Formulating The Matrix: A Foundational Introduction

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Abstract: Review of Matrix Theory – We present the definitions of matrices and introduce some common types.

Keywords: ***Matrix Basics, Matrix Order, Transpose, Matrix Addition, Matrix Subtraction, Scalar Multiplication, Matrix Multiplication, Laws of Matrix Operations, Determinant, Inverse, Minor & Cofactor, Adjoint & Inverse Formula***

1. INTRODUCTION

A matrix is a arrangement of elements organized in a rectangular form into a number of rows and columns enclosed in a square brackets.

2. ORDER OF A MATRIX

It is defined in very simple way as matrix is considered to have 'm' number of rows and 'n' number of columns, then it can be written as m × n.

Example $A=\begin{bmatrix}3 & 2\\1 & 2\end{bmatrix}$ $B=\begin{bmatrix}2 & 3 & 2\\2 & 1 & 1\\1 & 2 & 3\end{bmatrix}$

Transpose of a matrix: The matrix G is given as $(g_{ij})m\times n$ then its transpose is shown by g^T that is, $G^T=(g_{ij})n\times m$.

Example: $G = \begin{bmatrix}1 & 4 & 1\\2 & 1 & 2\\2 & 3 & 1\end{bmatrix}$ $G^T=\begin{bmatrix}1 & 2 & 2\\4 & 1 & 3\\1 & 2 & 1\end{bmatrix}$

3. OPERATION WITH MATRICES

Addition: If A and B have the same dimension, the sum G + H can be calculated by adding the respective entries. $(G+H)_{ij} = g_{ij} + h_{ij}$

$G=\begin{bmatrix}c & d\\e & f\end{bmatrix}$ $H=\begin{bmatrix}p & q\\r & s\end{bmatrix}$ $G+H= \begin{bmatrix}c+p & d+q\\e+r & f+s\end{bmatrix}$

Example: $A=\begin{bmatrix}1 & 2\\3 & 1\end{bmatrix}$ $B=\begin{bmatrix}2 & 1\\2 & 3\end{bmatrix}$ A+B= $\begin{bmatrix}1+2 & 2+1\\3+2 & 1+3\end{bmatrix}$ A+B= $\begin{bmatrix}3 & 3\\5 & 4\end{bmatrix}$

Subtraction: If A and B have the same dimensions, the difference between them G - H is calculated by subtracting the corresponding entries. $(G-H)_{ij} = g_{ij} - h_{ij}$

$A=\begin{bmatrix}c & d\\e & f\end{bmatrix}$ $B=\begin{bmatrix}p & q\\r & s\end{bmatrix}$ A-B= $\begin{bmatrix}c-p & d-q\\e-r & f-s\end{bmatrix}$

Example: $A=\begin{bmatrix}1 & 10 & 9\\7 & 8 & 9\\5 & 4 & 2\end{bmatrix}$ $B=\begin{bmatrix}1 & 8 & 7\\7 & 5 & 4\\5 & 3 & 1\end{bmatrix}$ A-B= $\begin{bmatrix}1-1 & 10-8 & 9-7\\7-7 & 8-5 & 9-4\\5-5 & 4-3 & 2-1\end{bmatrix}$ A-B= $=\begin{bmatrix}0 & 2 & 2\\0 & 3 & 5\\0 & 1 & 1\end{bmatrix}$

4. SCALAR MULTIPLICATION

Consider $F = (f_{ij})_{m\times n}$ as a matrix and 'r' as a scalar used for multiplication to a matrix F, denoted as $F = r[f_{ij}]m_{\times}n$.

$A=\begin{bmatrix}c & d\\e & f\end{bmatrix}$ $pA=\begin{bmatrix}pc & pd\\pe & pf\end{bmatrix}$

Example: p=4 $A=\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}$ $pA=\begin{bmatrix}8 & 12\\4 & 8\end{bmatrix}$

Multiplication:

If A be the m×n and B be the n×p then the product AB is defined m×p. The entry $(AB)_{ij}$is obtained by multiplying corresponding entries together and then adding the result

$A_{i1}=(a_{i1}\ a_{i2}\ \ldots\ldots\ldots\ldots\ a_{im})$

$B2j =(\ b_{1j}\ b_{2j}\ \ldots\ldots\ b_{mj})$

$(AB) = a_{i1}b_{1j}+a_{i2}b_{2j}+\ldots+a_{im}b_{mj}$

Example: $A=\begin{bmatrix}1 & 3\\2 & 2\end{bmatrix}$ $B=\begin{bmatrix}2 & 2\\3 & 1\end{bmatrix}$

$A\times B=\begin{bmatrix}1*2+3*3 & 1*2+3*1\\2*2+2*3 & 2*2+2*1\end{bmatrix}$

A×B = $\begin{bmatrix} 11 & 5 \\ 10 & 6 \end{bmatrix}$

5. LAWS OF MATRIX

The matrix addition, subtraction and matrix multiplication have the properties.

Commutative Law for Addition:
P + Q = Q + P (Addition)

PQ ≠QP (Multiplication)

Associative Laws:
P + (Q+ R) = (P+ Q) + R (Addition)

(PQ) R = P(QR) (Multiplication)

Distributive Laws:
P(Q + R) = PQ+ PR

6. DETERMINANT OF MATRIX

The element is a square matrix and the determinant is a square matrix of numbers. The number of rows in the determinant is always equal to the number of columns.

A= $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ IAI= $\begin{vmatrix} p & q \\ r & s \end{vmatrix}$

IAI = (p×s)- (q×r)

Example: A= $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$ IAI= $\begin{vmatrix} 3 & 7 \\ 2 & 4 \end{vmatrix}$ IAI =(3×4)-(7×2) IAI =(12-14) IAI=2

7. INVERSE OF A MATRIX

A unique matrix matching the connection is an inverse matrix A^{-1} that can only be found given a square and a non-singular matrix A.

$AA^{-1} = A^{-1}A = I$

$A^{-1} = \frac{1}{IAI}$ (adj A)

NOTE: If IAI= 0 then Matrix A is Singular Matrix.
If IAI ≠0 then Matrix A is non-singular Matrix.

A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ IAI= ad-cb

Minor of $(A)_{ij}$= M_{11} is d
M_{12} is c
M_{21} is b
M_{22} is a

C.F. $(a_{ij}) = (-1)^{i+j} M(a_{ij})$

C.F. = Minor; if i+j=Even number

C.F. = -Minor; if i+j=Odd number

I = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

Example: A= $\begin{bmatrix} 8 & 1 \\ 2 & 4 \end{bmatrix}$

IAI = $\begin{vmatrix} 8 & 1 \\ 2 & 4 \end{vmatrix}$ IAI =32-2 IAI =30

Minor of $(A)_{ij}$= 8_{11} is 4
1_{12} is 2
2_{21} is 1
4_{22} is 8

C.F. $(8_{11}) = (-1)^{1+1}$ (4) = 4

C.F. $(1_{12}) = (-1)^{1+2}$ (2) = -2

C.F. $(2_{21}) = (-1)^{2+1}$ (1) = -1

C.F. $(4_{22}) = (-1)^{1+1}$ (8) = 8

C.F. = $\begin{bmatrix} 4 & -2 \\ -1 & 8 \end{bmatrix}$

$[C.F.]^T$ = Adj A= $\begin{bmatrix} 4 & -1 \\ -2 & 8 \end{bmatrix}$

$A^{-1} = \frac{1}{IAI}$ adj A

$A^{-1} = \frac{1}{30} \begin{bmatrix} -4 & -1 \\ -2 & 8 \end{bmatrix}$

8. CONCLUSION

In this foundational overview of matrix theory, we have introduced the essential concepts and operations involving matrices. Beginning with the basic definition and structure of a matrix, we explored how matrices are classified by their order (rows × columns) and how operations like transpose, addition, subtraction, scalar multiplication and matrix multiplication are performed.

We also covered the properties and laws that matrices obey, such as the commutative, associative, and distributive laws in the context of addition and multiplication. Moreover, we delved into the computation of the determinant of square matrices, which plays a critical role in determining whether a matrix is singular or non-singular.

REFRENCE

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