Understanding Big Crunch by Poincaré Patch

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Abstract: A biggest challenge for today's astrophysicist is to solve the mystery of primordial evolution of the universe. This is also known as belongs to Planck's era. In this stage space-time singularity and inflationary phase all occur when universe was just of the size 10^{-5}m. All this evolution occurs in a tiny fraction of a second. This description is made by joining the cosmological principle of isotropy and homogeneity, the Hubble law, and the Einstein's field equations of General Relativity in the Big Bang Theory or Standard Cosmological Model (SCM). The SCM is failed to describe first moment in which universe emerges out in presence of singularity. Due to failure of above model quantum theory of gravitation is needed to understand the evolution of early universe. Quantum gravity (QG) is a field of theoretical physics that need to describe gravity by the principles of quantum mechanics, and where quantum effects cannot be ignored, such as in the vicinity of black holes or similar compact astrophysical objects where the effects of gravity are strong (such as neutron stars). Though the theory is able to unfold many mysteries but the observational results to prove it are still not existing. The Quantum Cosmology provides some simulating models to understand the primordial evolution by adding some hypothesis. The SCM well explain the observational data with consistency but have some limitations. In present paper Anti de Sitter and Conformal Field Theory are combined together to study condensed matter made up of extra dimensional particles like Dilaton and Strings which are going to decide ultimate fate; the Big Crunch of our Universe. The AdS/CFT correspondence is used to understand the transition from expanding universe to non-relativistic transformation to obtain Big Crunch. We also discuss how the Poincaré Patch may improve understanding about Big Crunch-Big Bang transition.

Index Terms- Big Bang, Big Crunch Conformal Field Theory, Poincaré Metric, Poincaré Patch.

I. INTRODUCTION

A biggest challenge for today's astrophysicist is to solve the mystery of primordial evolution of the universe. This is also known as belongs to Planck's era. In this stage space-time singularity and inflationary phase all occur when universe was just of the size 10^{-5}m. All this evolution occurs in a tiny fraction of a second. This description is made by joining the cosmological principle of isotropy and homogeneity, the Hubble law, and the Einstein's field equations of General Relativity in the Big Bang Theory or Standard Cosmological Model (SCM). The SCM is failed to describe first moment in which universe emerges out in presence of singularity. Due to failure of above model quantum theory of gravitation is needed to understand the evolution of early universe. Quantum gravity (QG) is a field of theoretical physics that need to describe gravity by the principles of quantum mechanics, and where quantum effects cannot be ignored,[1] such as in the vicinity of black holes or similar compact astrophysical objects where the effects of gravity are strong(such as neutron stars). Though the theory is able to unfold many mystery but the observational results to prove it are still not exist. The Quantum Cosmology provides some simulating models to understand the primordial evolution by adding some hypothesis [2-6]. The SCM well explain the observational data with consistency but have some limitations. In present investigation Anti de Sitter and Conformal Field Theory are combined together to study condensed matter made up of extra dimensional particles like Dilaton and Strings which are going to decide ultimate fate; the Big Crunch of our Universe. The dilaton appear in theory when we study volume of compact dimensions which vary in extra dimension. The cosmic singularity is a point where function takes infinite value for example it exist at centre of black hole. The Conformal Field Theory is a type of quantum field theory which remains invariant and follows Liouville's theorem. The present work is based on 2-dimensional dilaton gravity-scalar theories. The 2-dimensional dilaton gravity helps to understand Big Crunch cosmological singularities. The two dimensional model of gravity convert higher dimensional difficult calculation into

simplest way to solve the problems. The gravitational theories are defined in a higher dimensional space-time containing at least the d dimension of particle theory plus one extra dimension of infinite extent. They often include number of finite dimensions in the form of sphere for example depending on the context, the correspondence is known as a Gauge/Gravity duality, Gauge/String duality or AdS/CFT (Anti de Sitter/ Conformal Field Theory) correspondence[7]. The AdS/CFT correspondence is used to understand the transition from expanding universe to non relativistic transformation to obtain Big Crunch. AdS is the theory of quantum gravity while CFT is the theory of quantum field theory.

II. BIG CRUNCH IN BRIEF

The expansion of universe started some 15 billion year ago with Big Bang. The studies of red shift in the spectrum of galaxies in near vicinity suggest the expansion of our universe. The experimental studies of Cosmic Microwave Background (CMB) also suggest that Universe is originated due to Big Bang. Early of the twentieth century some astrophysicist started to propound that how long this expansion will last and what is going to be happened with this universe. There are three different fate of our universe suggested in popular literature which are (i) Big Rip, (ii) Big Freez and (iii) Big Crunch[8]. Here we are discussing the Big Crunch as other two are not part of our study.

Fig1: The Beginning to the end of the Universe The Big Crunch vs Big Freeze by Eric Betz "Astronomy" 31 Jan 2021.

The Fig 1 above is described by [8] and above Fig 2 on right is by WMAP (Wilkinson Microwave Anisotropy Probe experiment) which shows that the fate of Universe is ultimately depend on density of

dark matter and dark energy. In the next section we discuss the Anti de Sitter and Conformal Field theory and its aspects to explain the Big Crunch.
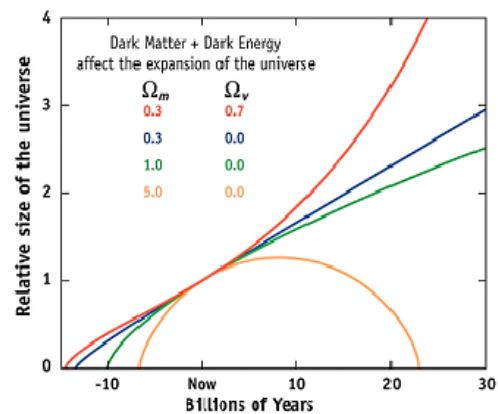

Fig 2 : Fate of Universe in the view of DM and DE Courtesy NASA WMAP Collaboration [12]

Anti de Sitter/ Conformal Field Theory (AdS/CFT) & Big Crunch

The idea of AdS/CFT duality arises from superstring theory. As we know that superstring theory is a major tool of Grand Unification Theory (GUT). The GUT unify four fundamental forces in nature which are gravity, electromagnetic force, strong force and weak force. The AdS/CFT duality claims following equivalence between two theories:

Since AdS/CFT claims relation between four dimensional physics and five dimensional physics therefore it is also called a holographic theory. We know that an optical hologram encodes a three dimensional image on a two dimensional object. Similarly the holographic theory encodes 5-dimensional theory by a 4-dimansional theory. The gauge theory well explain the three forces namely electromagnetic (by $U(1)$), Strong force ($SU(3)$) also known as QCD (Quantum Chromo Dynamics) and weak force but gravity is not explained by gauge theory.

> Strongly-coupled 4-dimensional gauge theory
>
> = Gravitational theory in 5-dimensional AdS spacetime

The strong force is really strong that we are not able to understand it for strong coupling by gauge theory. For the same we use AdS/CFT duality which explain

strong coupling using curved space time namely AdS spacetime.

De Sitter solved the Einstein's equation by considering a space as sphere with constant positive curvature (For expansion). On the other hand Anti de Sitter (AdS) space time have negative curvature (For contraction!). The gauge theory live on 4-dimensional boundary of space time [9]. The AdS/CFT is first adopted for explaining singularity in black hole. Using a black hole one can get glimpse of holography. The AdS/CFT duality may also give details of singularity in big crunch.

Fig3: The above is the famous cows figure used to explain effect of negative curvature. All the cows are having same shape (invariance) and proper size but due to negative curvature they appear smaller near boundary. This happened because of projection of 3 dimensional space on 2 dimensional paper. [7]

AdS/CFT enables one to analyze a strongly-coupled gauge theory using the AdS spacetime. However, there are a number of significant differences between the realistic SU(3) gauge theory and the gauge theory studied in AdS/CFT.

First, AdS/CFT typically considers a $SU(N_c)$ gauge theory. In such a theory, N_c plays a role of a parameter, and the "strong coupling" is the so-called large-N_c limit, where one tunes N_c in an appropriate way.

Second, AdS/CFT typically considers a supersymmetric gauge theory which has supersymmetry. In particular, the $\mathcal{N} = 4$ Super-Yang-Mills theory (SYM) provides the simplest example of AdS/CFT. Here, $\mathcal{N} = 4$ denotes the number of supersymmetries the theory has. The theory also has the scale invariance since the theory has no dimension parameter. Furthermore, the theory has a larger symmetry known as the conformal invariance which contains the Poincaré invariance and the scale invariance. Such a theory is in general known as a conformal field theory, or CFT. This is the reason why the duality is called AdS/CFT.

III.Poincaré Coordinate System of AdS Spacetime

In this section we discuss the Poincaré coordinates (t,r) however Static coordinate and Conformal coordinate can also be used for consideration but that is beyond the scope of this work. The Poincaré coordinate system is defined as:

$$Z = \frac{Lr}{2}\left(-t^2 + \frac{1}{r^2} + 1\right)$$

$$X = Lrt$$

$$Y = \frac{Lr}{2}\left(-t^2 + \frac{1}{r^2} - 1\right)$$

When $(r > 0, t: -\infty \to \infty)$. The metric become

$$\frac{ds^2}{L^2} = -r^2 dt^2 + \frac{dr^2}{r^2}$$

Above is most often used coordinate system in AdS/CFT duality. AdS_{p+2} spacetime has $SO(2,p+1)$ invariance which are defined by:

$$ds_{p+3}^2 = -dX_0^2 + dX_1^2 + \cdots + dX_{p+2}^2$$

$$-X_0^2 + X_1^2 + \cdots + X_{p+2}^2 = +L^2$$

For $p = 0$, we set $X_0 = Z, X_1 = X, and\ X_2 = Y$ just like dS_2 the dS_{p+2} spacetime becomes S^{p+2} by switching X to its complex variable.

AdS Spacetime Summary [9]:

- The AdS spacetime is the spacetime of constant negative curvature.

- TheAdS_5 spacetime has the (3+1)-dimensional timelike boundary known as the AdS boundary.

- The AdS_5 spacetime has the $SO(2,4)$ invariance. This symmetry is the same as the (3+1)-dimensional conformal invariance of the $\mathcal{N} = 4$ SYM.

- The AdS_5 spacetime in Poincaré coordinates coincides with the near-horizon limit of the D3-brane.

- In AdS/CFT, the gauge theory time is the coordinate time not the proper time.

- Given a proper energy of an excitation, the gauge theory energy is larger if the excitation is nearer the AdS boundary.

IV. QUANTUM EXTREMAL SURFACES AND SINGULARITY IN BIG CRUNCH

The Big Bang Singularity is a very striking and cosmological solvable phenomenon introduced by Einstein. In the year 1959, the question regarding Cosmological Singularity (CS) and its relation to Einstein's equation was first formulated by L. Landau. In the rear 1969, this question was answered by V. Belinski, I. Khalatnikov, E. Lifshitz (BKL). According to BKL, singularity is a property of a generic cosmological solution of the classical gravitational equations and it behaves in chaotic manner. The chaotic behavior can be explained in terms of "Cosmological Billiard" system, in which cosmological evolution is described at each spatial point as the relativistic motion of the fictious billiard ball in the Lorentzian space of the logarithmic scale factors. In 1965, Roger Penrose stated that under some conditions incomplete geodesics in space-time can't be avoided. Both BKL conjecture and Penrose represent two sides of phenomenon but the links between these two can't be established. The general solution with singularity can be applied both to the Big Crunch and Big Bang although both are physically different. The higher dimensional background were studied as time dependent deformation of AdS/CFT via gauge/gravity duality into singularity of Big Bang or Big Crunch[10]. When bulk spacetime develops a cosmological big crunch singularity and breakdown, then in AdS_5 case the AdS/CFT duality living on the space that crunch itself have strong time-dependent gauge coupling. The scaling assumptions considered for singularity of big crunch are:

$$X^2 = t^k r^m, e^f = \frac{1}{r^2} = t^a r^b, e^\emptyset = t^\alpha r^\beta$$

Then

$$ds^2 = \frac{e^f}{X^{2(d_i-1)/d_i}}(-dt^2 + dr^2) + X^{4/d_i} dx_i^2$$

Above equation is for simulation of Big Crunch (or Big Bang) [9-11].

V. POINCARÉ PATCH

In Poincaré patch the AdS metric have coordinates:

$$ds^2 = \frac{1}{z^2}\left(dt^2 - dz^2 - \sum_{i=1}^{d-1} x_i^2\right)$$

Here coordinate of z ranges from $0 < z < \infty$. If we multiply above by z^2 the metric obtained is for flat d+1 dimensional hence Penrose diagram must look the same as half of the Minkowski space time. A very commonly used alternative version re-writes dy = dz/z so that $z = e^y$.
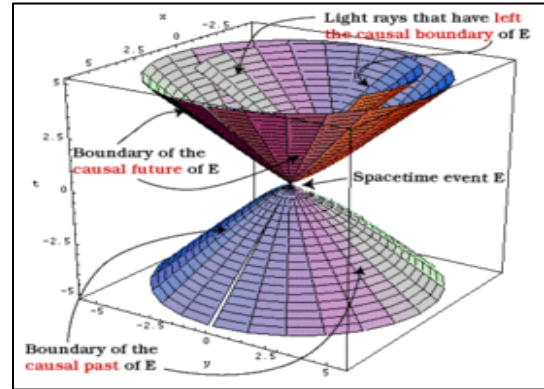

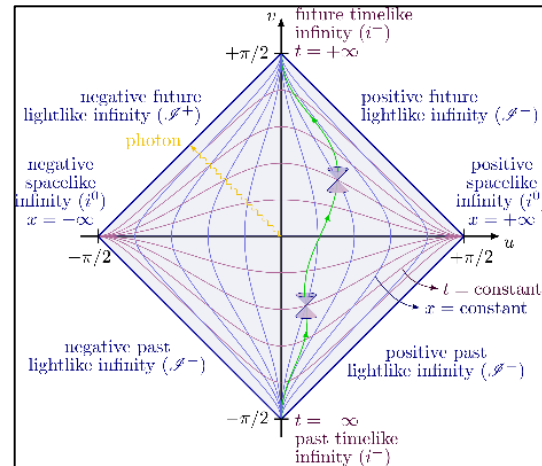Fig 4: Minkowski spacetime (courtesy: Web: Einstein Relatively Easy)
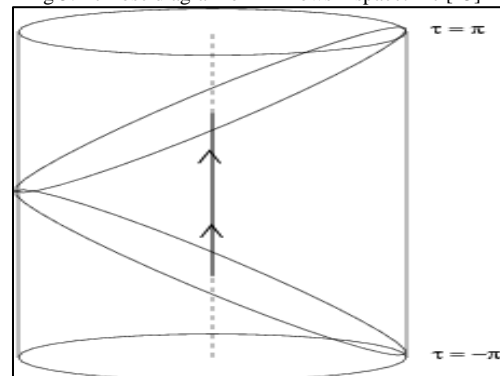

Fig 5: Penrose diagram of Minkowski spacetime [13]
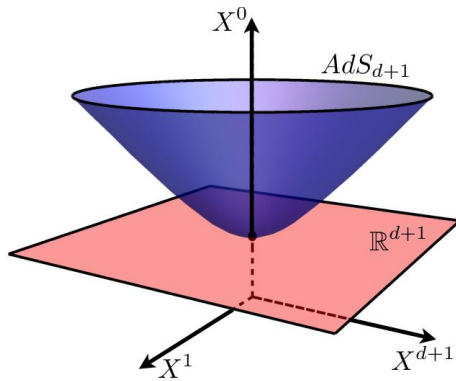

Fig 6: Poincaré patch for AdS_3

Fig 7: Euclidean AdS (Pic Courtesy: Emilio Trevisani)

V. CONCLUSION

Anti de Sitter and Conformal Field Theory are combined together to study condensed matter made up of extra dimensional particles like Dilaton and Strings which are going to decide ultimate fate; the Big Crunch of our Universe. The AdS/CFT correspondence is used to understand the transition from expanding universe to non-relativistic transformation to obtain Big Crunch. The Poincaré patch aids us to understand the big crunch.

VI. ACKNOWLEDGEMENT

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