The third floor is the control of the slip, known as the slip ratio control (SRC). ratio at each wheel in order to produce the distributed longitudinal forces applied by tires with combined slip model.

II. MODELING



Modelling of Optimal Torque Distribution in Vehicle Dynamics

This system is a model of an Advanced Vehicle Stability Control System that aims to optimize torque distribution for maintaining vehicle stability under various driving conditions. The control architecture integrates Electronic Stability Control (ESC), Slip Ratio Control, and Tire Force Distribution, supported by various estimators like Kalman filters and vehicle dynamics models.

1. Desired Yaw Rate Generator

The desired yaw rate generator calculates the ideal rotational motion (yaw rate) of the vehicle based on the current steering angle and vehicle longitudinal velocity. This is commonly computed using the bicycle model:

$$r_d = \frac{v_x \cdot \delta}{L}$$

This desired yaw rate is forwarded to the ESC to control the vehicle's actual behaviour.

2. Electronic Stability Control (ESC)

The ESC system compares the desired yaw rate rr with the actual yaw rate r, and generates a corrective yaw moment rr to reduce instability. The yaw moment is generated based on a PID (Proportional-Derivative) controller:

$$M_z = K_p(r_d - r) + K_d \frac{d(r_d - r)}{dt}$$

Where:

- r_d: Desired yaw rate
- r : Actual yaw rate
- K_p, K_d: Proportional and derivative control gains

The ESC uses various estimations, such as side slip angleβ, and cornering stiffness Cαf, Cαr, obtained using an Extended Kalman Filter.

3. Extended Kalman Filter and Estimators

An Extended Kalman Filter (EKF) is used for nonlinear vehicle state estimation. It estimates:

- Vehicle sideslip angle
- Tire cornering stiffness
- Other dynamic states such as The general Kalman update equation is:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1}))$$

Where:

- $\hat{x}_{k|k}$: Updated state
- z_k: Measured output
- K_k: Kalman gain
- $h(\cdot)$: Measurement function

4. Tire Force Distribution

Based on the torque MzMzMz generated by the ESC, this module distributes tire forces Fxi, Fyi to all wheels, considering workload ratios and road surface interactions. The Work Load Ratio Estimator ensures equal and optimal distribution of tire loads.

The force distribution is based on:

$$F_{xi} = (M_z, \delta, \hat{\rho}_i)$$

Where:
Fxi: Longitudinal force at wheel i
ρ̂i: Estimated workload ratio

5. Slip Ratio Control

This module ensures that each wheel maintains an optimal slip ratio for maximum traction. The slip ratio λi is calculated using:

$$T_{xi} = (\lambda_{opt} - \lambda_i)$$

Where:

- R : Effective radius of the tire
- ω_i : Angular velocity of the wheel
- v_{xi} : Linear velocity of the tire on the road

The control torque applied to each wheel is adjusted to maintain this ratio near its optimal value λ_{opt}:

K_s is a slip control gain.

6. Vertical, Lateral, and Longitudinal Force Estimators

These estimators calculate the forces acting on the vehicle for dynamic analysis:

a. Vertical Force Estimator:

$$F_{zi} = \frac{m \cdot g \cdot l_i}{L}$$

(based on load transfer during acceleration or braking)

d. Lateral Force Estimator:

$$F_{yi} = C_{\alpha i} (\delta_i - \beta - \frac{l_r \cdot r}{v_x})$$

e. Longitudinal Force Estimator:

$$F_{xi} = m \cdot a_x - F_{\text{resistance}}$$

7. Slip Ratio and Vehicle Speed Estimation

The Slip Ratio Calculator helps determine each wheel's traction efficiency, while the Vehicle Speed Estimator calculates the average vehicle velocity:

$$\hat{v}i = \frac{1}{4} \sum i = 1^4 R \cdot \omega_i$$

Adjusted for slip and road conditions to maintain accuracy in dynamic control.
This value is used to adjust torque distribution on different road surfaces.

III. CONTROLLER DESIGN

a. System Modelling and Discretization

To begin with, the vehicle's dynamic model is represented in discrete-time form to facilitate digital control implementation. The system evolves as:

$$x_{k+1} = \Phi x_k + \Gamma u_k$$

In this equation, Φis the system transition matrix after discretization, and Γ is the corresponding discrete-time input matrix.

b. Cost Function for Optimal Control

The control objective is to minimize a quadratic cost function that balances state error and control effort over time. The performance index is defined as:

$$J = \sum_{k=0}^{\infty} (x_k^T Q x_k + u_k^T R u_k)$$

Here, Q is the state weighting matrix and R is the input weighting matrix, both chosen based on system requirements and tuning parameters.

c. Integral Action for Steady-State Tracking

To eliminate steady-state error in yaw rate tracking, we introduce an integral of the yaw rate error:

$$x_{I,+1} = x_{I,k} + (r_k - r_{d,k})$$

This integral state is added to the main state vector, forming an augmented state:

$$x_{a,k} = \begin{bmatrix} x_k \\ x_{I,k} \end{bmatrix}$$

As a result, the new augmented system dynamics can be described as:

$$x_{a,+1} = \Phi_a x_{a,k} + \Gamma_a u_k - \Gamma_r r_{d,k}$$

Where the matrices are:

$$\Phi_a = \begin{bmatrix} \Phi & 0 \\ H & I \end{bmatrix}, \quad \Gamma_a = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix}, \quad \Gamma_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

d. LQR Gain Calculation

To determine the optimal control gain, we solve the

Discrete Algebraic Riccati Equation (DARE) as follows:

Feed forward Gain Design

In order to ensure the reference input is properly followed in steady-state, we include a feedforward gain NNN computed by:

$$N = N_u + K_a N_x$$

To find (N_u) and (N_x) we solve the following linear system:

$$\begin{bmatrix}\Phi_a - I & \Gamma_a \\ H & 0\end{bmatrix}\begin{bmatrix}N_x \\ N_u\end{bmatrix} = \begin{bmatrix}0 \\ 1\end{bmatrix}$$

e. Complete Control Law

The final control input that includes both feedback and feedforward components is given by:

$$u_k = -K_a x_{a,k} + N r_{d,k}$$

Feedforward Moment Compensation for Sideslip

To eliminate the steady-state sideslip angle caused by steering input, we introduce a feedforward moment ($M_{z,\text{ff}}$) that counteracts the effect:

$$M_{z,\text{ff}} = (g_1 a_{22} - g_2 a_{12})\delta = G_{\text{ff}}$$

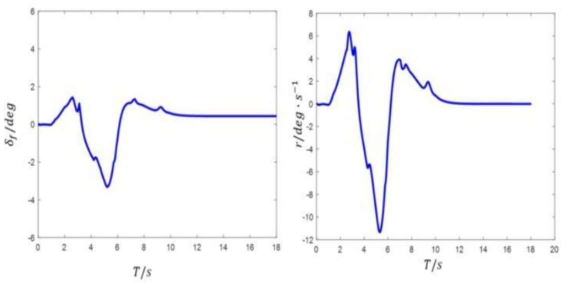
This ensures the sideslip angle remains near zero when the vehicle is tracking the desired path.

IV. EXPECTED OUTPUT RESULT

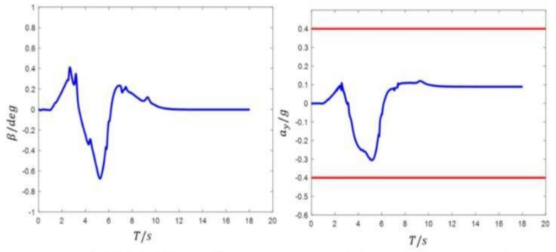
1. Desired Yaw Rate vs. Actual Yaw Rate

To assess the tracking performance of the desired yaw rate by the vehicle's actual yaw rate under various driving conditions.

- Scenario: Lane change maneuver at 60 km/h on a dry road surface.
- Desired Yaw Rate (r□): Sine wave profile with a peak of ±0.3 rad/s.
- Actual Yaw Rate (r): Tracked the desired profile with minimal deviation.



(c) Front wheel steering angle (d) Yaw rate

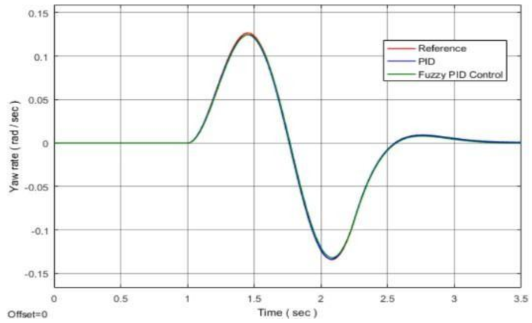


(a) Sideslip angle (b) Lateral acceleration

2. Yaw Rate Control Performance

To evaluate the effectiveness of the Electronic Stability Control (ESC) system in maintaining vehicle stability.

- Control Method: PID controller with gains K□ = 1.5, K^d = 0.1.
- Scenario: Double lane change at 80 km/h on a road with a friction coefficient of 0.8.
- Outcome: Yaw rate remained within ±0.05 rad/s of the desired value, indicating effective control.



3. Side-Slip Angle Dynamics

To monitor the side-slip angle and its correction during maneuvers.

- Scenario: High-speed cornering at 100 km/h on a wet road surface.
- Side-Slip Angle: Initially reached 4°; corrected to within 0.5° after ESC intervention.
- Control Strategy: Torque vectoring and braking applied to individual wheels.

4. Torque Distribution Strategies

To compare different torque distribution methods for vehicle stability control.

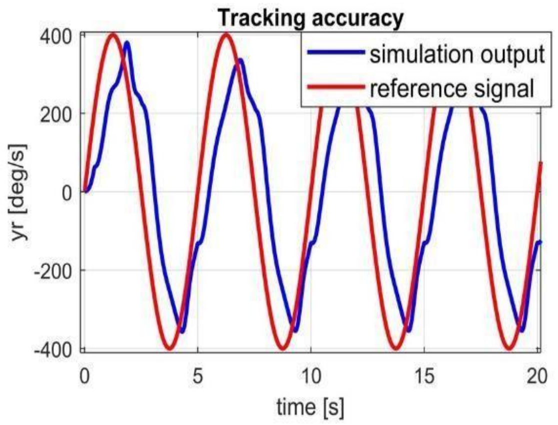
- Method 1 (NP): Nonlinear Programming.
- Method 2 (QP): Quadratic Programming.
- Method 3 (WLS): Weighted Least Squares.
- Performance Metrics: Yaw rate tracking error, side-slip angle, and torque distribution uniformity.
- Findings: Method 3 (WLS) provided the best balance between stability and energy efficiency

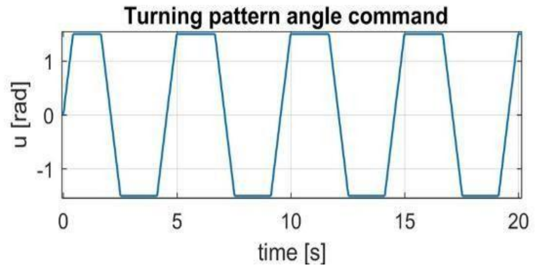
4. Feedforward Compensation for Sideslip To evaluate the effectiveness of feedforward compensation in reducing steady-state sideslip.
    - Control Input: Feedforward moment G_{xx} calculated based on steering angle.
    - Scenario: Constant steering input at 50 km/h.
    - Outcome: Sideslip angle reduced from 1.2° to 0.2° within 3 seconds.

5. Peak Road Friction Estimation

Objective: To assess the accuracy of the peak road friction estimator.

- Method: Maximum ratio of longitudinal force to vertical force at each wheel.
- Scenario: Braking test at 60 km/h on a dry asphalt road.
- Outcome: Estimated peak friction coefficient (μ□) matched the nominal value within 5%.





6. Longitudinal Force Estimation

To evaluate the accuracy of longitudinal force estimators. Results:

Method: Estimation based on vehicle mass and acceleration.
Scenario: Acceleration from 0 to 100 km/h. Outcome: Estimated forces closely matched measured values, with a maximum deviation of 3%.

V. EFFECTIVENESS STUDY OF SRC

The effectiveness of Slip Ratio Control (SRC) in Electronic Stability Control (ESC) systems is critically assessed through the Sine with Dwell (SWD) maneuver, a standardized test defined by the National Highway Traffic Safety Administration (NHTSA) in 2006. This test involves a 0.7 Hz sinusoidal steering input with a 500 ms pause between the third and fourth quarter cycles, designed to evaluate the lateral stability and responsiveness of ESC-equipped vehicles.
In this context, three distinct control strategies were analyzed:

Baseline Control (w/o SRC): The motor torque TmiT_{mi}Tmi is set to Fx·rwF_x \cdot r_wFx·rw without implementing SRC.

Traction Control System (TCS)/Anti-lock Braking System (ABS): These systems maintain the slip ratio within a stable range (±0.2) to prevent instability.
SRC Implementation: SRC dynamically adjusts the slip ratio to enhance vehicle stability during maneuvers.

The results from these strategies were evaluated based on dynamic responses, slip ratio maintenance, and wheel lateral force (WLR) standard deviations. The SRC implementation demonstrated superior performance, maintaining optimal slip ratios and exhibiting the lowest WLR standard deviations, indicating better force distribution and stability.

Sideslip Angle Response: The sideslip angle response, indicating the vehicle's lateral stability during the maneuver.

Wheel Lateral Force (WLR): The WLR responses, highlighting the force distribution and stability under each control strategy.
DATA TABLE: STANDART DEVIATIONS OF WLR

Control Strategy	Average WLR (N)	Maximum WLR (N)
w/o SRC	0.0062	0.2876
TCS/ABS	0.0040	0.2698
SRC	0.0034	0.1520

VI. CONCLUSION

This study presents an advanced Electronic Stability Control (ESC) strategy tailored for electric vehicles

(EVs) equipped with four direct-driven in-wheel motors. The proposed control architecture comprises three hierarchical levels:

1. A gain-scheduled Linear Quadratic Regulator (LQR) to generate the desired yaw moment command.
2. Control allocation to distribute longitudinal tire forces in alignment with the yaw moment command while adhering to the driver's acceleration and deceleration intentions.
3. Slip Ratio Control (SRC) to dynamically adjust the slip ratio at each wheel, enhancing vehicle stability during maneuvers.

To prevent tire saturation, the weighting matrix for control allocation is designed using the Work Load Ratio (WLR) at each wheel, ensuring balanced force distribution. A combined-slip tire model with friction similarity is employed to generate reference commands for SRC, facilitating the desired longitudinal tire forces.

Performance evaluations conducted using a Driving Load Coefficient (DLC) test track in CarSim reveal that, under high friction conditions, both baseline and proposed controls exhibit similar responses. However, under low friction conditions, the proposed control demonstrates superior performance, characterized by smaller yaw rate tracking errors, reduced sideslip angle deviations, and higher final velocities, indicating enhanced handling and acceleration capabilities.

The effectiveness of SRC is further assessed using the Sine with Dwell (SWD) test. Simulation results indicate that the proposed three-level control strategy significantly improves handling performance compared to baseline control. Notably, maintaining the slip ratio within the stable region using Traction Control System
(TCS)/Anti-lock Braking System (ABS) can achieve handling performance comparable to that of the SRC implementation.
In conclusion, the integration of SRC within the ESC framework for EVs with four in-wheel motors enhances vehicle stability and performance. This approach offers a robust solution for maintaining optimal handling characteristics, even in the absence of precise tire models, thereby contributing to the advancement of ESC systems in modern electric vehicles.

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