Effects of Parameters of Friedmann Equations on Big Crunch

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***Abstract*—A biggest challenge for today's astrophysicist is to solve the mystery of primordial evolution of the universe. This is also known as belongs to Planck's era. In this stage space-time singularity and inflationary phase all occur when universe was just of the size 10^{-5}m. All this evolution occurs in a tiny fraction of a second. This description is made by joining the cosmological principle of isotropy and homogeneity, the Hubble law, and the Einstein's field equations of General Relativity in the Big Bang Theory or Standard Cosmological Model (SCM). By solving equation proposed by Albert Einstein in the general relativity the cosmological singularity proposed by Alexander Friedmann explain well the Big Bang model of origin of our universe. The expansion of universe started some 15 billion year ago with Big Bang. The studies of red shift in the spectrum of galaxies in near vicinity suggest the expansion of our universe. The experimental studies of Cosmic Microwave Background (CMB) also suggest that Universe is originated due to Big Bang. Early of the twentieth century some astrophysicist started to propound that how long this expansion will last and what is going to be happened with this universe. There are three different fate of our universe suggested in popular literature which are: Big Rip, Big Freez and Big Crunch. In the present work we proposed solution of Friedmann equations and effect of their parameters on Big Crunch the ultimate fate of our universe.**

Index Terms*—Big bang, Big crunch, Cosmological Constant, Friedmann Equations.

I. INTRODUCTION

Einstein gravitational field equations are used to describe the general evolution of time after big bang till today [1,2]. Using field equations proposed by Einstein, Friedmann proposed cosmic singularity by which origin of the universe is explained. In the year of 1965 a remarkable discovery of Cosmic Microwave Background by Arno Penzias and Robert Wilson [3], prove the dynamic nature of universe proposed by Friedmann. Using Robertson-Walker metric with the warp function $R(t)$, which is cosmic scale factor, one can find solution of Friedmann equation to propose Big Bang [4,5]. Using the ideas proposed by Friedmann, Georges Lemaitre said that the universe is originated very hot and close state called primordial atomic state and exploded due to big bang[6]. The Friedmann-Lemaitre universe was homogenous and isotropic in nature. It is given by Robertson-Walker metric (i, j = 1,2,3):

$$ds_{RW}^2 = c^2dt^2 - R^2(t)\gamma_{ij}dx^i dy^j \qquad 1$$

Here γ_{ij} is third spatial metric of constant gaussian curvature parameter k.

Solution of Friedmann Equations: As we know that Einstein's gravitational field equation is

$$G^{\mu\nu} - \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4}T^{\mu\nu} \qquad 2$$

The left-hand side of the equation 2 is depends on $g^{\mu\nu}$ and is geometrical and shows that gravitation is playing significant role in space-time geometry. The right-hand side of equation 2 is depend on energy momentum tensor $T^{\mu\nu}$.The energy momentum tensor can be determined by distribution of mass/energy of universe. If a perfect fluid is considered which is moving with relativistic velocity then

$$T^{\mu\nu} = (\epsilon + p)\frac{u^\mu u^\nu}{c^2} - pg^{\mu\nu} \qquad 3$$

Here ϵ is energy density, p is pressure and u^μ is four velocity and defined as $\frac{dx^\mu}{d\tau}$ where $(d\tau = \frac{1}{c}ds)$, obviously

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{8\pi G}{c^4}T^{\mu\nu}$$

In right hand side first term is representing Ricci tensor and the R in second term give scalar curvature. By inserting equation 1 and 3 in 2 the Friedmann equation is obtained as

$$\dot{R}^2 + kc^2 = \frac{8\pi G}{3c^2}\epsilon_{tot}R^2 + \frac{\Lambda}{3}c^2R^2 \qquad 4$$

By differentiating again, the above equation gives

$$\ddot{R} = -\frac{8\pi G}{3c^2}(\epsilon_{tot} + 3p_{tot})R + \frac{\Lambda}{3}c^2R \qquad 5$$

Here ϵ_{tot} and p_{tot} are total energy density respect to total pressure by adding up various components. In addition to above the energy conservation is obtained

$$\dot{\epsilon_k} = -3(\epsilon_k + p_k)\frac{\dot{R}}{R} \qquad 6$$

The above is obtained by vanishing $T^{\mu\nu}$. Since equation 4,5 and 6 are not independent thus they are not sufficient to determine the $R_t(t)$, $\epsilon_{tot}(t)$ and $p_{tot}(t)$. Thus a fourth relation is introduced which give pressure p_k of the k^{th} component in the terms of energy density ϵ_k and called equation of state.

$$p_k = \omega_k \epsilon_k \qquad 7$$

Here $k = r, m, \Lambda$ with

$$\omega_r = \frac{1}{3}, \quad \omega_m = 0, \omega_\Lambda = -1 \qquad 8$$

Thus pressure p_k is linear function of energy density ϵ_k and a constant parameter ω_k. Solution of equation 6 is obtained by keeping ω_k = constant, thus

$$\epsilon_k(t) = \epsilon_{k,0}(\frac{R_0}{R(t)})^{3(1+\omega_k)} \qquad 9$$

By inserting the value of equation 7 and 9 in equation 4 we will get a nonlinear differential equation which determine the cosmic scale factor $R(t)$.

As it is considered that at the time of Big Bang $t = 0$ we get $R(0) = 0$, and hence $\dot{R}(0) > 0$. The interesting results obtained if we put $t = \infty$ by considering that Universe will come to an end at the time which is known as Big Crunch.

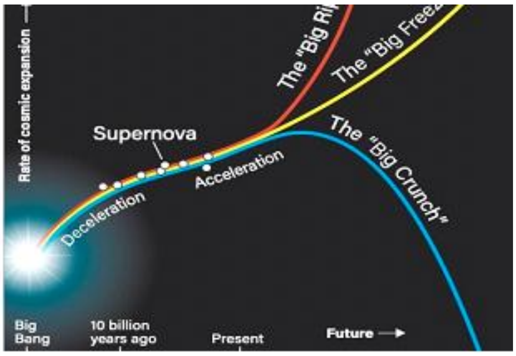

Fig1: The Beginning to the end of the Universe The Big Crunch vs other theories Courtesy Big Freeze by Eric Betz "Astronomy" 31 Jan 2021

Parameters in Friedmann' equations affecting big crunch: In this part we discuss consequences of various parameters of Friedmann' equations on big crunch.

As we know that Friedmann' two equations are

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \qquad 10$$

And

$$\frac{\ddot{a}}{a} = -\left(\frac{4\pi G}{3}\right)(\rho + 3p) + \frac{\Lambda}{3} \qquad 11$$

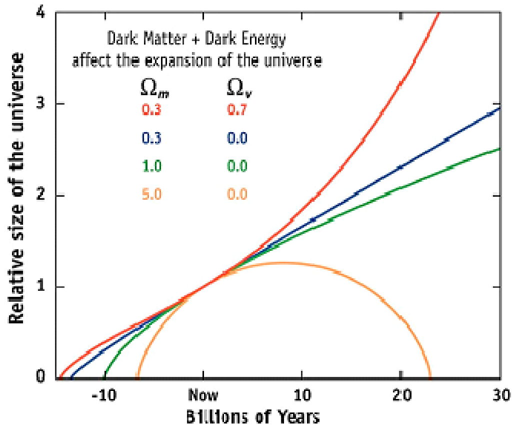

Fig 2 : Fate of Universe in the view of DM and DE

Where, $\dot{a}$ is time derivative of scale factor. $a(t)$ is the scale factor of universe at time t. ρ is total energy of matter and radiations and k is curvature of universe which decide open, closed or flatness of the universe and Λ is cosmological constant.

Properties of cosmological constant:

Dark Energy: Lambda represents a form of energy density that permeates space. Unlike matter, it does not dilute as the universe expands; instead, its effects grow stronger over time, leading to accelerated cosmic expansion.

Negative Pressure: The pressure associated with the cosmological constant is negative, which leads to the repulsive gravitational effect necessary for accelerating the expansion of the universe. This can be described by the equation of state:

$$P = -\rho c^2 \qquad 12$$

Here symbols have usual meaning.

Conformal Time: Conformal time is a crucial concept in cosmology, providing a means of simplifying the equations governing the dynamics of the universe. It is defined in such a way that the expansion of the universe is represented more straightforwardly than in standard cosmic time. Relation between conformal time η and cosmic time t by:

$$d\eta = \frac{dt}{a(t)} \qquad 13$$

Integrating this expression leads to the transformation between cosmic time and conformal time whereby standard time intervals are weighted by the scale factor a(t). This transformation is especially useful when studying the early universe when the scale factor is significantly smaller than one. Using conformal time, the Friedmann equations can be rewritten to bring forth a clearer interpretation of the universe's dynamics during expansion or collapse. For example, in conformal coordinates, the Friedmann equations can express the time evolution of the scale factor in terms of energy densities. This allows cosmologists to analyse the behaviour of the universe at different epochs effectively. We use Python© enabled coding and Google Colab© to analyse various parameters effecting big crunch. Here are some results obtained by our team, we focus basically on cosmological constant and value of density parameter to evaluate scale factor.

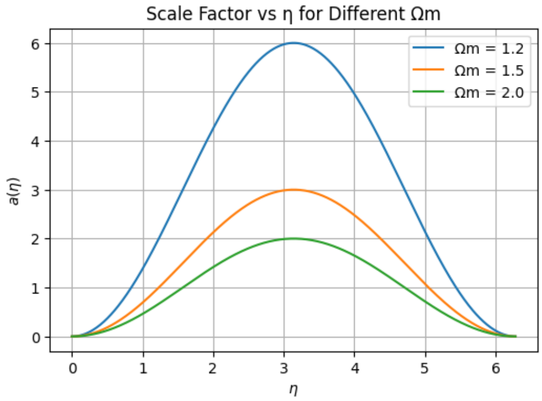

Fig 3: Scale factor for various values of Ω_m

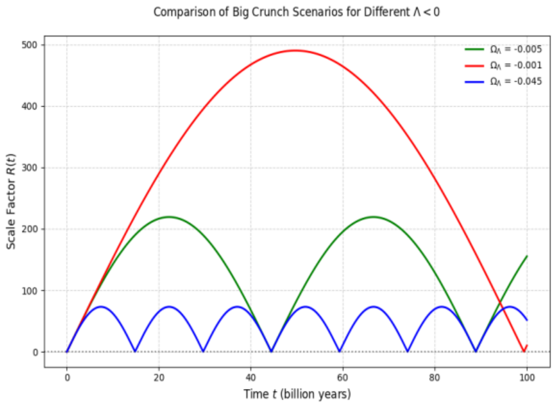

Fig 4: Comparison of big crunch scenario for various negative values of cosmological constant.

Discussion: In figure 3 the plot of scale factor against conformal time shows significance change in scale factor as the value of density parameter changes. In figure 4 scale factor and time is plotted, it clearly indicated that the time of dooms day will be decided by energy density parameter. Figure 5 shows change in time of dooms day decided by pressure and value of cosmological constant. In figure 6 evolution of scale factor is plotted which shows how the value change as universe transit between Big Bang and Big Crunch.

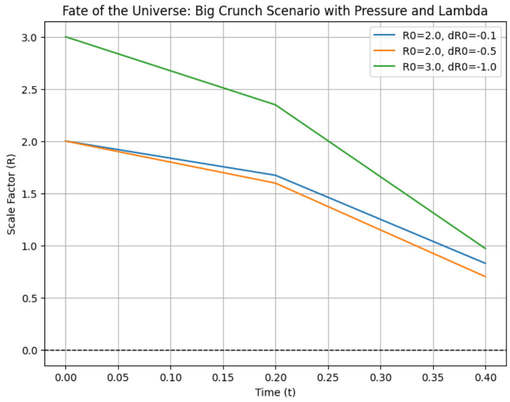

Fig 5: The fate of universe in big crunch scenario with considering pressure and cosmological constant is plotted

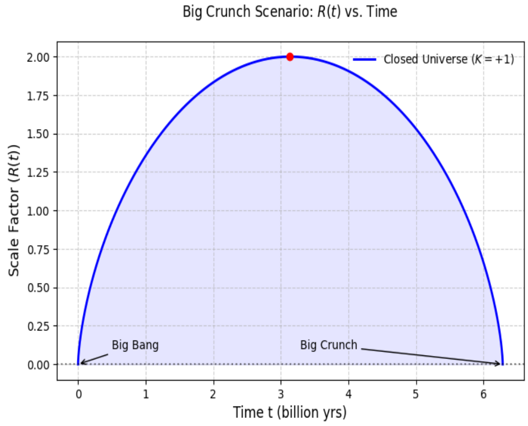

Fig 6: Big crunch scenario with considering scale factor R(t) vs time.

II. CONCLUSION

The Friedmann equations and their various parameters play very significant role to understand the physical chemistry of our universe. The equations and their parameters describe what are the elements by which the universe is made, it also helps to understand how the universe is evolved. In the present work we present an idea how the universe come to an end by giving a nonlinear differential equation. The above solution give light on the understanding about Big Crunch which can be the ultimate fate of the universe and fit well with the idea of cyclic universe proposed by Albert Einstein in the beginning of twentieth cent ury [8].

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