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# A novel method for solving transcendental equations

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***Abstract*— Finding the roots of transcendental equations is a fundamental problem in numerical analysis. There are many methods to solve transcendental equations. These methods have different approaches with advantages and disadvantages. In this article, a new method for finding the roots of equation is introduced. This method uses tangents at the end points of the intervals and the intersection of these tangents. This point of intersection gives the subsequent approximation for root. The convergence and order of the method is also discussed. In special case this method reduces to Bisection method.**

***Index Terms*— Numerical method, Transcendental equations, Convergence, Order of method, Bisection method, Roots of equation.**

## I. INTRODUCTION

Numerical methods are extensively used for finding approximate roots of transcendental equations. These methods are popular as it provides approximate roots with required accuracy. In this article the new method is introduced to find the roots of the algebraic and transcendental equations. The order of the method is also obtained. This method is tested for different algebraic and transcendental equations. Few solved examples are also included in this article.

## II. PRELIMINARY

In the numerical methods, the convergence and the order of the method is more important. To prove the convergence of the method, some standard theorems are required. These theorems are given below.

Cantor Intersection Theorem [4]:

Let $\mathrm{I_n} = [\mathrm{a_n}, \mathrm{b_n}]$, $\mathrm{n} \in \mathrm{N}$, be a non-empty closed and bounded interval on R such that $\{\mathrm{I_n}\}$ is a nested sequence satisfying $\lim_{n\to\infty}(\mathrm{b_n} - \mathrm{a_n}) = 0$ then $\bigcap_{n=1}^{\infty} \mathrm{I_n}$ contains exactly one point.

Existence Theorem [4]:

If $f$ is a continuous function on closed interval [a, b] where $f(a) \le 0 \le f(b)$ or $f(a) \ge 0 \ge f(b)$, then $f(x) = 0$ has at least one root in the interval [a, b].

Intermediate Value theorem [4]:

If $f(x)$ is continuous function defined on $(a\ ,b)$ then $f(x)$ takes every value between $f(a)$ and $f(b)$.

Bisection Method [1, 2, 3, 5]:

Let $f(x) = 0$ be a transcendental equation defined in interval $(a, b)$ such that $f(a).f(b) < 0$ then the approximate solution to the root of $f(x) = 0$ is given by $x = \frac{a+b}{2}$.

## III. PROPOSED METHOD

In this section, a new method is introduced to find a root of algebraic and transcendental equation $f(x) = 0$ in interval $(a, b)$ such that $f(a).f(b) < 0$. Without loss of generality, assume that $f(a) < 0 < f(b)$. Tangents are drawn to the curve $f(x)$ at the points $(a, f(a))$ and $(b, f(b))$. The intersection of two tangents is obtained say$(x_{0,}y_0)$. This is the first approximation to the root of $f(x) = 0$.

Suppose root lies in interval (a, $x_0$) i.e. f(a). f($x_0$) < 0. The tangent has been drawn at points $(x_0, f(x_0))$ which intersects the tangent at point $(a, f(a))$ say ($x_1$, $y_1$). Continuing this procedure, we get the sequence of intersection points of tangents as $(x_0, y_0), (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n), \dots.$ The sequences of x coordinates of these intersection points $\{x_n\}$ and the sequence of its images $\{f(x_n)\}$ is obtained. The sequence $\{x_n\}$ converges to the root of the equation. Its convergence and order is discussed in next section.

Since the intersections of tangents is used to approximate the roots. So, this method can be called as Tangent Intersection Method (TIM) or Patil-Dasre-Gujarathi Method.

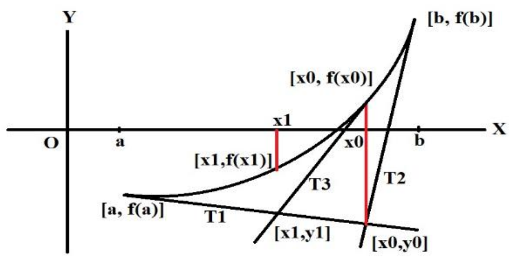



Fig (1) Tangent Intersection Method

Tangent Intersection Method (TIM):

Let $f(x)$ be a differentiable function in $(a, b)$ such that $f(a).f(b) < 0$ then the root lies in $(a, b)$.

To find the root of the equation $f(x) = 0$ by using Tangent intersection method, the tangent $T_1$ is drawn to the curve at point $(a, f(a))$ is given by,

$$T_1 \equiv\ y = f(a) + f'(a)(x - a) \quad \text{... (1)}$$

and the tangent $T_2$ is drawn to the curve at point $(b, f(b))$ is given by,

$$T_2 \equiv\ y = f(b) + f'(b)(x - b) \quad \text{... (2)}$$

Let (x, y) be the intersection point of two tangents. The x coordinate of tangents is calculated by solving equation (1) and (2) as

$$x = \frac{[f(b) - bf'(b)] - [f(a) - af'(a)]}{f'(a) - f'(b)} \quad \text{... (3)}$$

Let $x_0 = x$ be the first approximation to the root of the equation $(x) = 0$ .

If $f(x_0) \neq 0$ then a tangent at point $(x_0, f(x_0))$ is drawn and is given by, $T_3 \equiv\ y = f(x_0) + f'(x_0)(x - x_0)$.

If $f(a)f(x_0) < 0$ then the tangent $T_3$ will intersect tangent $T_1$ and if $f(x_0)f(b) < 0$ then tangent $T_3$ will intersect tangent $T_2$. The x coordinate of the intersection of two tangents is calculated by equation (3) and its x coordinate will be obtained say $x_1$.

On continuing this way, the sequence of x coordinates of intersection points of tangents $x_0, x_1, x_2, \ldots x_n, \ldots$ is generated.

The convergence of the sequence of intersection points of tangents is discussed in the following theorem.

Theorem: Let $f(x)$ be a differentiable function in $(a, b)$ and $f(a) \cdot f(b) < 0$ . The sequence of x coordinates $\{x_n\}$ of intersection of tangents generated by Tangent intersection Method [1] is converges and is the root of the equation $f(x) = 0$

Proof: Let $I_0 = (a, b)$ be the given interval. Then there are three cases as discussed below.

Case 1: Suppose $f(x_i) \in [f(a), 0],\ for\ all\ i$ and $0 < f(b)$.

The sequence $\{f(x_n)\}$ is generated as $f(a) < f(x_0) < f(x_1) < f(x_2) < \cdots < 0 < f(b)$. The sequence of intervals where the root lies is given by, $I_0 = [a, b]$, $I_1 = [x_1, b]$ , $I_2 = [x_2, b]$, ….. $I_n = [x_n, b]$… such that $I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots \supseteq I_n \supseteq \cdots$

Since, $f$ is differentiable function hence we have $f(I_0) \supseteq f(I_1) \supseteq f(I_2) \supseteq \cdots \supseteq f(I_n) \supseteq \cdots$

Therefore, by Cantor Intersection Theorem, $\bigcap_{n=1}^{\infty} I_n$ is a singleton point say $\eta$ i.e. $\bigcap_{n=1}^{\infty} I_n = \{\eta\}$ and as $0 \in f(I_n), \forall\ n \in N$ hence $\bigcap_{n=1}^{\infty} f(I_n)$ will be a singleton point 0.

Therefore $f(\eta) = 0$ and hence $\eta$ is a root of the equation $f(x) = 0$

Case 2: Consider $f(x_i) \in [0, f(b)], for\ all\ i$ and $f(a) < 0$.

The sequence $\{f(x_n)\}$ is generated as $f(a) < 0 < \cdots f(x_n) < \cdots f(x_2) < f(x_1) < f(x_0) < f(b)$. The sequence of intervals where the root lies is given by, $I_0 = [a, b]$, $I_1 = [a,\ x_1]$, $I_2 = [a,\ x_2]$, … $I_n = [a, x_n]$ such that $I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots \supseteq I_n \supseteq \cdots$ Since, $f$ is differentiable function hence we have $f(I_0) \supseteq f(I_1) \supseteq f(I_2) \supseteq \cdots \supseteq f(I_n) \supseteq \cdots$

Therefore, by Cantor Intersection Theorem, $\bigcap_{n=1}^{\infty} I_n$ is a singleton point say $\eta$ i.e. $\bigcap_{n=1}^{\infty} I_n = \{\eta\}$ and as

$0 \in f(I_n), \forall\, n \in N$ hence $\bigcap_{n=1}^{\infty} f(I_n)$ will be a singleton point 0.

Therefore $f(\eta) = 0$ and hence $\eta$ is a root of the equation $f(x) = 0$

Case 3: $f(x_n) \in [f(a), f(b)], for\ all\ n.$ Such that $f(a) < 0 < f(b)$

The sequence $\{f(x_n)\}$ such that $f(a) < f(x_n) < f(b)\ \forall\, n \in N$. This sequence can be divided into two sub sequences as follows $\{f(x_{n_i})\}\ and\ \{f(x_{n_j})\}$ Such that $f(a) < f(x_{n_i}) < 0\ < f(x_{n_j}) < f(b)$.

By Case 1, the sequence $\{f(x_{n_i})\} \to 0$ and from Case-2, $\{f(x_{n_j})\} \to 0$. Since both the subs-sequences $\{f(x_{n_i})\}\ and\ \{f(x_{n_j})\}$ of the sequence $\{f(x_n)\}$ are convergent and converges to 0. Therefore $\{f(x_n)\} \to 0$.

Since $f$ is continuous on $[a, b]$ hence the sequence $\{x_n\}$ will also converge.

Let sequence $\{x_n\} \to\ \eta \epsilon (a, b)$

Therefore $\{f(x_n)\} \to f(\eta)$ but $\{f(x_n)\} \to 0$.

Therefore $f(\eta) = 0$ which proves that $\eta$ is a root $f(x) = 0$ in (a, b).

Corollary: If $f(x) = px^2 + qx + r = 0, p \neq 0$ be any quadratic polynomial with root in [a, b] such that $f(a) \cdot f(b) < 0$ then Tangent Intersection Method (TIM) reduces to bisection method.

Proof: Let $f(x) = px^2 + qx + r = 0, p \neq 0$ be any quadratic polynomial $f'(x) = 2px + q$

At $x = a, f(a) = pa^2 + qa + r = 0$ and $f'(a) = 2pa + q$

At $x = b, f(b) = pb^2 + qb + r = 0$ and $f'(b) = 2pb + q$

By Tangent Intersection Method (TIM),

$$x = \frac{[pb^2 + qb + r - 2pb^2 - q] - [pa^2 + qa + r - 2pa^2 - q]}{2pa + q - 2pb - q}$$

$$= \frac{-pb^2 + r + pa^2 - r}{2pa - 2pb}$$

$$= \frac{pa^2 - pb^2}{2p(a-b)}$$

$x = \frac{a^2 - b^2}{2(a-b)} = \frac{a+b}{2}$ which leads to the formula of Bisection Method.

In other words, TIM is generalization of Bisection Method.

Order of Tangent Intersection method:

Let $\xi$ be the root of the equation $f(x) = 0$ so

$$f(\xi) = 0$$

Let $\epsilon_k$ be the error in the kth iteration i.e.

$$\epsilon_k = x_k - \xi$$

$\therefore x_k = \xi + \epsilon_k$ and $x_{k+1} = \xi + \epsilon_{k+1}$ ,

$x_{k-1} = \xi + \epsilon_{k-1}$, where $\epsilon_{k+1} and\ \epsilon_{k-1}$ are the errors in (k+1)th and (k-1)th iteration respectively.

$f(x_k) = f(\xi + \epsilon_k)$ , $f(x_{k-1}) = f(\xi + \epsilon_{k-1})$ and $'(x_k) = f'(\xi + \epsilon_k)$ , $f'(x_{k-1}) = f'(\xi + \epsilon_{k-1})$

Expanding all above functions by using Taylor's Series,

$$f(x_k) = f(\xi + \epsilon_k) = f(\xi) + \epsilon_k f'(\xi) + \frac{{\epsilon_k}^2}{2!} f''(\xi) + \frac{{\epsilon_k}^3}{3!} f'''(\xi) + \ldots$$

$$f(x_{k-1}) = f(\xi + \epsilon_{k-1}) = f(\xi) + \epsilon_{k-1} f'(\xi) + \frac{{\epsilon_{k-1}}^2}{2!} f''(\xi) + \frac{{\epsilon_{k-1}}^3}{3!} f'''(\xi) + \ldots$$

and

$$f'(x_k) = f'(\xi + \epsilon_k)$$
$$= f'(\xi) + \epsilon_k f''(\xi) + \frac{{\epsilon_k}^2}{2!} f'''(\xi) + \ldots$$

$$f'(x_{k-1}) = f'(\xi + \epsilon_{k-1})$$

$$= f'(\xi) + \epsilon_{k-1} f''(\xi) + \frac{{\epsilon_{k-1}}^2}{2!} f'''(\xi) + \ldots$$

Consider,

$$f(x_k) - x_k f'(x_k) = \left[f(\xi) + \epsilon_k f'(\xi) + \frac{\epsilon_k^2}{2!} f''(\xi) + \frac{\epsilon_k^3}{3!} f'''(\xi) + \ldots\right] - \left[x_k f'(\xi) + \epsilon_k x_k f''(\xi) + \frac{\epsilon_k^2}{2!} x_k f'''(\xi) + \ldots\right]$$

$$f(x_k) - x_k f'(x_k)$$

$$= f(\xi) + (\epsilon_k - x_k) f'(\xi) + \left(\frac{\epsilon_k^2}{2} - \epsilon_k x_k\right) f''(\xi) + \ldots \quad (1)$$

Similarly,

$$f(x_{k-1}) - x_{k-1} f'(x_{k-1}) = \left[f(\xi) + \epsilon_{k-1} f'(\xi) + \frac{\epsilon_{k-1}^2}{2!} f''(\xi) + \frac{\epsilon_{k-1}^3}{3!} f'''(\xi) + \ldots\right] - \left[x_{k-1} f'(\xi) + \epsilon_{k-1} x_{k-1} f''(\xi) + \frac{\epsilon_{k-1}^2}{2!} x_{k-1} f'''(\xi) + \ldots\right]$$

$$f(x_{k-1}) - x_{k-1} f'(x_{k-1})$$

$$= f(\xi) + (\epsilon_{k-1} - x_{k-1}) f'(\xi) + \left(\frac{\epsilon_{k-1}^2}{2} - \epsilon_{k-1} x_{k-1}\right) f''(\xi) + \ldots \quad \ldots (2)$$

Subtracting equation (2) from (1)

We get,

$$[f(x_k) - x_k f'(x_k)] - [f(x_{k-1}) - x_{k-1} f'(x_{k-1})]$$
$$= [(\epsilon_k - x_k) - (\epsilon_{k-1} - x_{k-1})] f'(\xi) + \left[\frac{\epsilon_k^2}{2} - \epsilon_k x_k - \frac{\epsilon_{k-1}^2}{2} + \epsilon_{k-1} x_{k-1}\right] f''(\xi) + \cdots$$

$$[f(x_k) - x_k f'(x_k)] - [f(x_{k-1}) - x_{k-1} f'(x_{k-1})] = (\xi - \xi) f'(\xi) + \left[\frac{\epsilon_k^2}{2} - \frac{\epsilon_{k-1}^2}{2} - (\epsilon_k + \xi)\epsilon_k + (\epsilon_{k-1} + \xi)\epsilon_{k-1}\right] f''(\xi) + \cdots$$

$$= \left[\frac{\epsilon_k^2}{2} - \frac{\epsilon_{k-1}^2}{2} - \epsilon_k^2 - \xi\epsilon_k + \epsilon_{k-1}^2 + \xi\epsilon_{k-1}\right] f''(\xi) +$$

$$= \left[-\frac{\epsilon_k^2}{2} + \frac{\epsilon_{k-1}^2}{2} - \xi\epsilon_k + \xi\epsilon_{k-1}\right] f''(\xi) + \cdots$$

$$= \left[-\frac{\epsilon_k^2}{2} + \frac{\epsilon_{k-1}^2}{2} - \xi\epsilon_k + \xi\epsilon_{k-1}\right] f''(\xi) + \cdots$$

$$= \left[\frac{1}{2}(\epsilon_{k-1} - \epsilon_k)(\epsilon_{k-1} + \epsilon_k) + \xi(\epsilon_{k-1} - \epsilon_k)\right] f''(\xi) + \cdots$$

$$= (\epsilon_{k-1} - \epsilon_k)\left[\frac{1}{2}(\epsilon_{k-1} + \epsilon_k) + \xi\right] f''(\xi) + \cdots \quad \ldots\ldots\ldots\ldots(3)$$

$$f'(x_{k-1}) - f'(x_k)$$

$$= \left(f'(\xi) + \epsilon_{k-1} f''(\xi) + \frac{\epsilon_{k-1}^2}{2} f'''(\xi)\right) - \left(f'(\xi) + \epsilon_k f''(\xi) + \frac{\epsilon_k^2}{2} f'''(\xi)\right)$$

$$= (\epsilon_{k-1} - \epsilon_k) f''(\xi) + \frac{1}{2}(\epsilon_{k-1}^2 - \epsilon_k^2) f'''(\xi) + \cdots$$

$$= (\epsilon_{k-1} - \epsilon_k) f''(\xi)\left[1 + \frac{1}{2}(\epsilon_{k-1} + \epsilon_k)\frac{f'''(\xi)}{f''(\xi)}\right] + \ldots (4)$$

Now dividing equation (3) and (4) we get,

$$x_{k+1} = \frac{[f(x_k) - x_k f'(x_k)] - [f(x_{k-1}) - x_{k-1} f'(x_{k-1})]}{f'(x_{k-1}) - f'(x_k)} = \frac{(\epsilon_{k-1} - \epsilon_k)\left[\frac{1}{2}(\epsilon_{k-1} + \epsilon_k) + \xi\right] f''(\xi) + \cdots}{(\epsilon_{k-1} - \epsilon_k) f''(\xi)\left[1 + \frac{1}{2}(\epsilon_{k-1} + \epsilon_k)\frac{f'''(\xi)}{f''(\xi)}\right] + \cdots}$$

Therefore,

$$\xi + \epsilon_{k+1} = \frac{\frac{1}{2}(\epsilon_{k-1} + \epsilon_k) + \xi}{\left[1 + \frac{1}{2}(\epsilon_{k-1} + \epsilon_k)\frac{f'''(\xi)}{f''(\xi)} + \cdots\right]}$$

$$\xi + \epsilon_{k+1} = \left[\frac{1}{2}(\epsilon_{k-1} + \epsilon_k) + \xi\right]\left[1 + \frac{1}{2}(\epsilon_{k-1} + \epsilon_k)\frac{f'''(\xi)}{f''(\xi)} + \cdots\right]^{-1}$$

$$\xi + \epsilon_{k+1} = \left[\frac{1}{2}(\epsilon_{k-1} + \epsilon_k) + \xi\right]\left[1 - \frac{1}{2}(\epsilon_k + \epsilon_{k-1})\frac{f'''(\xi)}{f''(\xi)} + \frac{1}{4}(\epsilon_{k-1} + \epsilon_k)^2\frac{f'''(\xi)}{f''(\xi)} + \cdots\right]$$

$$\xi + \epsilon_{k+1} = \frac{1}{2}(\epsilon_{k-1} + \epsilon_k) + \xi - \frac{1}{4}(\epsilon_k + \epsilon_{k-1})^2\frac{f'''(\xi)}{f''(\xi)} + \cdots$$

$$\epsilon_{k+1} = \frac{1}{2}(\epsilon_k + \epsilon_{k-1}) + \emptyset(\epsilon_k)$$

Discarding higher power terms of $\epsilon_k$

We get, $\epsilon_{k+1} = \frac{1}{2}(\epsilon_k + \epsilon_{k-1})$

$$\epsilon_{k+1} = \frac{1}{2}\epsilon_k\left(1 + \frac{\epsilon_{k-1}}{\epsilon_k}\right)$$

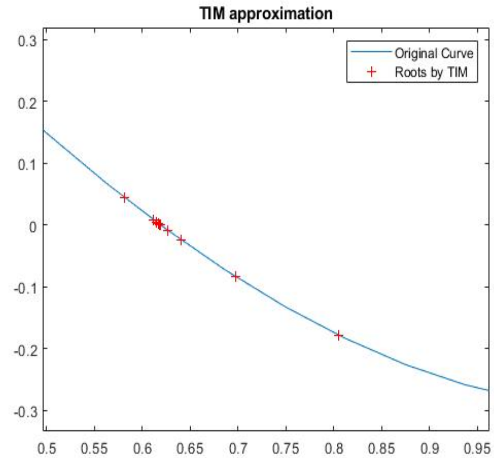
$\epsilon_{k+1} = A\epsilon_k$ , where $A = \frac{1}{2}\left(1 + \frac{\epsilon_{k-1}}{\epsilon_k}\right)$

This shows that order of the Tangent intersection method is 1. This method is of linear order.

## IV. EXAMPLES USING TIM

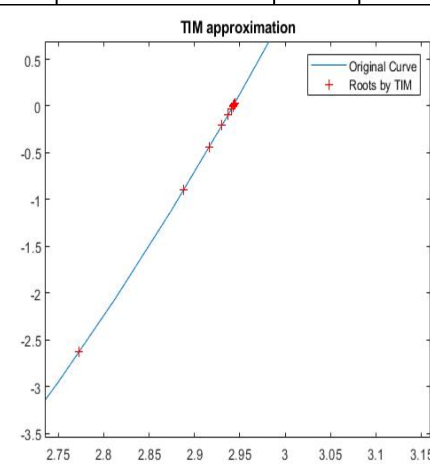
Example 1: Let $f(x) = e^x - 3x = 0$ be the function of x. The root $f(x) = 0$ lies in $(0,1)$ since $f(0) = 1 > 0$ and $f(1) = -0.28172 < 0$. The roots obtained by TIM are as given in table which are correct up to $10^{-4}$

| Sr. No. | TIM | Sr. No. | TIM |
|---|---|---|---|
| 1 | 0.581976707 | 10 | 0.61825639 |
| 2 | 0.805508075 | 11 | 0.618715864 |
| 3 | 0.697902784 | 12 | 0.618945549 |
| 4 | 0.641059399 | 13 | 0.619060377 |
| 5 | 0.611808933 | 14 | 0.619117789 |
| 6 | 0.626505464 | 15 | 0.619089083 |
| 7 | 0.619175198 | 16 | 0.61907473 |
| 8 | 0.615496587 | 17 | 0.619067554 |
| 9 | 0.61733702 | | |



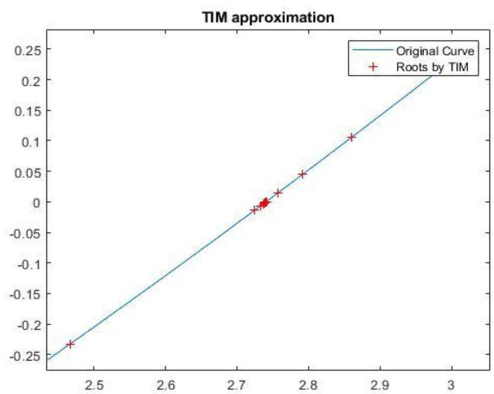

Example 2: Let $f(x) = x^3 - 9x + 1 = 0$ be the function of x. The root $f(x) = 0$ lies in $(2,3)$ since $f(2) = -9 < 0$ and $f(3) = 1 > 0$. The roots obtained by TIM are as given in table which are correct up to $10^{-4}$.

| Sr. No. | TIM | Sr. No. | TIM |
|---|---|---|---|
| 1 | 2.533333333 | 10 | 2.943529076 |
| 2 | 2.773226238 | 11 | 2.943091891 |
| 3 | 2.888097741 | 12 | 2.942873282 |
| 4 | 2.944403317 | 13 | 2.942763973 |
| 5 | 2.916341123 | 14 | 2.942818628 |
| 6 | 2.930394614 | 15 | 2.942845955 |
| 7 | 2.937404533 | 16 | 2.942832291 |
| 8 | 2.940905313 | 17 | 2.94282546 |
| 9 | 2.942654662 | | |



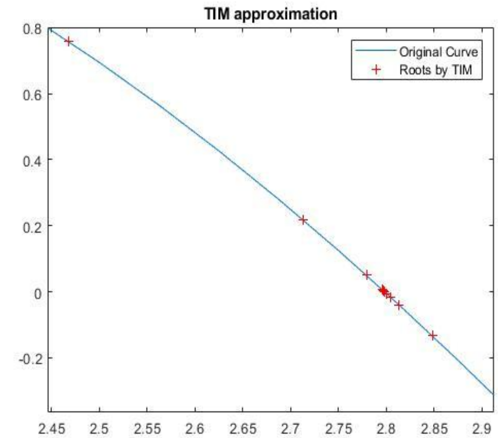
Example 3: Let $f(x) = xlog_{10}x - 1.2 = 0$ be the function of x. The root $f(x) = 0$ lies in (2,3) since $f(2) = -0.59 < 0$ and $f(3) = 0.23 > 0$. The roots obtained by TIM are as given in table which are correct up to $10^{-4}$.

| Sr. No. | TIM | Sr. No. | TIM |
|---|---|---|---|
| 1 | 2.466303462 | 10 | 2.740105094 |
| 2 | 2.724445065 | 11 | 2.740628126 |
| 3 | 2.860010455 | 12 | 2.740889667 |
| 4 | 2.791679188 | 13 | 2.740758895 |
| 5 | 2.757925539 | 14 | 2.74069351 |
| 6 | 2.741151225 | 15 | 2.740660818 |
| 7 | 2.732789634 | 16 | 2.740644472 |
| 8 | 2.736968301 | 17 | 2.740652645 |
| 9 | 2.73905923 | | |



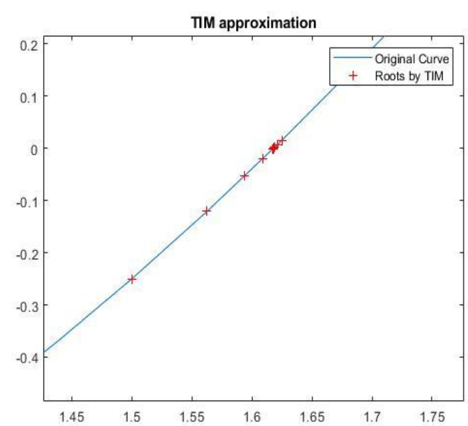

Example 4: Let $f(x) = xsinx + cosx = 0$ be the function of x. The root $f(x) = 0$ lies in (2,3) since $f(2) = 1.40 > 0$ and $f(3) = -0.567 < 0$. The roots obtained by TIM are as given in table which are correct up to $10^{-4}$.

| Sr. No. | TIM | Sr. No. | TIM |
|---|---|---|---|
| 1 | 2.468215642 | 10 | 2.797337732 |
| 2 | 2.713499201 | 11 | 2.797869453 |
| 3 | 2.848242576 | 12 | 2.798135387 |
| 4 | 2.779372784 | 13 | 2.798268372 |
| 5 | 2.813376565 | 14 | 2.798334869 |
| 6 | 2.796274877 | 15 | 2.798368119 |
| 7 | 2.80479984 | 16 | 2.798384744 |
| 8 | 2.800531008 | 17 | 2.798393057 |
| 9 | 2.79840137 | | |




Example 5: Let $f(x) = x^2 - x - 1 = 0$ be the function of x. The root $f(x) = 0$ lies in $(1, 2)$ since $f(1) = -1 < 0$ and $f(2) = 1 > 0$. The roots obtained by TIM are as given in table which are correct up to $10^{-4}$.

| Sr. No. | TIM | Sr. No. | TIM |
|---|---|---|---|
| 1 | 1.5 | 10 | 1.618164063 |
| 2 | 1.75 | 11 | 1.617675781 |
| 3 | 1.625 | 12 | 1.617919922 |
| 4 | 1.5625 | 13 | 1.618041992 |
| 5 | 1.59375 | 14 | 1.617980957 |
| 6 | 1.609375 | 15 | 1.618011475 |
| 7 | 1.6171875 | 16 | 1.618026733 |
| 8 | 1.62109375 | 17 | 1.618034363 |
| 9 | 1.619140625 | | |



For above Ex.5, the roots obtained by TIM and Bisection method are exactly same for all iterations as the function is quadratic polynomial. The table of iterations is given below. This is obvious from corollary.

| Sr. No. | TIM | Bisection Method |
|---|---|---|
| 1 | 1.5 | 1.5 |
| 2 | 1.75 | 1.75 |
| 3 | 1.625 | 1.625 |
| 4 | 1.5625 | 1.5625 |
| 5 | 1.59375 | 1.59375 |
| 6 | 1.609375 | 1.609375 |
| 7 | 1.6171875 | 1.6171875 |
| 8 | 1.62109375 | 1.62109375 |
| 9 | 1.619140625 | 1.619140625 |
| 10 | 1.618164063 | 1.618164063 |

Comparison of TIM and Bisection Method:
The iterations for above examples by TIM are compared with Bisection method (BM).

| Function | $f(x) = e^x - 3x$ | |
|---|---|---|
| Iteration | TIM | BM |
| 1 | 0.581977 | 0.5 |
| 2 | 0.805508 | 0.75 |
| 3 | 0.697903 | 0.625 |
| 4 | 0.641059 | 0.5625 |
| 5 | 0.611809 | 0.59375 |
| 6 | 0.626505 | 0.609375 |
| 7 | 0.619175 | 0.617188 |
| 8 | 0.615497 | 0.621094 |
| 9 | 0.617337 | 0.619141 |
| 10 | 0.618256 | 0.618164 |

Table for Ex. 1

| Function | $f(x) = x^3 - 9x + 1$ | |
|---|---|---|
| Iteration | TIM | BM |
| 1 | 2.533333 | 2.5 |
| 2 | 2.773226 | 2.75 |
| 3 | 2.888098 | 2.875 |
| 4 | 2.944403 | 2.9375 |
| 5 | 2.916341 | 2.96875 |
| 6 | 2.930395 | 2.953125 |
| 7 | 2.937405 | 2.945313 |
| 8 | 2.940905 | 2.941406 |
| 9 | 2.942655 | 2.943359 |
| 10 | 2.943529 | 2.942383 |

Table for Ex. 2

| Function | $f(x) = xlog_{10}x - 1.2$ | |
|---|---|---|
| Iteration | TIM | BM |
| 1 | 2.466303 | 2.5 |
| 2 | 2.724445 | 2.75 |
| 3 | 2.86001 | 2.625 |
| 4 | 2.791679 | 2.6875 |
| 5 | 2.757926 | 2.71875 |
| 6 | 2.741151 | 2.734375 |
| 7 | 2.73279 | 2.742188 |
| 8 | 2.736968 | 2.738281 |
| 9 | 2.739059 | 2.740234 |
| 10 | 2.740105 | 2.741211 |

Table for Ex. 3

| Function | $f(x) = xsinx + cosx$ | |
|---|---|---|
| Iteration | TIM | BM |
| 1 | 2.468216 | 2.5 |
| 2 | 2.713499 | 2.75 |
| 3 | 2.848243 | 2.875 |
| 4 | 2.779373 | 2.8125 |
| 5 | 2.813377 | 2.78125 |
| 6 | 2.796275 | 2.796875 |
| 7 | 2.8048 | 2.804688 |
| 8 | 2.800531 | 2.800781 |
| 9 | 2.798401 | 2.798828 |
| 10 | 2.797338 | 2.797852 |

Table for Ex. 4

## IV. CONCLUSION

The proposed numerical method is implemented to find the roots of the algebraic and transcendental equations. The order of the proposed method is 1. The bisection method is the special case of this method if the function is quadratic equation as proven in corollary.

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REFERENCES


[1] K. Sikorski, Bisection is optimal, Numerische Mathematik , 1982, Vol. 40, pp. 11-117.

[2] Michael N. Vrahatis, Generalizations of the intermediate value theorem for approximating fixed points and zeros of continuous functions, Numerical computations: Theory and Algorithms, NUMTA 2019. Lecture Notes in Computer Science Vol. 11974 Springer, Cham.

[3] James F. Epperson, An Introduction to Numerical Methods and Analysis, 2nd Edition, Wiley and Sons, 2021 Ch.3, pp.55-99.

[4] Walter Rudin, Principal of Mathematical Analysis, 3rd Edition, McGraw-Hill, 1976, Ch.3, pp-47-78.

[5] M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods, 8th Edition, New Age International Publication, 2022, Ch.1, pp-1-13.