DIFFERENTIAL-TRANSEFORM-METHOD FOR SOLVING LOTKA - VOLTERRA EQUATIONS BY USING HOLLING TYPE III FUNCTIONAL RESPONSE

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Abstract: The goal of the current study is to solve the one-species Lotka-Volterra model and its modified form by introducing Holling type III functional response using Differential-Transform-Method (DTM). The solution obtained by DTM is compared with the Adomian Decomposition Method (LADM) and it was found that DTM is most potent numerical techniques for non-linear differential equation. Results are validate with exact solution in limiting case.

Keywords: *Laplace Adomian Decomposition Method, Differential-Transform-Method, Lotka-Volterra model, Holling Type III Functional Response.*

1. Introduction:

Lotka-Volterra model:

The Lotka-Volterra equations describe an arbitrary number of ecological competitors (or predator-prey) model which is dynamic in nature [22]. The model was framed keeping in view the ecological system but gradually gained its popularity in the engineering fields. The simple prey-predator model is among the most popular models used to demonstrate a simple non-linear control system.

Numerical techniques:

In the concerned field of science and technology, numerous significant physical phenomenon are frequently modeled by nonlinear differential equations. Such equations are often stiff or impractical to solve analytically. Yet, analytical approximate methods to obtain fairly accurate solutions have gained much significance in recent years [18]. There are numerous methods, undertaken to find out approximate solutions to nonlinear problems. Homotopy Perturbation method (HPM), Homotopy Analysis method (HAM) [21], Differential Transform method (DTM), Variational Iteration method (VIM) [11], Adomian Decomposition method (ADM), Laplace Adomian Decomposition method (LADM) and Runge-Kutta-Fehlberg method (RKF) [19-20] and Chebyshev Spectral methods [13, 20] are some proven instances. The purpose of this paper is to bring out the analytical expressions of Lotka-Volterra single species population and the solution of nonlinear differential equations by using the new approach to Differential Transform method (DTM) [1-4, 14-16] in an elegant way. Thus, all these methods entail to multidimensional aspects.

Motivation:

The accurate solutions of population growth models may become a difficult task either if the equations are stiff (even with a small number of species) or when the number of species is large. To overcome the situation there are few literatures in existence. So to fill up the gap, here we consider and developed the accurate solutions of population models by applying such a reliable, efficient and more comfortable numerical technique (e.g. LADM, DTF).

Novelties:

The principal aim of this paper is to perform systematic analysis of the comparisons among exact solution and some reliable numerical techniques on the dynamics of the non-autonomous Lotka-Volterra model which shall be made to determine the performance of the method which is more acceptable and reliable for solving such kind of problem. The inclusions are described below

(i) Then we introduce two new methods called LADM and DTM for solution of the single species Lotka-Volterra equation. Here we highlight the numerical solutions of Lotka–Voltera single species model by using these methods.

(ii) Analysis of the comparisons among exact solution, Laplace Adomian Decomposition method (LADM) and Differential Transform method (DTM) on the Lotka-Volterra single species model.

(iii) The behavior of the method in-different numerical technique is illustrated graphically.

Moreover, we can say all these developments can help the researchers who engage with nonlinear differential equation and mathematical biology.

Structure of the paper:

The paper is organized as follows: In "Numerical Solution of Nonlinear Differential Equation" section we the proposed Laplace Adomian Decomposition Method (LADM) and Differential Transform method (DTM) in nonlinear **e**quation. "Analysis of multispecies Lotka–Volterra equations" section is followed by a numerical example. In Sec In sec "Result and Discussion" is illustrated the error term of these methods. Finally conclusions and future research scope of this article are drawn in last section, "Conclusion" section.

2. Numerical Solution of Nonlinear Differential Equation:

Laplace Adomian Decomposition Method (LADM) for Nonlinear Differential Equation:

The Laplace Adomian Decomposition Method (LADM) [22, 17] was firstly introduced by Suheil A. Khuri and has been successfully used to find the solution of linear and nonlinear differential equations. The significant advantage of this method is that it is a combination of two powerful techniques viz. Laplace transform and Adomian Decomposition Method [6, 7, 8, 9]. The Laplace transform is an elementary but useful technique for solving linear ordinary differential equations i.e., widely used by scientists and engineers for tackling linearized models. But it is totally incapable to solve non-linear equation and to overcome this shortcoming here we use the term Adomian polynomials from Adomian Decomposition Method which decompose the nonlinear term and make it easier to calculate. The main thrust of this method is that the solution of this method expressed in ∞- series which converges first to the exact solution and will not take too much time for compute.

Consider the following nonlinear differential equation

$$Lu(t) + Ru(t) + Nu(t) = g(t)\,. \tag{1}$$

where L is a linear operator of the highest-order derivative which is assumed to be invertible easily, R is the remaining linear operator of order less than L and N is a nonlinear operator and $g(t)g(t)$ is a source term.
Taking Laplace transform on both sides of above equation, we get

$$\mathcal{L}[Lu(t)] + \mathcal{L}[Ru(t)] + \mathcal{L}[Nu(t)] = \mathcal{L}[g(t)g(t)], \tag{2}$$

Using the differential property of Laplace transform and using the initial condition, we get

$$s^n\mathcal{L}[u(t)] - s^{n-1}u(0) - s^{n-2}u^{'}(0) - \cdots\cdots - u^{n-1}(0) + \mathcal{L}[Ru(t)] + \mathcal{L}[Nu(t)] = \mathcal{L}[g(t)g(t)],$$

$$\text{or, } \mathcal{L}[u(t)] = \frac{u(0)}{s} + \frac{u^{'(0)}u'(0)}{s^2} + \cdots\cdots + \frac{u^{n-1}(0)}{s^n} - \frac{1}{s^n}\mathcal{L}[Ru(t)] - \frac{1}{s^n}\mathcal{L}[Nu(t)] + \frac{1}{s^n}\mathcal{L}[g(t)g(t)] \tag{3}$$

Now we represent the unknown functions $u(t)u(t)$ by an infinite series of the form

$$u(t) = \sum_{n=0}^{\infty} u_n(t)\ , \tag{4}$$

Here the components $u_n(t)u_n(t)$ are usually determined recurrently and the nonlinear operator $N(u)N(u)$ can be decomposed into an infinite series of polynomials given by

$$N(u) = \sum_{n=0}^{\infty} A_n,$$

where A_n are Adomian polynomials of $u_0, u_1, \dots \dots \dots, u_n$ defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, n = 0,1,2, \dots \dots \dots$$

Therefore,

$$\mathcal{L}[\textstyle\sum_{n=0}^{\infty} u_n(t)u_n(t)] = \frac{u(0)}{s} + \frac{u'(0)}{s^2} + \dots \dots \dots \dots + \frac{u^{n-1}(0)}{s^n} - \frac{1}{s^n}\mathcal{L}[R\{\sum_{n=1}^{\infty} u_n(t)\}R\{\sum_{n=1}^{\infty} u_n(t)\}] - \frac{1}{s^n}\mathcal{L}[\sum_{n=1}^{\infty} A_n] + \frac{1}{s^n}\mathcal{L}[g(t)g(t)].$$

In general, the recursive relation is given by

$$\mathcal{L}[u_0(t)u_0(t)] = \frac{u(0)}{s} + \frac{u'(0)}{s^2} + \cdots \dots \dots \dots \dots + \frac{u^{n-1}(0)}{s^n} + \frac{1}{s^n}\mathcal{L}[g(t)g(t)], \tag{5}$$

and

$$\mathcal{L}[u_{n+1}(t)u_{n+1}(t)] = -\frac{1}{s^n}\mathcal{L}[R(u_n(t))] - \frac{1}{s^n}\mathcal{L}[A_n]\,. \tag{6}$$

Applying the inverse Laplace transform to both sides of (5) and (6), we obtain $u_n, (n \geq 0)\ \ u_n, (n \geq 0)$, which is then substituted into (4).
For numerical computation, we get the expression as

$$\phi_n(t) = \sum_{k=0}^{n} u_k(t)u_k(t). \tag{7}$$

which is the n^{th} term approximation of $u(t)u(t)$ and the obtained series solution converges to the exact solution and the convergence of the method is established by [24-28].

3. Differential-Transform Method (DTM) for Nonlinear Differential Equation (One-Dimensional Differential Transform):

In this section, we first give some basic properties of one-dimensional differential transform method. Differential transform of a function $y(x)y(x)$ is defined as follows:

$$Y(k) = \frac{1}{k!}\left.\frac{d^k y}{dx^k}\right|_{x=0} \tag{7}$$

where $y(x)y(x)$ is the original function and $Y(k)$ is the transformed function for $k = 0, 1, 2, 3, \ldots$. The differential inverse transform of $Y(k)$ is defined as

$$y(x) = \sum_{k=0}^{\infty} x^k \ Y(k) y(x) = \sum_{k=0}^{\infty} x^k \ Y(k) \tag{8}$$

From Equations (7) and (8) we get

$$y(x) = \sum_{k=0}^{\infty} x^k \ \frac{1}{k!}\left.\frac{d^k y}{dx^k}\right|_{x=0} \tag{9}$$

which implies that the concept of DTM is derived from Taylor series expansion, but the method does not evaluate the derivative symbolically. However, relative derivatives are calculated by an iterative procedure which is described by the transformed equations of the original functions. In this work, we use the lower-case letters to represent the original functions and upper-case letters to represent the transformed functions.

The operation properties of differential Transformation:

If u(x) and v(x) are two uncorrelated functions of x where U(k) and V(k) are the transformed functions corresponding to u(x) and v(x) then we can easily proof the fundamental mathematics operations performed by differential transformation,

1. $If \ \ y(x) = u(x) \pm v(x) \ \ then \ \ Y(k) = U(k) \pm V(k)$
2. $If \ y(x) = cu(x) \ then \ Y(k) = cU(k)$. Where c is any constant.
3. $If \ y(x) = \frac{du}{dx} \ then \ Y(k) = (k+1)U(k+1) \ If \ y(x) \ = \frac{du}{dx} \ then \ Y(k) = (k+1)U(k+1)$
4. $If \ y(x) = \frac{d^m u}{dx^m} \ then \ Y(k) = (k+1)(k+2) \ldots \ldots \ldots \ldots (k+m)U(k+m)$.
5. $If \ y(x) = u(x)v(x) then \ Y(k) = \sum_{r=0}^{k} U(r)V(k-r) \ If \ y(x) \ = u(x) \ v(x) \ then \ Y(k) = \sum_{r=0}^{k} U(r)V(k-r)$
6. $If \ y(x) = \exp(\lambda x) \quad then \ Y(k) = \frac{\lambda^k}{k!} \ \ If \ y(x) \ = \exp(\lambda x) \quad then \ Y(k) = \frac{\lambda^k}{k!}$

7. $If\ y(x) = x^m\ then\ Y(k) = \delta(k-m) \qquad where\ \delta(k-m) = \begin{cases} 1 & if\ k = m \\ 0, & if\ k \neq m \end{cases}$ $If\ y(x) = x^m\ then\ Y(k) = \delta(k-m) \qquad where\ \delta(k-m) = \begin{cases} 1 & if\ k = m \\ 0, & if\ k \neq m \end{cases}$

8. $If\ y(x) = (1+x)(1+x)^m\ \ then\ Y(k) = \frac{m(m-1)(m-2)\ldots\ldots\ldots(m-k+1)m(m-1)(m-2)\ldots\ldots\ldots(m-k+1)}{k!}$ $If\ y(x) = (1+x)(1+x)^m\ \ then\ Y(k) = \frac{m(m-1)(m-2)\ldots\ldots\ldots(m-k+1)m(m-1)(m-2)\ldots\ldots\ldots(m-k+1)}{k!}$

9. $If\ y(x) = sin(\omega x + \alpha)\ then\ Y(k) = \frac{\omega^k}{k!} sin\left(\frac{\pi k}{2} + \alpha\right)$ $If\ y(x) = sin(\omega x + \alpha)\ then\ Y(k) = \frac{\omega^k}{k!} sin\left(\frac{\pi k}{2} + \alpha\right)$

10. $If\ y(x) = cos(\omega x + \alpha)\ then\ Y(k) = \frac{\omega^k}{k!} cos\left(\frac{\pi k}{2} + \alpha\right)$ $If\ y(x) = cos(\omega x + \alpha)\ then\ Y(k) = \frac{\omega^k}{k!} cos\left(\frac{\pi k}{2} + \alpha\right)$

2.2.2. Two dimensional Differential Transform:

The basic definitions and fundamental operations of the two dimensional differential transform are defined in [4-7]as follows. Consider a function of two variable $w(x,y), w(x,y)$, be analytic in the domain K and let $(x,y) = (0,0)(x,y) = (0,0)$ in this domain. The function $w(x,y)w(x,y)$ is then represented by one series whose centre at located at (0,0). (0,0). The differential transform of the function $w(x,y)\ w(x,y)$is the form

$$W(k,h) = \frac{1}{k!\,h!}\left[\frac{\partial^{k+h}\ w(x,y)\partial^{k+h}\ w(x,y)}{\partial x^k \partial y^k}\right]_{(0,0)(0,0)}$$

Where $w(x,y)w(x,y)$ is the original function and $W(k,h)W(k,h)$ is the transformed function. The differential inverse transform of $W(k,h)W(k,h)$ is defined as

$$w(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} W(k,h)\ (x - x_0)(x - x_0)^k (y - y_0)(y - y_0)^h$$

1. $If\ \ w(x,y) = u(x,y) \pm v(x,y)\ \ then$

   $W(k,h) = U(k,h) \pm V(k,h)$

2. $If\ \ w(x,y) = cu(x,y)\ then$

   $W(k,h) = c\ U(k,h)$

3. $If\ \ w(x,y) = \frac{\partial u(x,y)}{\partial x}\ \ then$

$W(k,h) = (k+1)U(k+1,h)W(k,h) = (k+1)\ U(k+1,h)$

4. $If\ \ w(x,y) = \frac{\partial u(x,y)}{\partial y}\ \ then$

$W(k,h) = (k+1)U(k,h+1)W(k,h) = (k+1)\ U(k,h+1)$

5. $If\ \ w(x,y) = \frac{\partial^{r+s} u(x,y)}{\partial x^r \partial y^s}\ \ then$

$W(k,h) = (k+1)(k+2)(k+3)\ldots(k+r)(h+1)(h+2)\ldots(h+s) \times U(k+r,h+s)$

6. $If\ \ w(x,y) = u(x,y)v(x,y)\ \ then$

$W(k,h) = \sum_{x=0}^{k}\sum_{x=0}^{h}\ U(r,h-s)V(k-r,s)\ U(r,h-s)V(k-r,s)$

8. $If\ w(x,y) = x^m y^n\quad then\ If\ w(x,y)\ = x^m y^n\quad then$

$W(k,h) =\ \delta(k-m,h-n)\ = \begin{cases} 1 & if\ k=m\ and\ h=n \\ 0, & otherwise \end{cases} W(k,h) =\ \delta(k-m,h-n)\ = \begin{cases} 1 & if\ k=m\ and\ h=n \\ 0, & otherwise \end{cases}$

9. $If\ \ w(x,y) = \frac{\partial u(x,y)}{\partial x}\frac{\partial v(x,y)}{\partial x}\ \ then$

$W(k,h) =\ \sum_{x=0}^{k}\sum_{x=0}^{h}(r+1)(k-r+1)\ U(r+1,h-s)V(k-r+1,s)(r+1)(k-r+1)\ U(r+1,h-s)V(k-r+1,s)$

10. $If\ \ w(x,y) = \frac{\partial u(x,y)}{\partial y}\frac{\partial v(x,y)}{\partial y}\ \ then$

$W(k,h) =\ \sum_{x=0}^{k}\sum_{x=0}^{h}(h-s+1)(s+1)\ U(r,h-s+1)V(k-r,s+1)(h-s+1)(s+1)\ U(r,h-s+1)V(k-r,s+1)$

11. $If\ \ w(x,y) = \frac{\partial u(x,y)}{\partial x}\frac{\partial v(x,y)}{\partial y}\ \ then$

$W(k,h) =\ \sum_{x=0}^{k}\sum_{x=0}^{h}(k-r+1)(h-s+1)\ U(k-r+1,s)V(r,h-s+1)(k-r+1)(h-s+1)\ U(k-r+1,s)V(r,h-s+1)$

12. $If\ \ w(x,y) = u(x,y)v(x,y)p(x,y)\ thenIf\ \ w(x,y) = u(x,y)v(x,y)p(x,y)\ \ then$

$W(k,h) =\ \sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U(r,h-s-p)V(t,s)P(k-r-t,p)U(r,h-s-p)V(t,s)P(k-r-t,p)$

13. $If\ \ w(x,y) = u(x,y)\frac{\partial v(x,y)}{\partial x}\frac{\partial p(x,y)}{\partial x}\ \ thenIf\ \ w(x,y) = u(x,y)\ \frac{\partial v(x,y)}{\partial x}\frac{\partial p(x,y)}{\partial x}\ \ then$

$$W(k,h) = \sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(t+1)(k-r-t+1)U(r,h-s-p)V(t+1,s)P(k-r-t+1,p)(t+1)(k-r-t+1)U(r,h-s-p)V(t+1,s)P(k-r-t+1,p)$$

14. $If\ \ w(x,y) = u(x,y)v(x,y)\frac{\partial^2 p(x,y)\partial^2 p(x,y)}{\partial x^2}$

$$W(k,h) = \sum_{r=0}^{k}\sum_{t=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s}(k-r-t+2)(k-r-t+1)U(r,h-s-p)V(t,s)P(k-r-t+2,p)(k-r-t+2)(k-r-t+1)U(r,h-s-p)V(t,s)P(k-r-t+2,p)$$

3. Analysis of multispecies Lotka–Volterra equations:

Mathematical models of population growth have been formed to provide an inconceivable significant angle of true ecological situation. The meaning of each parameter in the models has been defined biologically. For *n* species, we consider the following [12, 23] general Lotka–Volterra model

$$\frac{dN_i}{dt} = N_i\left(b_i - \sum_{j=1}^{n} a_{ij}N_j\right), i = 1, 2, \dots\dots\dots\dots\dots..n.$$

These equations may represent either predator–prey or competition cases.

3.1 Model I (Single species):

In case of one-species, Eq. (10) is written for a given limited source of food,

$$\frac{dN}{dt} = N(b-aN), \qquad b>0, \qquad a>0, \qquad N(0)>0$$

where a and b are positive constants. This equation has an exact solution

$$N(t) = \begin{cases} \dfrac{be^{bt}}{\left(\dfrac{b-aN(0)}{N(0)}\right)+ae^{bt}} & for\ b \neq 0 \\[2ex] \dfrac{N_0(t)N_0(t)}{1+atN_0(t)1+atN_0(t)} & for\ b = 0. \end{cases}$$

where $N(0)$ is the initial condition.

Solving Eq. (10) by LADM yields the following recursive algorithm

$$N_0 = N(0),\ \ N_{n+1}(t) = bL^{-1}\left\{\frac{1}{s}L\big(N_n(t)\big)\right\} - aL^{-1}\left\{\frac{1}{s}L\big(A_{1,n}\big)\right\}\ \ ,\ \ \ n \geq 0$$

where the Adomian Polynomials $A_{1,n}$ are given by

$$A_{1,n} = \sum_{i=0}^{n} N_i N_{n-i} \ .$$

We now solve Eq. (10) by Differential - Transform Method (DTM) with the initial condition $N(0) = 0.1$. Applying differential transform, we have

$$(k+1)N_1(k+1) = bN_1(k) - a\sum_{r=0}^{k} N_1\,(r)N_1(k-r)(k+1)N_1(k+1)$$
$$= bN_1(k) - a\sum_{r=0}^{k} N_1\,(r)\,N_1(k-r)$$

$$N_1(\mathrm{k}+1) = \frac{1}{(k+1)(k+1)}\left[\mathrm{b}N_1(\mathrm{k}) - a\sum_{r=0}^{k} N_1 \circledR N_1(\mathrm{k}-\mathrm{r})\mathrm{b}N_1(\mathrm{k}) - a\sum_{r=0}^{k} N_1 \circledR N_1(\mathrm{k}-\mathrm{r})\right] N_1(\mathrm{k}+1)$$
$$= \frac{1}{(k+1)(k+1)}\left[\mathrm{b}N_1(\mathrm{k}) - a\sum_{r=0}^{k} N_1 \circledR N_1(\mathrm{k}-\mathrm{r})\mathrm{b}N_1(\mathrm{k}) - a\sum_{r=0}^{k} N_1 \circledR N_1(\mathrm{k}-\mathrm{r})\right]$$

Where $N_1(\mathrm{k})$ is the differential transform of $N(t)$.

Now the initial condition is

$$N(0) = 0.1 \Rightarrow N_1(0) = 0.1$$

Putting $k = 0,1,2,3, \ldots \ldots ..$ in equation (2) we get,

$$N_1(1) = \left[bN_1(0) - a\sum_{r=0}^{0} N_1\,(r)N_1(-r)bN_1(0) - a\sum_{r=0}^{0} N_1\,(r)N_1(-r)\right]$$

$$\Rightarrow N_1(1) = 0.1b - 0.01a$$

$$N_1(2) = \frac{1}{2}\left[bN_1(1) - a\sum_{r=0}^{1} N_1\,(r)N_1(1-r)bN_1(1) - a\sum_{r=0}^{1} N_1\,(r)N_1(1-r)\right]$$

$$\Rightarrow N_1(2) = \frac{1}{2}[0.1b - 0.03ab + 0.002a^2]$$

$$N_1(3) = \frac{1}{3}\left[bN_1(2) - a\sum_{r=0}^{2} N_1\,(r)N_1(2-r)bN_1(2) - a\sum_{r=0}^{2} N_1\,(r)N_1(2-r)\right]$$

$$\Rightarrow N_1(3) = \frac{1}{3}[0.05b^3 - 0.035ab^2 + 0.006a^2b - 0.0003a^3]$$

$$N_1(4) = \frac{1}{4}\left[bN_1(3) - a\sum_{r=0}^{3} N_1\,(r)N_1(3-r)\right]$$

$$\Rightarrow N_1(4) = \frac{1}{4}[0.017b^4 - 0.0254ab^3 + 0.0084a^2b^2 - 0.001a^3b + 0.00004b^4]$$

Using the inverse differential transform, we get

$$N(t) = \sum_{k=0}^{\infty} t^k N_1(k)$$

$$\Rightarrow\; N(t) = N_1(0) + tN_1(1) + t^2N_1(2) + t^3N_1(3) + t^4N_1(4) + \cdots$$

$$\begin{aligned}\Rightarrow\; N(t) = 0.1 + t(0.1b - 0.01a) + \frac{t^2}{2}(0.1b^2 - 0.03ab + 0.002a^2) \\ + \frac{t^3}{3}(0.05b^3 - 0.035ab^2 + 0.006a^2b - 0.0003a^3) \\ + \frac{t^4}{4}(0.017b^4 - 0.0254ab^3 + 0.0084a^2b^2 - 0.001a^3b \\ + 0.00004a^4) +\end{aligned}$$

Results and Discussion:

The numerical solutions obtained by using the DTM and LADM are compared with the exact solution (for single-species). Table 1 shows comparison among the DTM, 3-term LADM and the exact solution for the single species in the case $b = 1, a = 3$ and $N\,(0) = 0.1, h = 0.1$. The results show error free calculation between exact solution and DTM whereas there are some amount of error in the calculation between exact solution and LADM.

Table 1: Numerical Comparison when initially we have $N(0) = 0.1, a = 3, b = 1$.

t	Exact Sol.	DTM	LADM (3-iteration)	EDTM	ELADM
0	0.1	0.10000000	0.10000000	0.00E+00	0.00E+00
0.1	0.10713679	0.10713678	0.10714000	1.00E-08	-3.21E-06
0.2	0.11453291	0.11453279	0.11456000	1.20E-07	-2.71E-05
0.3	0.12216385	0.12216320	0.12226000	6.50E-07	-9.61E-05
0.4	0.13000114	0.12999876	0.13024000	2.38E-06	-2.39E-04
0.5	0.13801261	0.13800583	0.13850000	6.78E-06	-4.87E-04
0.6	0.1461629	0.14614634	0.14704000	1.66E-05	-8.77E-04
0.7	0.15441399	0.15437778	0.15586000	3.62E-05	-1.45E-03
0.8	0.16272591	0.16265327	0.16496000	7.26E-05	-2.23E-03
0.9	0.1710575	0.17092148	0.17434000	1.36E-04	-3.28E-03
1	0.17936718	0.17912667	0.18400000	2.41E-04	-4.63E-03
1.1	0.18761383	0.18720869	0.19394000	4.05E-04	-6.33E-03
1.2	0.19575756	0.19510298	0.20416000	6.55E-04	-8.40E-03
1.3	0.2037605	0.20274054	0.21466000	1.02E-03	-1.09E-02
1.4	0.21158743	0.21004799	0.22544000	1.54E-03	-1.39E-02
1.5	0.21920638	0.21694750	0.23650000	2.26E-03	-1.73E-02
1.6	0.22658907	0.22335684	0.24784000	3.23E-03	-2.13E-02
1.7	0.23371122	0.22918937	0.25946000	4.52E-03	-2.57E-02
1.8	0.24055276	0.23435402	0.27136000	6.20E-03	-3.08E-02
1.9	0.24709782	0.23875530	0.28354000	8.34E-03	-3.64E-02
2	0.25333471	0.24229333	0.29600000	1.10E-02	-4.27E-02

EDTM → Error term of DTM
ELADM → Error term of LADM.

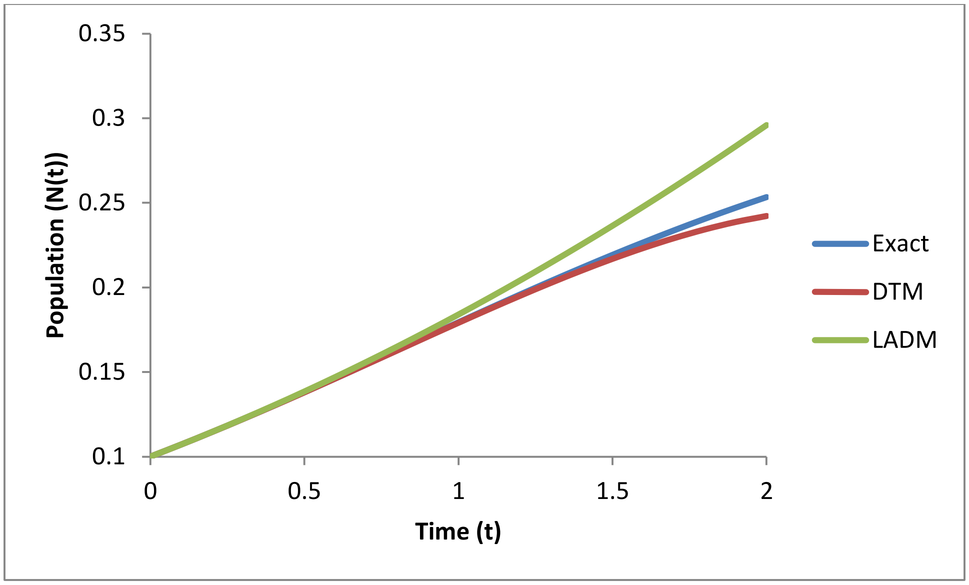

Figure 1: Evaluation between the exact solution and the solutions obtained by using LADM and DTM methods for model I.

Conclusions:

This paper aims to present the numerical solution of Lotka – Voltera model by virtue of a popularly techniques called DTM. The solution obtained by this technique show high accuracy in compare to previously available solution. The obtained solution is also compared with exact solution for a limiting case (single species case) and the LADM solution. The above observation establishes the reliability and accuracy of DTM technique for the solution of linear and no-linear population models. The solution obtained by DTM also reasonably accurate after sufficiently large time. Secondly DTM techniques not require evaluation of Adomian polynomials, which is needed for the case of LADM. Hence it provides an efficient numerical solution.

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