Anisotropic Bianchi Type III Cosmological Model with magnetized domain wall in f(R) Theory of Gravity

K.V. Somwanshi[1], S.N. Bayaskar[2]
[1.] *S.S.S.K.R. Innani.Mahavidyalaya, Karanja (Lad)-444105*
[2.] *Adarsha Science, J.B. Art and Birla Commerce Mahavidyalaya, Dhamangaon Rly-444709*

Abstract- **The paper deals with a spatially homogeneous and totally anisotropic Bianchi type-III cosmological model in the presence of Magnetized domain walls within the frame work of $f(R)$theory of gravitation. We assume that F_{13} is only the non-vanishing component ofF_{ab}.To obtain deterministic model, we assume relations $B^n = C$and $\rho = P$. The physical and geometrical aspects of the model are discussed.**

Keywords- **Bianchi type-III, Domain walls, Electromagnetic Field,$f(R)$ gravity.**

I. INTRODUCTION

Even after nearly a century, General Relativity (GR) remains our most reliable theory of gravity. GR allows us to derive straightforward cosmological models, such as the Friedman or Lemaitre models, which effectively describe the evolution of our Universe. So far, GR has successfully withstood every experimental test we have conducted. However, in recent decades, we have been compelled to introduce the concept of dark matter to explain astrophysical observations regarding the rotation curves of spiral galaxies. Furthermore, we have had to introduce the notion of dark energy to account for the accelerated expansion of the Universe, as suggested by the redshift of supernovae. Throughout history, there have been numerous attempts to modify GR for various reasons, such as the desire to quantize it and unify it with the other three elementary forces: electromagnetic, weak, and strong nuclear forces. One potential way to modify general relativity (GR) is by incorporating higher order invariants into the conventional Einstein-Hilbert action, resulting in what is known as higher-order theories of gravity. One specific class of these theories is referred to as $f(R)$ gravity, which is derived from GR by including additional terms involving higher powers of the Ricci scalar into the standard GR actions.

Domain walls are the two-dimensional objects that form when discrete symmetry is broken at phase transition. A network of domain walls effectively partitions the universe into various cells. Domain walls have some rather peculiar properties, for example, the gravitational field of a domain wall is repulsive rather than attractive. Thus, the study of domain walls is motivated by the fact that the properties of domain walls are object of intense investigation for different reasons. One is that domain walls are objects formed at the early stages of the universe (Kibble et al. [1]) and have been studied intensively due to their implications to cosmology. Other reason is that study of topological defects has wider applicability in many areas of physics. In cosmological area, defects have been put forward as a possible mechanism for structure formation (Vilenkin and Shellard [2]). Goetz [3], Mukherjee [4], Wang [5], Rahaman *et al.* [6], Reddy and Rao [7], Rahaman and Bera [8] and Chakraborty [9] have studied domain walls in alternative theories of gravitation in four and five dimensions. Adhav et al. [10,11] discussed four-dimensional non-static domain walls in Brans–Dicke theory and Saez–Ballester theory of gravitation respectively. Reddy et al. [12] investigated five-dimensional domain walls in Saez–Ballester theory. Katore et al. [13] obtained Bianchi type-I cosmological model in Barber’s second self-creation theory of gravitation for cosmic domain walls.

So far, a considerable amount of work has been done on domain walls. Vilenkin [27], Isper and Sikivie [19], Windraw [28], Goetz [17], Mukherjee [24], Rahmann [25], Reddy and Subba Rao [26], Adhav et al. [14] are some of the authors who have investigated several aspects of domain walls. Also, the occurrence of magnetic fields on galactic scale is well established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zel’dovich et al. [29]. Harrison [18] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic field in the energy-momentum tensor of the universe. The presence of primordial magnetic fields in the early stage of the evolution of the universe has been discussed by Misner et al. [23], Asseo and Sol [15], Kim et al. [21], Melvin [22]. Also, Iwazaki [20] and Cea, P.,Tedesco [16] gives the interesting phenomena as the magnetization of domain walls and the dynamical generation of massive ferromagnetic domain walls.

When the interaction of domain walls with other kind of matter is small then they get accelerated. If they are moving their effective equation of state becomes (Vilenkin, 1994; Kolb, 1990)$P = \left(V^2 - \frac{2}{3}\right)\rho$ [32, 33]. Bayaskar et al. (2009) [31] investigated the plane symmetric models of interacting fields in General Relativity. Patil et al. (2010) [30] have studied plane symmetric cosmological models of domain wall in General Relativity. Agrawal et al. (2017) [49] have studied magnetized Bianchi Type V cosmological model in,$f(R,T)$ theory of gravity.

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovichet al. [34] Also Harrison [35] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of the field equations leads to the cosmological models more general than Robertson Walker model. The presence of primordial magnetic fields in the early stages of the evolution of the universe has been discussed by several authors. [36−48] Electric current exist due to the occurrence of magnetic field and strong magnetic fields can be created due to adiabatic compression in clusters of galaxies. Asseo and Sol[37] speculated the large-scale inter galactic magnetic field and is of Primordial origin at present measure 10−8 G and gives rise to a density of order 10−35 gcm−3. The present- day magnitude of magnetic energy is very small in comparison with the estimated matter density; it might not have been negligible during early stage of evolution of the universe. Bianchi type-III space-time has a fundamental role in describing early stages of evolution of the universe.

Motivated the situations discussed above, the paper is devoted to study the anisotropic Bianchi type III space-time in presence of magnetic field in the frame work of,$f(R)$ theory of gravity. Here we have obtained an exact solution of Einstein's field equations. The paper is organized as follows. Sect.2 focuses on the,$f(R)$ gravity Formalism and energy momentum tensor. The metric and the field equations are presented in Sect. 3. In Sect. 4, we deal with solution of the field equations with magnetized domain wall. In Sect. 5 we describe some physical and geometric properties of the model with brief discussion of the results. In Sect.6 we discuss cosmological parameters in terms of redshift. Finally, in Sect. 7, concluding remarks are given.

II. $f(R)$ GRAVITY FORMALISM AND METRIC

$f(R)$Theories of gravity are extended theories of gravity which simplify to general relativity in the most elementary case of the function $f(R)$. The idea of understanding this theory stemmed in two ways – one by considering a variation of the Einstein-Hilbert action with the metric (the metric formalism), and the other by varying the metric and an independent connection (called the Palatini formalism) [50]. Each of these methods describe a form of an extended theory of gravity by considering some function of the Ricci scalar – the field equations in each case (i.e. the field equations for an$f(R)$ theory of gravity for some form of the function) can be derived by either of these two formalisms. The nature of the actions of,$f(R)$ theories of gravity was first studied extensively in [51].

We consider a class of modified gravity in which modifies Einstein-Hilbert action by replacing Ricci curvature scalar, $R$by an arbitrary function of curvature,$f(R)$ as follows

$$S = \int \sqrt{-g}\left(\frac{f(R)}{2} + kL_m\right) d^4x \qquad (1)$$

Where $g$is the metric determinant, $L_m$is the matter Lagrangian

The field equation of,$f(R)$ is given by

$$F(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - \nabla_i\nabla_j F(R) + g_{ij}F(R) = k^2 T_{ij} \qquad (2)$$

Where, $F=\frac{df}{dR}$
, ∇ *is a covariant derivative*, $\Box=\nabla^{\alpha}\nabla_{\alpha}$ is the D'Alembertian, T_{ij} is the matter energy-momentum tensor

The standard FRW model is in good agreement with the present-day universe. However, it does not give a clear description of the early stage of evolution of the universe. Bianchi type models provide a physically realistic description of the initial universe. It is well known that, near the Big Bang, the universe is neither isotropic nor spherically symmetric. Thus, anisotropic models play an important role in cosmology.

The energy momentum tensor for a system of domain walls and magnetic field is given by;

$$T_{bc}^{D} = \rho(g_{bc} + w_b w_c) + p w_b w_c + E_{bc} \;,$$

$$w_b w_c = -1 \qquad (3)$$

Where ρ is energy density of domain walls, p is the pressure in the direction normal to the plane of the wall and w_b is a unit space like vector in the same direction. and E_{bc} is the electromagnetic field tensor, which is given by Lichnerowicz et al. (1967)

$$E_c^b = \bar{\mu}\left[|h|^2\left(u_b u^c + \tfrac{1}{2}c\right) - h_b h^c\right] \qquad (4)$$

$\bar{\mu}$ is magnetic permeability, h_b is magnetic flux vector defined as

$$h_b = \frac{\sqrt{-g}}{2\bar{\mu}} \epsilon_{bcde}\, f^{de} u^c \qquad (5)$$

In the above u_b is the flow vector satisfying

$$g_{bc}U^b U^c = -1$$

F^{de} is dual electromagnetic field tensor and ϵ_{bcde} is Levi-Civita tensor density

The incident magnetic field is taken along y-axis.

Hence, $h_2 \neq 0,\; h_1 = h_3 = h_4$

The only non-vanishing component of F_{ab} is F_{13}

The first sets of maxwell's equation are given as:

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0$$

Gives rise to

$F_{13} = L$(Constant)

Here $F_{12} = F_{14} = F_{23} = F_{24} = F_{34} = 0$ due to assumption of infinite conductivity are explained by Roy.M(2000)

For $b = 2$ equation (5) gives

$$h_2 = -\frac{BL}{2\bar{\mu}AC} \qquad (6)$$

Since $|h|^2 = h_a h^b = h_2 h^2 = g^{22}(h_2)^2$

We obtain

$$|h|^2 = -\frac{L^2}{4\bar{\mu}^2A^2C^2} \qquad (7)$$

Using equation (6) and (7) in equation (4), The components of E_b^a for the line element (12) are given by;

$E_1^1 = -\frac{L^2}{8\bar{\mu}A^2C^2} = -\frac{L_1}{A^2C^2}$,

where $L_1 = \frac{L^2}{8\bar{\mu}}$

Similarly, $E_2^2 = \frac{L_1}{A^2C^2}$, $E_3^3 = -\frac{L_1}{A^2C^2}$ and

$$E_4^4 = \frac{L_1}{A^2C^2} \qquad (8)$$

$$T_1^1 = \rho + E_1^1 = \rho - \frac{L_1}{A^2C^2} \qquad (9)$$

$$T_2^2 = -p + E_2^2 = -p + \frac{L_1}{A^2C^2} \qquad (10)$$

Similarly, $T_3^3 = \rho - \frac{L_1}{A^2C^2}$ and $T_4^4 = \rho + \frac{L_1}{A^2C^2}$ (11)

III. METRIC AND FIELD EQUATION

The spatially homogeneous and anisotropic Four-dimensional Bianchi Type -III metric is given by,

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2sx}dy^2 - C^2\,dz^2 \qquad (12)$$

where A, B, C are the metric potentials and functions of cosmic time t only.

For the metric (12), the field equation (2) together with (11) in co-moving co-ordinate leads to the following set of equation.

$$F\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{S^2}{A^2}\right] - \frac{f}{2} + \frac{B_4}{B}F_4 + \frac{C_4}{C}F_4 + F_{44} = k^2\rho - \frac{k^2L_1}{A^2C^2} \quad (13)$$

$$F\left[\frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} - \frac{S^2}{A^2}\right] - \frac{f}{2} + \frac{A_4}{A}F_4 + \frac{C_4}{C}F_4 + F_{44} = -k^2p + \frac{k^2L_1}{A^2C^2} \quad (14)$$

$$F\left[\frac{C_{44}}{C} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC}\right] - \frac{f}{2} + \frac{A_4}{A}F_4 + \frac{B_4}{B}F_4 + F_{44} = k^2\rho - \frac{k^2L_1}{A^2C^2} \quad (15)$$

$$F\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C}\right] - \frac{f}{2} + \left[\frac{A_4}{A}F_4 + \frac{B_4}{B}F_4 + \frac{C_4}{C}F_4\right] = k^2\rho + \frac{k^2L_1}{A^2C^2} \quad (16)$$

And $\frac{A_4}{A} - \frac{B_4}{B} = 0$ (17)

Solving equation (17), we get $A = B$

above equation becomes:

$$F\left[\frac{B_{44}}{B} + \frac{{B_4}^2}{B^2} + \frac{B_4C_4}{BC} - \frac{S^2}{B^2}\right] - \frac{f}{2} + \frac{B_4}{B}F_4 + \frac{C_4}{C}F_4 + F_{44} = k^2\rho - \frac{k^2L_1}{B^2C^2} \quad (18)$$

$$F\left[\frac{B_{44}}{B} + \frac{{B_4}^2}{B^2} + \frac{B_4C_4}{BC} - \frac{S^2}{B^2}\right] - \frac{f}{2} + \frac{B_4}{B}F_4 + \frac{C_4}{C}F_4 + F_{44} = -k^2p + \frac{k^2L_1}{B^2C^2} \quad (19)$$

$F\left[\frac{C_{44}}{C}+2\frac{B_4C_4}{BC}\right]-\frac{f}{2}+2\frac{B_4}{B}F_4+F_{44}= k^2\rho-\frac{k^2L_1}{B^2C^2}$ (20)

$F\left[2\frac{B_{44}}{B}+\frac{C_{44}}{C}\right]-\frac{f}{2}+\left[2\frac{B_4}{B}F_4+\frac{C_4}{C}F_4\right]= k^2\rho+\frac{k^2L_1}{B^2C^2}$ (21)

Equation $(21)-(18)$,we get

$F\left[\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{B_4{}^2}{B^2}-\frac{B_4C_4}{BC}+\frac{S^2}{B^2}\right]+\frac{B_4}{B}F_4-F_{44}=\frac{2k^2L_1}{B^2C^2}$ (22)

$F\left[\frac{B_{44}}{B}+\frac{B_4{}^2}{B^2}+\frac{B_4C_4}{BC}-\frac{S^2}{B^2}\right]-\frac{f}{2}+\frac{B_4}{B}F_4+\frac{C_4}{C}F_4+F_{44}= -k^2p+\frac{k^2L_1}{B^2C^2}$ (23)

$F\left[\frac{C_{44}}{C}+2\frac{B_4C_4}{BC}\right]-\frac{f}{2}+2\frac{B_4}{B}F_4+F_{44}= k^2\rho-\frac{k^2L_1}{B^2C^2}$ (24)

IV. SOLUTION AND THE MODEL

We have two equations in ρ, p, B, C, F five unknown. Therefore, to obtain the solution of the field equation two more condition is required for consistency.

Firstly, we assume the relation between metric potential $B^n = C$ (25)

Secondly, we assume that constant curvature scalar $F(R) = R_0$ = constant = 1

$F_4 = F_{44} = 0$ (26)

Using equation (25) and (26), equation (22) becomes,

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{B_4{}^2}{B^2}-\frac{B_4C_4}{BC}+\frac{S^2}{B^2}=\frac{2k^2L_1}{B^2C^2}$$

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{B_4{}^2}{B^2}-\frac{B_4C_4}{BC}+\frac{S^2}{B^2}=\frac{2k^2L_1}{B^{2(n+1)}}$$

$$\frac{B_{44}}{B}+n(n-1)\frac{B_4{}^2}{B^2}-n\frac{B_{44}}{B}-\frac{B_4{}^2}{B^2}-n\frac{B_4{}^2}{B^2}+\frac{S^2}{B^2}=\frac{2k^2L_1}{B^{2(n+1)}}$$

$$(1+n)\frac{B_{44}}{B}+[n(n-1)-1-n]\frac{B_4{}^2}{B^2}+\frac{S^2}{B^2}=\frac{2k^2L_1}{B^{2(n+1)}}$$

$$(1+n)\frac{B_{44}}{B}+(n^2-2n-1)\frac{B_4{}^2}{B^2}+\frac{S^2}{B^2}=\frac{2k^2L_1}{B^{2(n+1)}}$$

$$B_{44}+\frac{(n^2-2n-1)}{n+1}\frac{B_4{}^2}{B}+\frac{S^2}{B^{(n+1)}}=\frac{2k^2L_1}{B^{2(n+1)}(n+1)}$$

$$2B_{44}+\left[\frac{2(n^2-2n-1)}{n+1}\right]\frac{B_4{}^2}{B}=\frac{4k^2L_1}{(n+1)B^{2(n+1)}}-\frac{2S^2}{(n+1)B}$$

$$\frac{d}{dB}\left(B_4^2B^{\frac{2(n^2-2n-1)}{n+1}}\right)=\frac{4k^2L_1}{(n+1)}B^{\frac{2(n^2-2n-1)}{n+1}-2n-1}-\frac{2S^2}{(n+1)}B^{\frac{2(n^2-2n-1)}{n+1}-1}$$

Integrating we get

$$B_4=\left[\frac{4k^2L_1}{(-6n-2)B^{2n}}-\frac{S^2}{(n^2-2n-1)}+CB^{\frac{-2(n^2-2n-1)}{n+1}}\right]^{1/2}$$

$$\frac{dB}{\left[\frac{4k^2L_1}{(-6n-2)B^{2n}}-\frac{S^2}{(n^2-2n-1)}+CB^{\frac{-2(n^2-2n-1)}{n+1}}\right]^{1/2}}=dt \quad (27)$$

Now, equation (27) can be solved by using distinct values of n and C, for that we have following cases.

Case I: putting c = 0and n = 1 in equation (27) $(k^2 = s^2 = 1)$

$$\frac{dB}{\left[\frac{4L_1}{-8B^2}-\frac{1}{-2}\right]^{1/2}}=dt$$

Integrating, we get

$$B=\frac{L}{\sin[\cot^{-1}(t)]}$$

$A=\frac{L}{\sin[\cot^{-1}(t)]}$ and $C=-\frac{L}{\sin[\cot^{-1}(t)]}$

Case II: put c = 0 and n = $-\frac{1}{2}$

$$\frac{dB}{\left(\frac{4L_1}{B^{(-1)}}-\frac{1}{1/4}\right)^{1/2}}=dt$$

Integrating, we get,

$B=L_1t^2+\frac{1}{L_1}$ (28)

$A=L_1t^2+\frac{1}{L_1}$ and $C=\left(L_1t^2+\frac{1}{L_1}\right)^{-\frac{1}{2}}$ (29)

From cases I and II it is clear that equation (27) cannot give definitive solution for any arbitrary values of n and C. Hence for C = 0 and n = -1/2 we have made out its solution.

By using equation (28) and (29), the equation of metric can be written as,

$$ds^2=dt^2-\left(L_1t^2+\frac{1}{L_1}\right)^2dx^2-\left(L_1t+\frac{1}{L_1}\right)^2e^{-2Sx}dy^2-\left(L_1t+\frac{1}{L_1}\right)^{-1}dz^2$$

$$ds^2=dt^2-\left[\frac{(L_1t)^2+1}{L_12}\right]^2dx^2-\left[\frac{(L_1t)^2+1}{L_1}\right]^2e^{-2Sx}dy^2-\frac{L_1}{(L_1^2t+1)}dz^2 \quad (30)$$

V. PHYSICAL AND GEOMETRICAL PROPERTIES OF THE MODEL

We discuss the Physical and kinematical properties of the cosmological model (30). For the line element (12) the energy density ρ and pressure p are given as follows:

$$\rho = \frac{{L_1}^2[S^2 - K^2(1 + {L_1}^2 t^2)]}{K^2({L_1}^2 t^2 + 1)^2}$$

$$P = \frac{{L_1}^2[S^2 - 3K^2(1 + {L_1}^2 t^2)]}{K^2({L_1}^2 t^2 + 1)^2}$$

Volume $V = (ABC)$

$$V = \left(L_1 t^2 + \frac{1}{L_1}\right)^{3/2}$$

Hubble parameter $H = \frac{1}{3}\left(\frac{V_4}{V}\right)$

$$H = \frac{L_1^2 . t}{(L_1^2 t^2 + 1)}$$

Expansion scalar θ $= 3H$

$$\theta = \frac{3L_1^2 . t}{(L_1^2 t^2 + 1)}$$

Deceleration parameter $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$

$$q = -\frac{1}{(L_1^2 t^2)}$$

Anisotropy parameter $\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2$

$$\Delta = 2$$

Shear scalar $\sigma^2 = \frac{3}{2} H^2 \Delta$

$$\sigma^2 = 3\frac{{L_1}^4 t^2}{({L_1}^2 t^2 + 1)^2}$$

VI. COSMOLOGICAL PARAMETERS IN TERMS OF REDSHIFT

The scale factor "a" in terms of redshift parameter z is written as,

$$\frac{a_0}{a} = (1 + z)$$

Here, take $a_0 = 1$, where a_0 the present value of scale factor.

$$t(z) = [k_1(1 + z)^{-2} - k_2]^{\frac{1}{2}}$$

The parameter like deceleration, Hubble, density, pressure and EoS in terms of redshift can be written as;

$$q(z) = \frac{-(1 + z)^2}{L_1 - (1 + z)^2}$$

$$H(z) = (1 + z)[-(1 + z)^2 + L_1]^{\frac{1}{2}}$$

$$\rho = \frac{(1 + Z)^2 [\alpha^2(1 + Z)^2 - k^2 L_1]}{k^2 {L_1}^2}$$

$$p = \frac{(1 + Z)^2 [\alpha^2(1 + Z)^2 - 3k^2 L_1]}{k^2 {L_1}^2}$$

$$\omega = \frac{[\alpha^2(1 + Z)^2 - 3k^2 L_1]}{[\alpha^2(1 + Z)^2 - k^2 L_1]}$$

The expansion scalar, shear scalar and volume in terms of redshift is given by,

$$\theta = 3(1 + z)[-(1 + z)^2 + L_1]^{\frac{1}{2}}$$

$$\sigma^2 = 3(1 + Z)^2[-(1 + Z)^2 + L_1]$$

$$V = \frac{1}{(1+Z)^3}$$

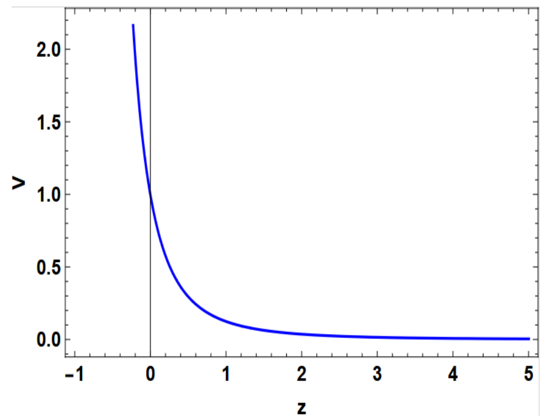

Fig. 1 Plot of Volume versus redshift z

From fig.1 it is clear that the volume decreases in the past and increases in the future and present.

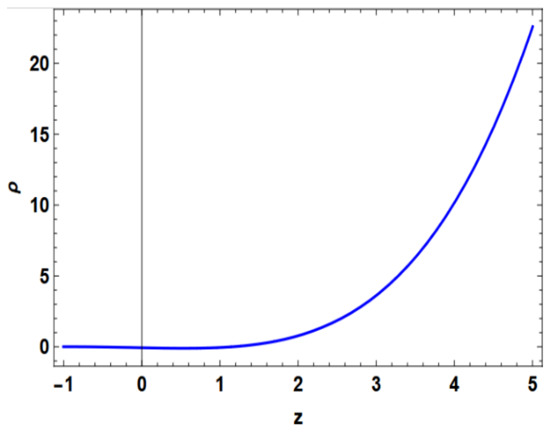

Fig. 2 Plot of matter energy density (ρ)versus redshift z

Fig. 2 shows that the evolution of energy density with respect to redshift z.

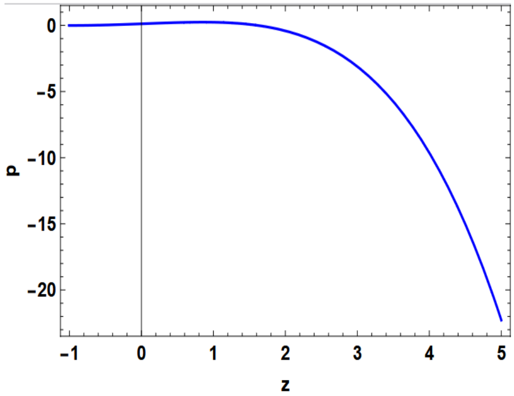

Fig. 3 Plot of matter energy pressure (p)versus redshift z

Fig.3 reveals that the evolution of energy pressure with respect to redshift z.

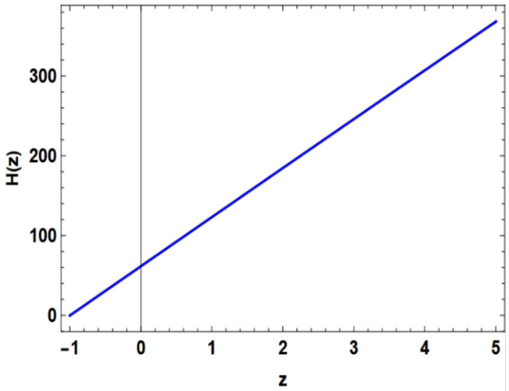

Fig. 4 Plot of Hubble parameter versus redshift z

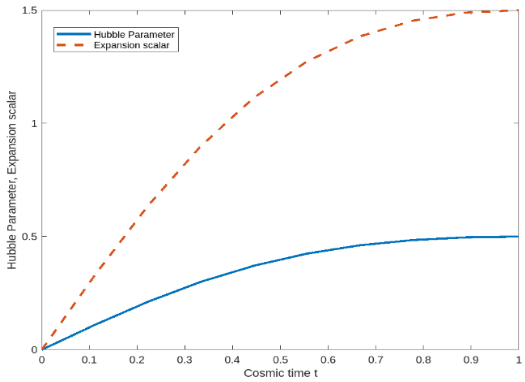

Fig. 5 Plot of Expansion scalar and Hubble parameter versus cosmic time t (Gyr)

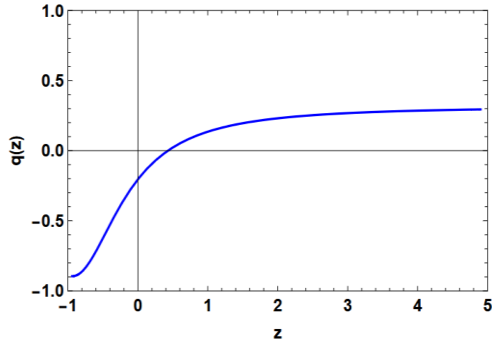

Fig. 6 Plot of Deceleration parameter versus red shift z.

Fig.6 reveals that the sign of deceleration parameter is negative in the future and present and positive in the past. This shows that the universe is accelerating in the future and present and decelerating in the past.

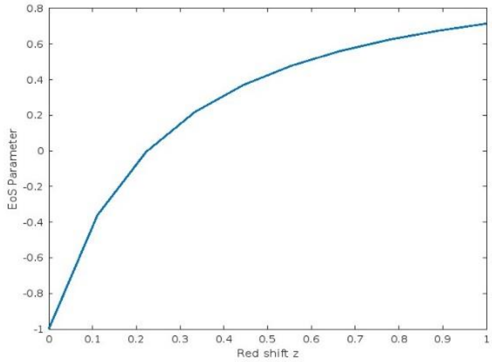

Fig. 7 Plot of EoS parameter versus red shift z.

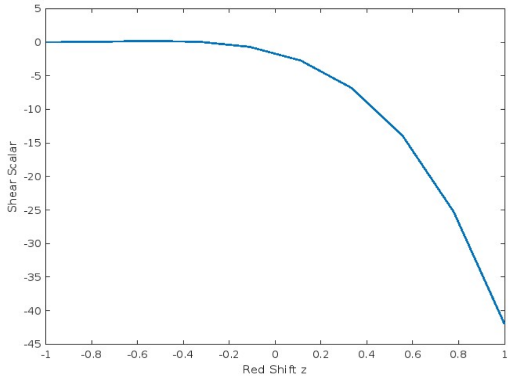

Fig. 8 Plot of Shear scalar versus red shift z.

VII. DISCUSSION AND CONCLUSION

The expression of volume V shows that the model universe does not start from zero volume at the initial epoch. As time progresses, the universe expands and when t approaches infinity, the volume V also approaches infinity. Additionally, when $t \rightarrow \infty, V \rightarrow \infty$.

The energy conditions pressure $p \geq 0$ and density $\rho \geq 0$ are satisfied for suitable constants.

In the model, we found that the expansion scalar (θ) and Hubble parameter (H) are infinite at time $t = 0$, indicating the maximum value of Hubble's parameter and accelerated expansion of the universe. However, as time progresses, both (θ) and (H) decrease gradually and eventually become zero when $t \rightarrow \infty$

This implies that the universe expands with time, but the rate of expansion decreases as time increases.

The deceleration parameter (q) is related to the rate at which the expansion of the universe is slowing down. If q is negative, this means that the expansion of the universe is accelerating rather than slowing down.

The model describes an accelerating model ($q < 0$). The mean anisotropic parameter Δ is uniform throughout the evolution of the universe as it does not depend on t.Shear scalar σ are infinite at $t = 0$ and tend to zero as time t approaches infinity.

At the initial epoch of time the pressure and energy density have infinite values whereas the same quantities tend to zero as time $t \to \infty$. In this case $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$ is found to be constant. Hence derived model does not approach to isotropy.

We have explored Bianchi type-III domain wall cosmological models with magnetic field in $f(R)$theory of gravity. We examined the models in the presence of electromagnetic field does not gives isotropic solution.

REFERENCES

[1] Kibble TWB, Lazarides G, Shafi Q (1982) Walls bounded by strings. Phys Rev D 26:435–439

[2] Vilekin (1983) Gravitational field of vacuum domain walls. Phys Lett B 133:177–179

[3] Goetz G (1990) The gravitational field of thick domain walls. J Math Phys 31:2683–2687

[4] Mukherjee M (1993) Gravitational field of cosmic membranes. Class Quantum Gravity 10:131–146

[5] Wang A (1994) Gravitational collapse of thick domain walls. Mod Phys Lett 39:3605–3609

[6] Rahaman F, Chakraborty S, Kalam M (2001) Domain wall in Lyra geometry. Int J Mod Phys D 10:735–739

[7] Reddy DRK, Rao RVS (2006) Axially symmetric cosmic strings and domain walls in Lyra geometry. Astrophys Space Sci 302:157–160

[8] Rahaman F, Bera J (2001) Higher dimensional cosmological model in Lyra geometry. Int J Mod Phys D 10:729–733

[9] Chakraborty S, Rahaman F, Biswas L (2000) Gravitational field of domain walls in higher dimension. Astrophys Space Sci 274:851–858

[10] Adhav KS, Nimkar AS, Naidu RL (2007) Axially symmetric non static domain walls in scalar tensor theories of gravitation. Astrophys Space Sci 312:165–169

[11] Adhav KS, Gadodia PS, Bansod AS (2011) Stiff domain walls in creation-field cosmology. Int J Theor Phys 50:275–288

[12] Reddy DRK, Rao PG, Naidu RL (2008) Five-dimensional domain walls in scalar tensor theory of gravitation. Int J Theor Phys 47:2966–2970

[13] Katore SD, Shaikh AY (2010) Bianchi type-I cosmological model in self-creation cosmology. Fiz B 19:161–168

[14] Adhav, K.S., Nimkar, A.S., Naidu, R.L.: Astrophys. Space Sci. 312, 165–169 (2007)

[15] Asseo, E., Sol, H.: Phys. Rev. 6, 148 (1987)

[16] Cea, P., Tedesco, L.: Phys. Lett. B 450, 61 (1999)

[17] Goetz, G.: J. Math. Phys. 31, 2683 (1990)

[18] Harisson, E.R.: Phys. Rev. Lett. 30, 188 (1973)

[19] Isper, J., Sikivie, P.: Phys. Rev. D. 30, 712 (1984)

[20] Iwazaki, I.: Phys. Rev. D 56, 2435 (1997)

[21] Kim, K.T., Tribble, P.G., Kronberg, P.P.: Astrophys. J. 379, 80 (1991)

[22] Melvin, M.A.: Ann. NY Acad. Sci. 262, 253 (1975)

[23] Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. Freeman, New York (1973)

[24] Mukherjee, M.: Class. Quantum Gravity 10, 131 (1993)

[25] Rahmann, F.: Astrophys. Space Sci. 282, 625 (2002)

[26] Reddy, D.R.K., SubbaRao, M.V.: Astrophys. Space Sci. 302, 157–160 (2006)

[27] Vilenkin, A.: Phys. Rev. D. 23, 852 (1981)

[28] Windraw, L.M.: Phys. Rev. D. 39, 3571 (1989)

[29] Zel’dovich, Y.B., Ruzmainkin, A.A., Sokoloff, D.D.: Magnetic Field in Astrophysics. Jordan and Breach, NewYork (1993

[30] Patil, V.R., Pawar, D.D., Deshmukh, A.G., 2010. Rom. Rep. Phys. 62, 722–730

[31] Bayaskar, S.N.,Pawar, D.D., Deshmukh, A.G., 2009. Rom. Journ. Phys. 54 (Nos. 78), 763–770

[32] Vilenkin, A.,Shellard, E.P.S., 1994. Cosmic strings and other topological defects. Cambridge University press, Cambridge.

[33] Kolb, E.W., Turner, M., 1990. Addison Wesley, Massachusetts
[34] Ya. B. Zeldovich, A.A. Ruzmainkin, and D.D. Sokoloff, Magnetic Field in Astrophysics, Gordon and Breach, New York (1983).
[35] E.R. Harrison, Phys. Rev. Lett. 30 (1973) 188.
[36] C.W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation Freeman, New York (1973).
[37] E. Asseo and H. Sol, Phys. Rep. 6 (1987) 148.
[38] R. Pudritz and J. Silk, Astrophys. J. 342 (1989) 650.
[39] K.T. Kim, P.G. Tribble, and P.P. Kronberg, Astrophys. J. 379 (1991) 80.
[40] R. Perley and G. Taylor, Astrophys. J. 101 (1991) 1623.
[41] P.P. Kronberg, J.J. Perry, and E.L. Zukowski, Astrophys. J. 387 (1991) 528.
[42] A.M. Wolfe, K. Lanzetta, and A.L. Oren, Astrophys. J. 388 (1992) 17.
[43] R. Kulsrud, R. Cen, J.P. Ostriker, and D. Ryu, Astro phys. J. 380 (1997) 481.
[44] J.D. Barrow, Phys. Rev. D 55 (1997), 7451.
[45] R. Bali, A. Pradhan, and H. Amirhashchi, Int. J. Theor. Phys. 47 (2008) 2594.
[46] R. Bali and S. Jain, Astrophys. Space Sci. 311 (2007) 401.
[47] A. Pradhan, V. Rai, and K. Jotania, Commun. Theor. Phys. 50 (2008) 279.
[48] A. Pradhan, H. Amirhashchi, and H. Zainuddin, Int. J. Theor. Phys. DOI: 10.1007/s10773-010-0493-0.
[49] Agrawal, P.K., Pawar,D.D.,New Astronomy 54 (2017) 56-60 DOI: 10.1016/j.newast.2017.01.006
[50] M. P. Hobson, G. P. Efstathiou, A. N. Lasenby,” General Relativity, An Introduction for Physicists”, Cambridge University Press, 2006.
[51] H.A. Buchdahl,” Non-linear Lagrangians and cosmological theory”, 1970, Mon. Not. Roy. Astr. Soc. 150, 1

a.