The Numerical Models for Simulating the propagation of an acoustic Waves in a turbulent Flows

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Abstract—**The behavior of an acoustic wave propagating across a turbulent medium is simulated numerically. Two components make up the technique: the creation of hypothetical, 3-D, random, isotropic velocity fields in terms of a set of discrete Fourier velocity modes, and the incorporation of the ray-trace equation to characterize the trajectories of the acoustic wave front and points tagging. The variation of travel time over the ensemble is computed by recording the propagation times for these points to travel set distances through each of an ensemble of random velocity fields. Acoustic travel-time variations in numerical ray-trace experiments through fields with average perturbation indices ≈0.01 are found to have a higher order dependence on travel distance R than the classical Chernov prediction, which states that R increases linearly. However, when the rays are restricted to axial paths, the Chernov result is obtained. Additional numerical experiments integrating the stochastic Helmholtz equation and its parabolic approximation yield time variance estimates consistent with the ray-trace results. After applying the predictions from these simulations to Blanc-Benon's laboratory investigations, it is discovered that there is qualitative agreement. In order to distinguish between source-receiver eigenray propagation and pre-assigned beginning direction ray propagation, a series of 2-D travel-time experiments are finally given.**

Index Terms—**numerical simulation, acoustic waves, turbulent flow, turbulent field**

1. INTRODUCTION

The fact that the medium through which sound travels has a significant impact on its propagation properties is one of its most intriguing aspects. Convention allows sound waves to be twisted, muted, amplified, or even rendered entirely incoherent by refraction and scattering. Understanding how an acoustic wave is impacted by its medium of passage is crucial since the coherence and integrity of acoustic waves are vital to noise control, sonic detection systems, and harmonic sound transmission.

The depiction of sound in the atmosphere is particularly interesting. Random inhomogeneities un temperature and velocity across a wide range of scales define this medium. However, because these natural settings are not well regulated, it is preferable to study how sound might be affected by them in a laboratory setting using a turbulent or thermal field created by a grid or jet.

The phase change when acoustic waves travel through the medium is a crucial indicator of the impact of random inhomogeneities. To date, two methods have been used to study this variance: a laboratory method that actually measures and interprets sound propagation fluctuations, and a purely mathematical method that simplifies and averages wave or ray propagation equations to account for the random environment.

The travel-time variance of an acoustic wave propagating over a random inhomogeneous medium (air) is the sole focus of the Chernov approximation numerical experiment described in this work. We have the flexibility to derive solutions to the ray-trace equations with or without simplifying assumptions in this experiment, partly due to our approach for following individual rays and partly due to the way we characterize the turbulent fields. By integrating ray-trace equations that have been adjusted to meet the simplifying assumptions stated in the corresponding theory, they simulate theoretical approximations. The Chernov approximation—propagation along a straight line—is the theory that is being studied.

2. Theory:

The traditional mathematical approaches are Chernov (1960) and Tatarski (1971). In recent years, Ishimaru (1978), Tatarski Zavorotnyi (1980), and Flatte (1983) have added to this body of writing. These methods use a random index associated with temperature or velocity changes to depict the interplay between turbulence and acoustics.

Blanc-Benon (1981) (B-B) and Ho and Kovasznay (1974) both use experimental methods to measure sonic propagation in variable velocity fields. In their 1986 study, Blanc-Benon et al. tackle the issue of temperature fluctuations.

We present a third method in this paper: an explicit numerical simulation of the motion of an acoustic ray moving through a fictitious inhomogeneous velocity field. The hypothetical field is made up of a few discrete, randomly oriented, Fourier velocity wave vectors with amplitudes selected to yield a distribution of velocities similar to those observed in experimental flows, such as the jet-generated turbulent flow of B-B and the grid-generated turbulent flow of Comte-Bellot and Corrsin (1971) (C-BC). We employ the ray-trace equations to numerically integrate the trajectories of acoustic rays under the premise of a "frozen" velocity field and a faint acoustic wave front that does not interact with the field. We collect statistics of the variation in arrival times for the rays traveling prescribed distances by performing such integrations over an ensemble of statistically similar velocity fields.

Two aspects of our method stand out. First, the typical source-receiver problem, which examines the characteristics of eigenrays traveling from source to receiver, is not modeled by the majority of our numerical trials. Instead, we investigate the propagation statistics of rays with preassigned initial directions that do not take into account their potential journey paths. Second, there is no intrinsic acoustic wavelength in our approach.

2. SIMULATION OF AN ACOUSTIC WAVE PROPAGATION

We use the geometrical acoustics principles, which hold true for small amplitude changes across an acoustic wavelength, a large wavefront radius of curvature relative to its wavelength, and a small wavelength relative to the characteristic length scale of the disturbing medium. (For instance, see Chernov, 1960). In our plan, we "tag" a point on an acoustic wave front, assume its beginning position and direction of propagation, then track the wave front's path through the velocity field—a procedure called "ray tracing."

The ray-tracing equations are expressed by and Candel (1977) in a way that is very appropriate for our use. In these innovations, the propagation properties of the tagged point on the wave front are described by a parametric variable, a "wave-slowness" vector s. In this case, s has a magnitude equal to the reciprocal of the wave front's speed and points in the direction of propagation. Six linked, nonlinear differential equations are needed, three of which describe the tagged point's motion and three of which describe how the slowness vector associated with that point evolves. The propagation field and its spatial gradients are the only parameters used to define the differential equations in this approach. From Pierce

$$\frac{dx_i}{dt} = \frac{c^2 s_i}{\Omega} + u_i$$
$$\frac{ds_i}{dt} = -\frac{\Omega}{c}\frac{\partial c}{\partial x_i} - \sum_{j=1}^{3} s_j \frac{\partial u}{\partial x_i} \qquad \text{--------------------(2.1)}$$

where $\Omega = 1 - u_i s_i$ and c(x) and u_i(x) are the sound-speed and velocity components of the field at the position of the tagged point on the wave front. Some of our experiments mimic theoretical results where the velocity disturbances are replaced by "equivalent" scalar refractive fields, even though our simulations are meant to involve fields with only velocity disturbances, so the terms involving refractive gradients could be omitted.

We can precisely compute the right-hand side of the differential equations at each point along a trajectory using this formulation. The spatial derivatives of our turbulent velocity fields can be obtained analytically at each position as they have been described in terms of a set number of discrete Fourier velocity modes. Therefore, the typical finite-difference approach to these derivatives will not result in any cumulative inaccuracy during an integration of a ray trajectory. This feature is essential to its success insofar as we integrate over great distances.

We follow a simple simulation process. In order to choose the random components of our Fourier modes, we first create a random velocity field. Next, select a point (s) in the field that we assume is on an acoustic wave front [e.g., x (0) = (0,0,0)], and then specify a slowness vector [e.g. s (x (0)) =

$\{1/[c_0 + u_1(x(0))], 0, 0\}$. to give the wave front there a propagation attribute. Next, using a fourth-order Runge-Kutta integration strategy, we numerically track the trajectory of that point and the slowness vector's change over time.

Double precision computations are performed using an integration time step of $\Delta t = 1/(c_0\, k_{max})$ --the largest step for which the results are identical to those from integrations of half that step size. Our simulations follow a trajectory within a "frozen" field, meaning that there is no time evolution of the field throughout our integrations, because we assume that the transit period of an acoustic wave is short in comparison to any time scale in the velocity field.

We record the propagation time at different distances R from the "source" over ensembles of 300 to 1000 such individual integrations, each employing a different random velocity field, and then we aggregate statistics. Generally, reliable acoustic time-variance estimations and realistic execution times for the computer runs were affected by setting the number of realizations to 1000. The point at which a ray will have propagated a distance R cannot be predicted a priori, with the exception of our studies in "one-dimensional" propagation. Therefore, the spherical shell of positions $\sqrt{x_1^2 + x_2^2 + x_3^2}$ = R was our definition of a propagation distance R.

3. NUMERICAL EXPERIMENT

(a)The Chernov Approximation – Straight-line ray Propagation.
Chernov estimates the acoustic ray passage time by presuming those variations from

$$t = \frac{1}{c_0} \int_0^R (x_1, x_2, x_3)\, dx_1 \quad \text{-----------------(3.1)}$$

where the medium's index of refraction along the ray is denoted by n (x_1, x_2, x_3). n = $1 + \mu$, where $\mu \approx -u_1/c_0$, for an inhomogeneous medium made up only of velocity variations. When R $\gg L_f$ the transit-time variance can then be represented by,

$$\langle t'^2 \rangle = \frac{2\langle \mu^2 \rangle R}{c_0^2} \int_0^\infty N(x_1, x_2, x_3)\, dx_1 \quad \text{------------(3.2)}$$

where the normalized correlation function of μ is $N(x_1, x_2, x_3)$. The integrand is reduced to $N(x_1, 0, 0)$ by Chernov since it is assumed that the deviations of ray paths from $x_2 = x_3 = 0$ are minimal [Chernov, 1960, Eq. (68)]. Regarding an isotropic field with only velocity variations

$$\int_0^\infty N(x_1, 0, 0)\, dx_1$$

is simply L_f, the integral length scale. Consequently, (3.2) can be expressed as

$$\langle t'^2 \rangle = \frac{2\langle u_1^2 \rangle R}{c_0^4} L_f \quad \text{--------------------------------(3.3)}$$

Additionally, we note that a smooth-perturbation analysis of the wave equation in the limits of both very large and very small values of the wave parameter D = 4R / (k_0^2) yields the same linear dependence 0f $\langle t'^2 \rangle$ with R (Tatarski,1971). The acoustic wave number in the medium at rest is k_0, the transit distance is R, and the mean magnitude of the field's inhomogeneities is d $\approx L_f$.

Since our fields had constant densities, we simplified our governing ray equations to exclude all nonaxial components and, of course, density-inhomogeneity terms in order to demonstrate the feasibility of our method for researching acoustic propagation. As a result, we could model the Chernov method. Our equations in this instance simplified to the following two equations:

$$\frac{dx_i}{dt} = \frac{c^2 s_1}{\Omega} + u_1$$
$$\frac{ds_i}{dt} = -s_1 \frac{\partial u_1}{\partial x_i} \quad \text{-------------------------------(3.4)}$$

with $\Omega = 1 - u_1.s_1$. We generated the variation of arrival durations at various distances from the origin by simulating unidirectional ray propagation over an ensemble of 1000 realizations. We show the result in Fig.1. The integral length scale L_f is used to express the propagation distance. The Chernov solution assessed at the distance R = $40L_f$ is used to normalize time variance.

Remember that the field's parameters are $u_1' = 2$ m/s and $L_f = 0.067$m.

The time-(phase) variance is predicted to expand linearly with propagation distance by the Chernov solution, which is thought to be valid for long propagation distances. Our simulations unequivocally support that conclusion.

The integral in (3.2), which becomes L_f at greater distances, integrates to a smaller value close to the origin. As a result, travel-time variance is less than the asymptotic result at short distances. In every subsequent experiment, this "dip" is visible.
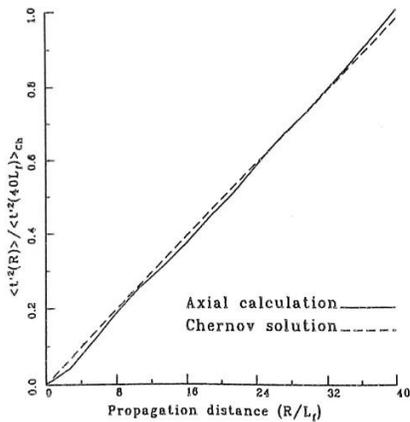
Fig. 1. Arrival time variance of simulated acoustic rays constrained to axial Propagation

(b). Ray-trace Simulations

We performed a second set of trajectory calculations using the entire ray-trace equations [Eq. (2.1)] and the identical field characteristics as before. In other words, we restored the equations that account for lateral excursions of the ray trajectories and kept the nonaxial velocity terms, unlike the simulation described above. The results are once more scaled L_f for the distance of propagation and normalized by the Chernov result at R= 40 L_f for the time variance in Figure 2. It is clear that time variance is not proportional to propagation distance when using full ray tracing. In reality, $\langle t'^2 \rangle$ is the best fit over the interval R = 0 to R = 20 L_f if a power law were applied to the result $R^{1.6}$. Although this result is surprising, it is compatible with the subsequent numerical trials, as we will show.
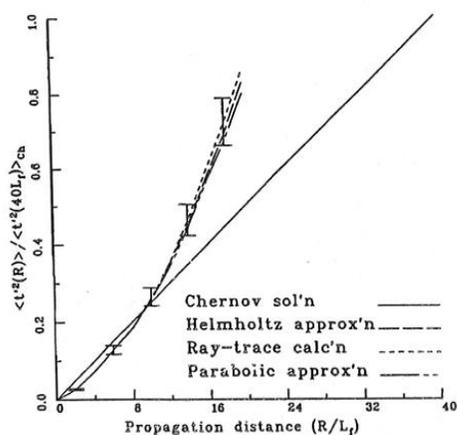

Fig.2 Arrival time variance in simulations using three different ray-trace approximations

(c). Stochastic Helmholtz Solutions

The so-called stochastic Helmholtz equation approximates a description of sound transmission across a turbulent field:

$$[\nabla^2 + k_0^2\, n^2]\, P(x) = 0$$

where P is the wave pressure and k_0 is the acoustic wave number in the medium at rest. The "perturbation" index ϵ can be used to characterize the refraction index n for forward scattering as follows: $n^2 = c_0^2/c_2 = 1 + \epsilon$, where $\epsilon = -\; 2u_1/\, c_0 - \tau'/\, T$. (The contribution caused by temperature inhomogeneities is denoted by the quantity τ'/T, which is not used in this study.) This explicit form necessitates a number of approximations (Tatara, 1971; Candel, 1979; Neubert, 1970; Neubert and Lumley, 1970).

Keep in mind that velocity disturbances appear to be scalar rather than vector disturbances, and that the influence of field disturbances is restricted to those in the axial direction. By representing our velocity fluctuations as variations in the refractive index and sending hypothetical rays through an equivalent inhomogeneous scalar field instead of our actual velocity field, we can replicate this approximation.

With $\Omega = 1$ in these circumstances, our ray equations for "scalar" inhomogeneities turn into

$$\frac{dx_i}{dt} = \frac{c^2 S_1}{(1 - 2u_1/c_0)}$$

$$\frac{ds_i}{dt} = -\frac{1}{c_0(1 - 2u_1/c_0)}\frac{\partial u_1}{\partial x_i}$$

The relation $n^2 = 1 - 2u_1/c_0$ can be approximated at first order as $c = c_0\,(1 + 2\,u_1/c_0)$. This gives an example of how the "Helmholtz" ray differs from the actual ray. A vectorial addition of $c_0 s\,/|s|$ and u, where $s/|s|$ is the unit normal to the wave front, yields the propagation velocity in the former scenario. In the latter, $(c_0 + u_1)\, s\,/|s|$ approaches the propagation velocity. The angular difference between the two procedures is negligible due to the tiny value of u compared to c_0, and the outcomes of the two approaches ought to be closely aligned.

Using these equations, we conducted a series of ray-tracing experiments once more, yielding the time-variance results shown above in Figure 2. In reality, time variance is completely consistent with the outcomes of the whole ray-tracing tests and is still not linearly correlated with journey distance.

(d). A Parabolic Approximation Solution

The parabolic approximation is a popular method for estimating the Helmholtz equation in acoustics. Since the acoustic pressure in the Helmholtz equation is expressed as $P = \exp(ik_0 x_i)\,\psi\,(x)$, and acoustic propagation is assumed to be primarily along a principal direction, such as the x_i axis, the second derivative of ψ with respect to x_i is insignificant in comparison to the first derivative, and the result assumes a parabolic form. Refer to the talks by Candel (1979) and Tappert (1977). This means that (7) reduces to

$$2ik_0 \frac{\partial \psi}{\partial x_1} + \frac{\partial^2 \psi}{\partial x_2^2} + \frac{\partial^2 \psi}{\partial x_3^2} + k_0^2(n^2 - 1)\psi = 0$$

The eikonal equation can be written in terms of the components if one gives ψ the form

$\psi \sim e^{i\,k_0[S(x) - x_i]}$, where S is connected to the slowness vectors s by $\nabla S = c_0 s$. $Q_i = \nabla_i S$

$$2Q_1 + Q_2^2 + Q_3^2 - n^2 - 1 = 0$$

Thus, $H(Q, S, x) = 0$ is the form of the eikonal relation. This is an example of a first-order, nonlinear problem that may be solved using the characteristics technique. Following Eqns. (2.82-2.88) of Whitham [(1974)] Using the auxiliary variable σ, we derive the ray-tracing equations. In this approximation, integration along a characteristic is typically not orthogonal to the acoustic wave front. Therefore, an extra equation and the auxiliary variable σ are needed to determine propagation time.

We obtain

$$\frac{dx_1}{d\sigma} = c_0$$

$$\frac{dx_1}{d\sigma} = c_0$$

$$\frac{dx_2}{d\sigma} = c_0 Q_2$$

$$\frac{dx_3}{d\sigma} = c_0 Q_3$$

$$\frac{dQ_1}{d\sigma} = c_0\, n\, \frac{\partial n}{\partial x_1}$$

$$\frac{dt}{d\sigma} = Q_1 + Q_2^2 + Q_3^2 = n^2 + 1 - Q_1$$

$S = c_0 t$ is used to generate the final equation pertaining to the propagation time t. The beginning conditions are $Q_1 = 0.5(n^2 + 1)$ and $Q_2 = Q_3 = 0$, which correspond to a horizontal ray. These equations were used to numerically track an ensemble of rays, producing the

final curve shown in Fig.2 which once more agrees with the whole ray-trace curve.

4. Experimental Results (Laboratory);

The lack of data suggests that it is difficult to acquire laboratory measurements of phase or time-of-flight variance. Only the B-B experiments are published, with the exception of H_0 and Kovasznay (1974), who performed such measurements across an air-jet (although over a very short propagation distance). Using a piston-like sound source, B-B produced a nearly plane acoustic wave for his experiments, which he then directed over jet-generated air flows with turbulence levels u = 2.4 m/s and 1.1 m/s. He examined the acoustic phase variance at four acoustic frequencies 31.25, 41.66, 50.00, and 83.33 kHz at a distance of one meter from the nozzle.

We employed fields with the same integral length scale and turbulence levels in a final set of numerical experiments to recreate these laboratory ones using our ray-tracing technique. Fig. 2 plots the results, B-B's data points, and the Chernov approximation. The relation $\omega t = \varphi$ has been used to translate laboratory phase variances to time variances. This connection is supported by the
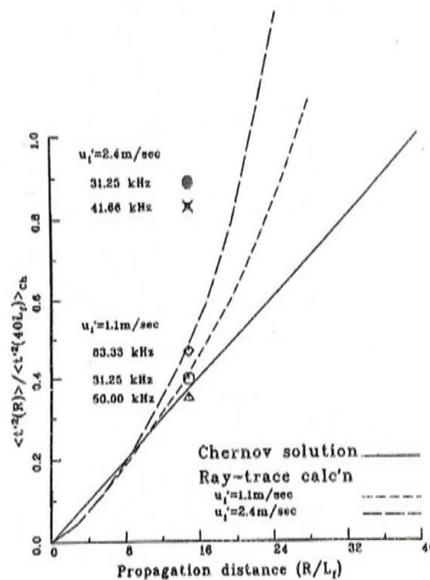

Fig.3 Arrival time variances of numerical experiments compared with the laboratory Experiments of Blanc-Benon.

quasilinear dependence of $\sqrt{\varphi^2}$ versus ω seen in the experiments (relative error < 5%). At his lower turbulence level, B-B's measurements closely match

the computer simulation and the Chernov forecast. The measurements do not follow either at his greater turbulence level. They do, however, qualitatively match the nonlinear structure that the ray-trace simulations predicted

For our ray-trace simulation scan, the applicability of B-B's experiments can be assessed using a $\wedge - \varphi$ diagram developed by Flatt et al. (1979). This technique divides the acoustic situation into "saturated," "partially saturated," and unsaturated" zones according to the diffraction parameter $\wedge = R / (L_f^2 \ k_0)$ and the rms value expected for phase variance, $ß\varphi = 2L_f k_0^2 \ R\langle\mu^2\rangle^{1/2}$. Zone boundaries for the five cases of B-B are determined for a Kolmogorov spectrum $E(k) \sim k^{-5/3}$ and are shown on such an A-• diagram in Figure 6. Here, we demonstrate that the geometric acoustics (unsaturated) regime encompasses the majority of the experimental data. The partially saturated zone is only reached in the cases where u' = 2.4 m/s and f = 41.66 kHz.

It is important to note that these zone boundaries are qualitative in nature. Furthermore, B-B's results might be somewhat tainted by effects that weak fluctuation theory does not account for because his trials took place around the region of partial saturation. Comparisons must therefore be made carefully.

4. CONCLUSION

We have presented a novel method for examining sound propagation across inhomogeneous media in this paper. We have given a way to analytically analyze the acoustic ray-trace equations at each location along a path by describing a field in terms of a set of Fourier modes. As a result, we are able to carry out precise numerical simulations that are comparable to lab tests. These simulations have the ability to map out the entire history of an acoustic wave, unlike laboratory investigations. Additionally, computer simulations can be carried out with precise prescriptions of field properties and acoustic wave-front characteristics, unlike laboratory tests.

There are numerous applications for the approach. We are currently expanding the methodology to investigate acoustic dispersion in density-inhomogeneous fields and the occurrence of caustics in velocity-inhomogeneous fields with varying spectral characteristics. Additionally, one may infer

acoustic intensity variability by choosing a small circle of "tagged" points to mark the perimeter of a hypothetical ray tube and tracking the propagation of these points.

Lastly, we have observed that other methods that offer acoustic pressure repartitions, like the parabolic approximation, can also be modified for use in random field simulations. Our approach may be especially helpful for forecasting changes in sound intensity in saturated or unsaturated situations.

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