A Systematic Review on Analysing and Improving the Performance of Energy Management Systems in Electric Vehicle

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***Abstract*- To gain a maximum amount of efficiency, performance, and sustainability of electric vehicles (EVs), including battery electric vehicles (BEVs), hybrid electric vehicles (HEVs), and plug-in hybrids (PHEVs), energy management systems (EMS) play a vital role. New studies have reported the various EMS strategies, including global optimization, fuzzy rule-based, real-time, and intelligent data-driven that have unique benefits and weaknesses according to the kind of vehicle and driving environment. Hybrid energy battery packs (which are battery and super capacitor) have been reported to enhance power density and battery life managed by powerful control technology like Pontryagin minimum principle. Smart EMS using artificial intelligence and machine learning are being developed as highly effective methods of adapting to the real life driving environment and even smart grids, but there are issues of data security and privacy that need to be addressed. The EMS combined with smart infrastructure and the coordination of the systems in home and in the parking lot can be both technologically and economically positive. The main issues are the complexities of controlling in real-time, keeping in consideration of the effects of driver states and the traffic as well as the prevention of battery wear out.**

***Index Terms*- Electrical vehicles, Smart EMS, Hybrid energy systems, smart grids.**

I. INTRODUCTION

Sequential or parallel with more working conditions that will allow the whole load to be multiplied considerably in the most complicated driving situations. Although the series route is avoided as less efficient, the main feature of this construction is the decoupling of the engine, generator and motor speeds that offers more flexibilities to the control. The power split is connected by the engine and the two electric machines to a power split device which can be either a planetary gear set that allows the engine power and electric machine power to be added both mechanically and electrically and therefore allows series or parallel operations to occur [8]. Compared to the parallel hybrid powertrain, the power-split architecture is the most flexible and its control is more effective to adjust the engine operating conditions, as it uses the concept of doubled energy conversion typical of a series operation, but little part of the total power is generated in this way only during a small part of the overall power demand therefore overall losses are less [2,8]. Figure 3 shows the power-split topology. It is also possible to combine the engine and the electric machines, both mechanically and electrically both in series and in parallel [8]. The power-split architecture is the most versatile compared to the parallel hybrid powertrain, which offers more control of the engine operating conditions and also employs the use of double energy conversion, which is common in the series operation, in a small part only. It is well-known that three main topologies are used mainly when it comes to Hybrid Electric Vehicles: their serial, parallel, and power-split design. The series hybrid powertrain is basically a direct extension of battery powered electric vehicle as it is powered only by a motor. Under this arrangement the engine will drive a generator that creates electrical energy. This energy can be added to the energy available in the energy storage system and this can be transmitted using an electric bus to the wheels via an electric motor (or motors) [8]. The primary attractive feature the series hybrid powertrain is that it needs only electrical connections among major power conversion units, which makes vehicle planning and packaging easy. Also, because the engine does not have a direct trail with the wheels, it will have a large freedom in the choice of speed and load which allows the engine to operate on high-efficiency range. Nevertheless, there are two energy conversions in the series hybrid powertrain, generator (mechanical to electrical) and motor (electrical to mechanical), that causes loss of efficiency, despite the fact that the engine could be directly mechanically connected with the wheels under the existing configuration. Therefore, a hybrid electric vehicle (series iteration) can consume more fuel than an ordinary vehicle, especially on a high way. Further, one of the two electromechanical energy converters shall be suited to the most significant demand of power of the vehicle because it is the main source of propulsion [2,8]. Figure 1 is the topology of the series.…Issues with energy shortages and air pollution have prompted researchers and auto manufacturing industries to be innovative in the new-energy vehicle segment, by creating battery electric vehicles, hybrid electric vehicles, and fuel cell electric vehicles [1]. Most people tend to agree that the use of plug-in hybrid electric cars (PHEVs) has a potential due to the generous energy consumption and gas savings, which goes hand in hand with it. This would be attained through a number of important technologies, in particular, through the way of optimizing design of components and management of energy [2]. The use of rule-based controls is quite satisfactory in modern cars since they are cheaper in processing time and memory [3]. These regulations are however based on some considerable amount of experience and calibration of the engineers, hence they are not as flexible to various driving environments and vehicle configurations as the data-driven approaches. Hence, optimization-based techniques are deemed as substitute ways to produce improved strategies of control. The techniques used in these approaches are called the global optimization techniques like the genetic algorithms (GA) [4], dynamic programming (DP), Pontryagin minimum principle [5], convex techniques and so on. Programming [6], game theory, simulated annealing, and particle swarm optimization [7], instantaneous optimization methods namely, sliding mode control [8], equivalent consumption minimization strategy [9], fuzzy logic control [10], neural network control [11], model predictive control [12], stochastic DP [13], and extremum seeking control [14]. Yet, quite a number of such optimal controls are usually not robust to uncertain driver intention and intricate driving behaviors. Driving events are generally characterized actions carried out in driving like steering, acceleration, deceleration and overtaking [15]. The driving conditions are interpreted to mean the speed profile and other facts implemented by the analysis of the speed, like maximum acceleration, average speed, power demand and distance travelled [16]. Driving conditions is a major factor influencing the energy management strategy of the PHEVs and could be categorized and interpreted with historical driving data. As a result, numerous approaches have been proposed to identify and detect driving conditions and drivers, and some of these are the Bayes classifier [17], Gaussian mixture model [18], and the K-means algorithm [19]. Recent researchers have pointed at a combination of multiple algorithms with complementary capabilities as a useful strategy to improve the performance of energy management strategies, and such augmenting methods tend to rely on machine learning solutions to enhance the accuracy of the classification and prediction of driving conditions.

II. LITERATURE REVIEW

These hybrid algorithms have the capability of adjusting to different driving styles and environmental conditions because they increase the correlations of diverse sensors and vehicle systems in real time. This real-time adapted comprehension of driving situations permits energy management in PHEVs to be conducted in a more efficient way, which might be translated into an increase in the fuel economy and lower emissions. Hybrid Electric Vehicles (HEVs) include multiple sources of energy and power converters, which are usually combined with an internal combustion engine (ICE) and an electric motor. These cars are already the most cost-efficient one and are destined to stay the same in the nearest future. The main idea behind the development of HEVs is to reduce the fuel usage and emissions, and satisfy the power demands of the drivers by investigating appropriate energy management practices (EMSs). Energy management aims at paving way toward an optimum balance of power in complicated driving scenarios, whereby the aim is to reduce fuel consumption and emissions. It is an open fact that improvement in fuel efficiency of HEVs and the subsequent reduction of their emissions largely depend on their energy management plans (EMSs) [1]. Complex structure and functionality of multi-source hybrid energy systems pose threat to the efficacy of EMSs. Irrespective of the powertrain architecture, the EMS goal is to coordinate power flows of energy converters in real-time such as to achieve control objectives [2]. Such control algorithms which lead to the most optimal way of controlling a given driving cycle are therefore the primary research activity in the energy management strategies. A great number of Energy Management Systems (EMSs) in Hybrid Electric Vehicles (HEVs) have been considered in the last few years. The literature reviews have identified the different categorizations of the energy management strategies [37]. There are generally three types of EMSs namely, rule-based EMSs, local optimization-based EMSs, and global optimization-based EMSs [3]. A presentation of EMSs of plug-in Hybrid Electric Vehicle is given in [4]. The taxonomy of energy management strategies is given in terms of rule and optimization-based control strategies according to their mathematical modeling and common methodology. In [5], EMSs are classified in rule-based and optimization-based systems to parallel HEVs, including the comparison between the pros and cons of each solution. Furthermore, a few real-time implementation issues are addressed on various grounds, critical area being the level of computations and optimum condition. In [6], the various categories of hybrid cars that emphasize on hydraulic drives are presented and elaborated upon. Several methods are identified and compared as offline strategies, online strategies, etc. The new EMSs have been evolved along the development of the intelligent transportation system (ITS) and the popularity of the machine learning techniques to respond to performances needs, including flexibility and time-sensitiveness. Nevertheless, the detailed survey of EMSs remains one of the areas where a deeper understanding of the state-of-the-art and possible further investigation should be carried out. In response to this, the current review unlike other EMS review papers has come up with a description of a detailed hierarchical classification scheme represented as the first such classification scheme. Under the first class, there exists offline EMSs with respect to driving level under the global optimization-based and rule-based global optimization-based and rule-based EMSs. Under this second category, the online EMSs are classified into instantaneous optimization-based, predictive, and learning-based EMSs. Due to the fact that the targeted solution goals are manifold, as well as being focused on the designed scheme, the area of the examined problem includes a wide range of literature studies, reviewed in detail. The remaining paper will be organized as follows. In section 2, a succinct discussion and comparison of different Hybrid Electric Vehicle powertrain is provided. In section 3, the authors present a tree of classification of EMSs. In section 4 and 5, it explores the different types of EMS which are offline EMS and online EMS and explains in details what each one is and how each is similar to the other using the principles and the advantages and the disadvantages. Having discussed only some of the current trends in EMSs, in Section 6 I will touch on some of the future trends.

III. RECENT ADVANCEMENT OF ELECTRIC VEHICLES

As it is well understood, Hybrid Electric Vehicles mostly apply three classes of topology namely series, parallel and power-split. The series hybrid drive train is a simple extension of a battery powered electric vehicle with the power being propelled by a motor alone. In such an arrangement, an engine drives a generator that produces electrical energy.

A. POWERTRAIN TOPOLOGIES OF HYBRID ELECTRIC CARS

This power may be used along with the power of the energy storage system and then fed, using an electric bus, into the electric motor(s) turning the wheels [8]. The primary advantage with the series hybrid powertrain is that it entails only electrical interconnections between the main power conversion elements thus making the design and packaging of the vehicle simple. Also, the engine is not coupled directly to the wheels, so there is a lot of flexibility in the speed and load as well as the engine is able to operate in a high-efficiency range. Nevertheless, series hybrid powertrain is associated with two kinds of energy conversion, between mechanical and electrical in the generator and electrical and mechanical in the motor, which cause the loss of efficiency, although the direct mechanical connection is maintained between the engine and the wheels in the present configuration. This means that a series hybrid electric powered vehicle in some specific conditions can consume more fuel than a conventional vehicle especially where there is only a short distance to be covered. highway driving. In addition, one of the two electromechanical energy converters has to be capable of carrying the maximum power demand of the vehicle since it is the main source of propulsion [2,8].

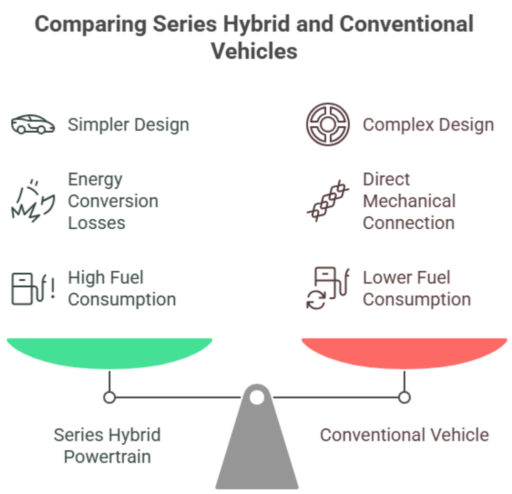

Fig.1 The Topology of EV series

Under a parallel topology, a mechanical coupling device connects the engine and power train whereas driving the vehicle is the role of the motor. The vehicle may be powered by either the engine or the motor depending on the load conditions which could greatly improve fuel efficiency. Motor kicks in at lower speeds (which requires less fuel use) and hence, this arrangement is more fuel-efficient (higher efficiency) and fuel-efficient. The power combination is mechanical not electrical and the engine and one or more electric machine apply torque to a gear set, chain or belt, so combining and transmitting the combined torque to the wheels [8]. With this configuration, no need exists to size either of the two electromechanical energy converters to meet the most power demanded by a parallel hybrid powertrain. The electric motors however are less powerful than in a series hybrid power train, unless greatly oversized, because not all mechanical power must go through them and thus limit the possibilities of regenerative braking. Also, the operating conditions of the engine cannot be set as arbitrarily as possible in a series hybrid powertrain because the speed of the engine is physically coupled with the speed of the vehicle via the transmission system [2,8].

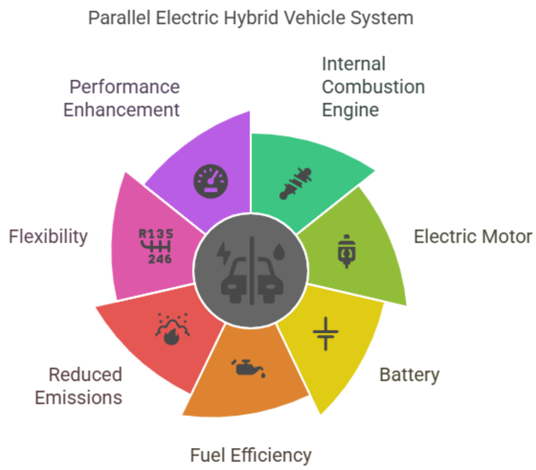

Fig.2 The Topology of parallel series EV

it is in either series or parallel, and it provides more operational modes to provide a very high efficiency overall in complex driving conditions. Although the series route is normally avoided, as it happens to be of less efficiency, the most important feature of this design is the non-coupling of the engine speed, the generator speed, and the motor speed, which allows so much more flexibility of control. Two electric machines and the engine are connected into a power split device, usually a planetary gear set, that allows combining the power of the engine and electric machines both mechanically and electrically, thereby allowing series and parallel operation [8]. Compared to the parallel hybrid powertrain, the power-split is the most adjustable and bears the best control over engine operating conditions yet performs double energy conversion a feature of series operation just a fraction of the total power demand hence minimizing the total losses [2,8].

B.EMSS CLASSIFICATION

During this research, we propose a new hierarchical classification of Energy Management Systems (EMSs) that can be applied to any kind of Hybrid Electric Vehicles. This framework will be organized into two main groups:

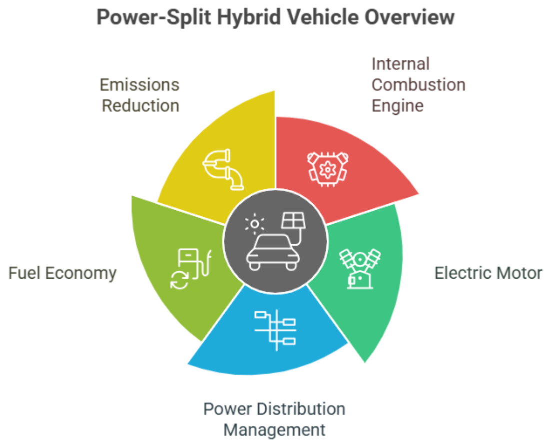

Fig.3 The Split Power connection

(1) offline EMSs: put in order according to the level of information about driving conditions they use: global optimization-based and rule-based; and (2) online EMSs: instantaneous optimization-based, predictive and learning-based EMSs. The EMS classification is graphically represented in figure 4. Of importance to mention is that a versatile EMS can employ a combination of mixed techniques (offline, and online) in order to develop a wholesome EMS to achieve optimum fuel efficiency and performance. This paper has therefore taken these combinations alongside other techniques when introducing a certain classification of EMS. In the case of the offline EMSs, two approximations are distinguished: global optimization based and rule based EMSs. The major purpose of the global optimization-based EMSs is to optimize the power distribution of a specific driving cycle and to provide the modified online EMSs. They are not appropriate to real-time control because they require too much computation and need prior knowledge of the complete driving cycle. They may however be used as a reference to change control parameters. Global optimization using known reactive methods, such as dynamic programming can be achieved using methods to optimize over defined driving cycles, but these methods cannot easily be applied to real vehicles. This method can therefore be applied to evaluate other optimization strategies so as to obtain control

regulations. Energy Management Systems (EMSs) developed using rules are said to be offline methods since the rules are worked out using pre-production testing. All these systems use preset rules of control to arrive at a power distribution, and they fail to realize the optimal power distribution that offline globally-optimized energy management does. On the contrary, online EMSs are local, causal and appropriate in real-time control. One of them is Instantaneous optimization EMSs has the goal to optimize fuel consumption, moment to moment, without prior information on the complete driving path and can only reduce results to a local optimum. Instantaneous optimization EMSs). These are (1) the equivalent consumption minimization strategy (ECMS); (2) adaptive-ECMS (A-ECMS); (3) robust. control (RC). One of the basic approaches, ECMS appears to be appropriate to be implemented in real-time owing to the flexibility it is associated with, which is referred to as the equivalent factor (EF). As it has been identified, ECMS performance is highly correlated to the corresponding factor that is why the choice of appropriate factor is one of the primary questions of concern of ECMS. That is why different techniques were introduced to modify the equivalent factor online and grant power according to ECMS, e.g. A-ECMS. Then the talk moves to predictive EMSs, in which the general idea is optimal power distribution depending on planned velocity across a particular horizon. Future power requirement within this horizon is estimated by the help of traffic data received with the help of ITS and GPS. With the current trend of applications of intelligent transportation system (ITS) technology application in traffic control, information about the roads ahead on communication links is very useful in establishing predictive controller which works as a means to optimally allocate power so as to maximize fuel consumption within a specific period of time. Therefore, the driving cycle prediction is important to predictive EMSs. Prediction Model predictive control (MPC) is a widely used solution technique and is based on the vehicle model prediction and can be used in the predictive HEVs control. EMSs EMSs based on learning performance their learning on training data to update control parameters that allow adapting to dynamic driving conditions in a first order of magnitude. The two can combine engine and electric machines in the mechanical as well as electrical circuits meaning they can operate series and parallel [8]. When compared to the parallel hybrid powertrain, the power-split architecture proves to be the most versatile and offering more control over the operating condition of engines and relies on the use of the double energy conversion, as is typical of series connections only in a limited fraction. required to calibrate and formulate. Of these, DP needs the most prior information about the future of the driving cycle and the fuel economy is best with this

IV. POWE MANAGEMENT STRATEGY

Dynamic programming the offline optimization method can produce a globally optimal solution to a particular driving cycle.

A.DYNAMIC PROGRAMMING (DP)

Nevertheless, it cannot directly be implemented into a real vehicle energy management system (EMS) since it is impossible to predict the subsequent driving conditions as far as the speed rate or the road slope and traffic flow are concerned. Also, DP demands much computational time to solve the optimal problem moving backward to the state in the future to come out with the initial control input in a feasible set. Computational load rises with number of states of the system. Still, being used as a benchmark it allows finding operating conditions that lead to best fuel consumption which can be in turn used to evaluate the performance of other energy management algorithms and draw up heuristic rules. Moreover, it can also be used to get an optimal solution under a prediction horizon in the case of model predictive control, as seen in [9]. The basic idea of DP is shown in Figure 5. The minimization of the process depends on the determination of the optimal cost function-A to F. The first thing to do is to break down the feasible region into a grid to calculate all possible routes between A and F. Once the optimal path was stable, at time t, from F to E, then, moving backward starting at F, the optimal path is also computed with time.

B. STOCHASTIC DYNAMIC PROGRAMMING (SDP)

Although dynamic programming (DP) can be said as an excellent approach of realizing a world optimum solution, it is not possible to know in advance the whole driving conditions like speed, slope of the road etc. In an attempt to overcome this, stochastic dynamic programming has been presented by the researchers. Power of a Markov chain is the fundamental idea of stochastic dynamic programming built of the assumption that future power requirements of drivers can be calculated through the formation of a state transition matrix map with the help of determining the sequential value based on the power of a Markov chain. The ask of this power demand sequence comes through the discretization of historical driving data at a certain time steps and the requires power demand is calculated depending with the speed of the vehicle. It is the Maximum likelihood estimation process that is used to estimate the probability of the systems state transition that is not only in the current state but also in the following time to distribute total power using discrete dynamic programming. This method has been successfully applied to the construction of the quasi-optimal policy that can be envisioned with online implementation and the availability of the same policy at the real-time environment since it merely needs historical driving data with no previous experience of the same.

Table 1: Recent developments of electric vehicles systems prior to previous reference

Approaches	Main Disadvantages	Main Advantages	References
DP	Less adaptability to changeable driving cycles highest computational complexity (3-level) prior knowledge of entire driving cycle	Achieves global optimal results Benchmark for other EMSs	[9–22]
SDP	Highest computational complexity(3-level) Requires driving cycle database	More adaptability Achieves near-optimal fuel economy	[23–28]
GA	Higher computational complexity (2-level) Less adaptability	Global optimality Good global Search performance	[29–33]
GT	Highest computational complexity(3-level) Poor adaptability	Trade of among conflicting objectives consider driver behaviours in EMSs	[34–45]
Pseudospectral method	Higher computational complexity(2-level) Requires analytic expressions for vehicle models	Global optimality More accurate numerical computation	[46–49]
Convex optimization	Requires convex models Limited applications	Fast computation Easy to implement	[50–54]
PMP	Complex mathematical models Require co-state estimate	Achieve near-optimal results Lower computational burden	[55–68]

information on the driving cycle. Nevertheless, there are inconsistencies between the power requirement in the Markov chain model and the real driver power requirement which have limited flexibilities to different driving cycle since real driving cycles are complex and at unpredictable. Also, the logistic calculation to address the stochastic dynamic programming is time consuming due to policy iterations. Future discounted costs are chosen on grounds of mathematical convenience and this makes them difficult to validate in the engineering problem statement. In [23], a stochastic dynamic programming of the power-split hybrid vehicle is suggested. The steps of this process include assigning a series of power demand of drives in different driving cycle utilising the Markov process to form a state transfer matrix of the power requirements of the driver. The goal of optimization seeks to optimize fuel and electricity efficiency as the objective function, a constraint domain, taps on the engine and motor torque, the charging and discharging power of the battery. Energy pricing is included in the objective function. The results of simulation are analysed and compared with the charge-depleting and charge sustaining (CD-CS) strategy in terms of fuel usage, engine control mechanics, engine start-stop controlling and the energy costing. The major aim of researchers is to minimize the fuel consumption through SDP. To encounter the issues of drivability, Opila et al. [24] have devised stochastic dynamic programming method to optimize fuel economy against the driveability, including start-stop and shift timing of the engine. Driver power demand is studied in the Markov process to model the driving cycle as a stochastic. Simulation Finding The proposed EMSs in FTP and NEDC produce 11% of the reduction of fuel consumption. enhancement in fuel economies. Influence of engine start-stop and gear shifting and timing on fuel efficiency is also provided and compared with default EMSs. In [25], an optimal energy management strategy of a series hybrid electric vehicle is offered on the basis of SDP with the respect to fuel consumption and emissions. The fact is that the computational complexity of SDP is difficult because of the large state space in this problem. Thus, new neurodynamic programming (NDP) solution is offered to fix this problem. Last but not the least, an SDP controller and an NDP controller are compared to a baseline controller and well, both SDP and NDP are capable of dramatically improving fuel economy over rule-based EMSs.

C. GENETIC ALGORITHM (GA)

Genetic algorithm (GA) represents a significant method in evolutionary computing, which has become well known within the modern family of optimization algorithms because of its ability of a global searching and reasonably high level of simplicity [29]. Being a random search based technique, the genetic algorithm follows the global search approach to solve an optimality problem where inspiration is provided by the biological evolution principles. Such properties render it especially appropriate to streamline rules, parameters, or measure of performance in EMS to improve performance [29]. Optimization represents the imitation of the biological processes, including genetic variation. GA can be used to generate globally optimal results in EMSs but it is very computationally demanding, particularly when high number of variables are involved as a result of successive searches, and it can be said that this method of optimization is offline. The technique assists a researcher to choose the best parameters, in the case of the HEVs, the size of the engine and battery. Zhou et al. [30] have applied GA to establish the favourable parameters and have studied the case of energy management of fuel cell Hybrid Electric Vehicles. Figure 6 shows the fundamental overview of genetic algorithm. Main action plans to do it are the following: GA encompass: (1) Initial population: select an initial population in a feasible optimal domain. (2) Genetic operation: To arrive at the global optimal solution create a new population by selecting the initial population, crossover and changing it to arrive at the global optimum solution. This is (3) to know whether the population meets the termination conditions that depend on the iterations of the intelligent optimal algorithm. Piccolo et al. [31] suggested an energy management approach to realize a global optimization approach which makes use of the genetic algorithm to optimize control parameters to reduce fuel consumption and emissions. It has the capability of calculating the global optimal solution and has a better robustness but is computationally expensive as compared to other EMSs. As one of the approaches to implementing the genetic algorithm, Liu et al. [32] proposed a hybrid genetic algorithm in the case of a series hybrid electric vehicle which promises a quicker converge and more adaptability than the classic GA that performs a random global search. The given algorithm has the ability of converging to a global solution quickly by making use of the quadratic programming algorithm. Besides, to address the challenge of energy management optimization, GA is used in combination with other algorithms. In [33], a genetic algorithm optimization-based fuzzy energy management strategy is presented. The genetic algorithm is used to optimize a membership function of fuzzy logic controller. Results of simulation show that both proposed EMSs have a strong positive effect on fuel consumption as well as a decrease in gas emissions as compared to deterministic rules that are not using GA modifications.

process

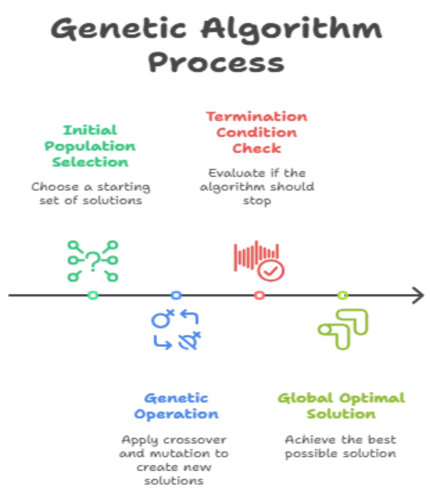

Fig 4 Genetic algorithms

D. GAMES THEORY (GT)

The game theory is an area of operational research that is commonly used to many multi-subject optimization problems by using some predicted and observed behaviours of those involved in a game. With the 1950s, the cooperative game theory reached its peak and gave rise to non-cooperative game theory [34]. John F. Nash, later a famous figure of this period, wrote his first known use of the language of mathematics precisely in his 1950 and 1951 essays [35] [36] and then made simpler terms in defining Nash Equilibrium which was a turning point in the history of game theory. The fundamental principle of the game theory is the application of a formal reasoning to calculating the strategies, which the players ought to follow in the attempt to rationally pursue their interests and the consequences, which would follow in such cases [37]. In the recent past Energy Management Systems (EMSs) based on game theory have been formulated, which respond to variation in vehicle parameters. Gielniak et al. [38] proposed system-based integration of game theoretical techniques on automotive electrical power and energy management. The performers desire to earn the greatest payoff which lies in the performance of their vehicle as well as the efficiency of the power train. The Yin et al. [39] were dedicated to the analysis of the energy management problem as a non-cooperative current control game, and the analytical derivation of Nash equilibrium is a balanced between patterns of varying preferences of independent devices. Dexireit et al. [40,41] developed a controller of parallel hybrid electric vehicle based on game theory, and their aim was on fuel economy and emission. In the first stage, vehicle operating conditions and the powertrain are considered to be two non-cooperative players (finite-horizon game). The metric called the cost function of this game is established by weighting the fuel usage, the NOx emissions, the difference between the state of charge (SOC) and setpoint values in the battery, and the difference between the vehicle operating conditions and their set value. Wheel speed, torque and battery SOC determine the policy to allow the control strategy of the engine, motors and battery to be selected. Unlike traditional This control policy does not rely on time or driving cycles as compared to energy management systems (EMSs), therefore it is better in a variety of driving situations. The results of the tests prove that the game theory controller is much better than the baseline controller with reference to the New European Driving Cycle (NEDC). Xu et al. [42] proposed a game-theoretic energy management technique that puts into consideration the vehicle velocity prediction of a hybrid electric car. They adopted a recurrent neural network model to predict future velocities and Nash equilibrium of game-theoretic energy management that is executed using best response functions. Chen et al. [43] came up with a game-theoretic approach to solving the comprehensive vehicle energy management problem involving hybrid heavy-duty vehicle (H truck) with a high-voltage battery and an electric refrigerated semi-trailer. This solution rests on a two-level single-leader multi-follower game model which shows optimal performance during the simulation. An adaptive game theoretical approach to the same issue is given in the work of Chen et al. [44], covering improvements of the strategy to adapt to the real driving behaviour. The findings give the fact that the adaptive game-theoretic provides both mature and superior performance in a wide variety of drive cycles as opposed to its non-adaptive counterpart. The complete vehicle energy management (CVEM) of a hybrid heavy-duty truck thus a game-theoretic solution concept of the problem described in [45] is given.

V. PSEUDO SPECTRAL & OPTIMIZATION

The pseudospectra approach [46], which is also known as the discrete variable representation approach, is a direct numerical approach to addressing optimal control problems. In solving the energy management problem, the optimal control method is used in order to intensify energy spread. This technique may be used as direct numerical technique to obtain optimal energy distribution. By discretising, the continuous energy management optimisation problem and solving it as a nonlinear programming problem, it can be solved easily. Hu et al. [47] presented a 2-objective charging optimization method to two kinds of lithium-ion battery considering the impact of battery charging period and the energy dissipation on HEV energy management. They developed a multi-objective optimal charging control problem, and then this was solved with the help of Radua pseudospectra method. In the study by Zhou et al. [48] the pseudospectra approach was used in solving an HEV energy management problem where they simultaneously optimized an energy management problem along with the co-state trajectory. Their results suggested a better computing efficiency of pseudospectra method than DP, a comparable performance being achieved in optimization. Wu, Yang, and Qin [49] proposed a Hierarchical EMS of Hybrid Electric Vehicles on the basis of a pseudospectra approach that comprises velocity planning and trades between fuel consumption and path following precision.

A. CONVEX OPTIMIZATION

Convex optimization is an algorithm to solve convex problems [3], which assume a convex objective as well as convex constraints. With those problems, local and global optimization solutions agree, making a solution process considerably simpler [50]. It ensures the optimal solutions are reached at a greater level than the other global optimization methods. The energy management optimization of HEVs may be considered a nonlinear programming problem which may be transformed in the nonlinear programming problem with the help of a convex optimization problem into a semi convex problem which has the point of easier calculation scheme and also a better optimization results. Murgovski et al. [51] proposed to use an EMS that uses convex optimization in plug-in hybrid electric bus. They converted issues to semi-convex problems by solving them using a convex optimization process to determine the effect that battery size, gear shifting and engine on/off have on energy management. In addition to this, optimal solutions to convex optimization were compared with those to dynamic programming. Nafisi et al. [52] investigated how the power grid would impact the energy management of plug-in HEVs and suggested a two-level approach to optimize the energy management by leveraging on convex optimization to reduce energy loss. Yet, the drawback of convex optimization is that the objective function and inequality constraint of the problem must be convex [53] which places its usage in a limited sphere. In the case of parallel HEVs, gearshift strategy should be established separately, i.e. not the gearshift and power split solutions should be optimized in combination with each other, as stated in [54].

B. MINIMUM PRINCIPLE OF PONTRYAGIN (PMP)

PMP is an analytical optimization method that works to solve optimal control situations by providing a necessary condition. Can be reformulated as an instantaneous Hamiltonian optimization problem which is a reformulation of DP by Hamiltonian optimization of a global optimization problem. In turn, the optimal solution can lead to the minimum of the instantaneous Hamiltonian, which includes fuel consumption and SOC of battery. As in ECMS, it is important to calculate an optimal co-state. It is common to use a shooting approach to determine the best co-state, as it is the case in [55]. Further researches can be found in [56 59]. PMP portrays the form of instantaneous optimization that allows the implementation of the control in real time. The basic principle is usually manifested in the form of the Equations (3) to (7). As far as it can be seen, it will be needed that the objective function is differentiable in order to establish the minimum, though a continuous Hamiltonian of Hybrid Electric Vehicles, and especially a parallel HEV, is not easily reached. To overcome this, in [60], a simplified PMP was presented to skip the co-state adaptation mechanism with focus to real-time applications. The main weakness (or the main drawback) of the control idea is that the PMP-based EMS does not guarantee that there will be the optimality of the control in case the PMP is used without consideration of the future states of the driving conditions [61].

C. RULE-BASED EMSS

Rule-based EMSs commonly imply the establishment of pre-established logical rules according to the features and working regimes of the HEV system. These regulations are created following a rule of the following kind, considering such aspects as battery SOC, driver power demand, and vehicle speed. With these rules, the distribution of power can be regulated in such a way that they will meet the power requirements of the driver and maintain the SOC in a certain range. This method is based on reasoning and local requirements, instead of knowing driving cycle beforehand, thus its flexibility to varying driving behaviours is confined by being unable to modify control parameters in subsequent control without having any detail of future driving cycle. In the following sections, methods that are commonly used like deterministic rule based control and fuzzy rule based techniques are discussed.

In this strategy, the rules of logic are predetermined with the help of the engine map and motor efficiency map so that the power is distributed between the engine and the motor keeping in mind the efficiency of the motor and efficiency of the engine along with the state of charge of the battery. Such control regulations can be easily enforced on the Web implemented as look-up table, thus, becoming economical in applications in commercial vehicle controllers. The regulations may be formulated on particular driving cycles (e.g., ECE) and different traffic conditions will make it less flexible to be used in different driving cycles. Peng et al. [69] give an account of a rule-based EMS in a parallel hybrid electric vehicle. Thus, the real power management cannot be achieved through the traditional rule-based system, because there is no single way of developing logical rules. This, in most instances, is dependent on the driving cycles, and the experience of an engineer.

Rule-based approaches such as on/off incentives and power follower EMSs are discussed more in the following subsections. In this strategy, the battery SOC is maintained at all times within the minimum and maximum limits that are set out by switching the engine ON and OFF. The basic control rules are the following:

- The engine is used in full efficiency mode or sub-optimal emissions region and offers steady power when battery SOC is below the set minimum value. A fraction of engine power is transferred to the motor to meet the power demand whereas the rest of the power is used to charge the battery.
- The vehicle switches off the engine and alone uses the battery, when the state of charge (SOC) of the battery has reached the maximum predetermined value.

With these energy management systems (EMSs) in some cases a sudden surge of instant power can be pulled out of the battery and this can lead to the battery having a reduced charging and discharging cycle with increased engine start-stop. The main advantage is that the average efficiency of the engine will be raised and the charging-discharging episodes of the battery will be decreased. This, however, has the disadvantage that power loss during frequent engine start-stop is higher, energy efficiency of the whole system is lower and batteries have a shorter service life [70].

The power follower EMSs The power provided by the engine and the time when to start or stop the engine are set, based on the SOC of the battery, and the load of the vehicle, so that it supplies the power as required by the driver. Control rules are as follows:

- when the needed power is below the maximum power that the engine can supply at its operating speed, then the operation point is moved to operate at the lowest output power line.
- When the battery SOC is between the minimum set value and the maximum value and the power demand of the driver exceeds the minimum value and it is smaller than the battery capacity and at the same time it is higher than the maximum power of the engine at the operating speed, the engine operates on the maximum output power line with the additional power demand covered by the battery.
- In case the battery SOC exceeds the set maximum value and has an opportunity to cover the power demanded, the engine is to be shut down.

The key benefit of such an approach is that it allows decreasing the number of battery charges and discharges and improving the system energy consumption along with the battery lives. This approach provides a more flexibility in achieving engine output power to suit the needs of requiring power but this increases the operating range of the engine; thus decreasing the overall efficiency. The rule-based EMSs Systems are quite easy to deploy on the Internet, but they are not optimal and cannot guarantee optimum behaviour under various driving cycles. They are also unable to manipulate control parameters to obtain the most optimum fuel economy because driving conditions are complicated.

D. FUZZY LOGIC BASED EMSS

Fuzzy logic control theory combines the fuzzy set theory and fuzzy logic. The first represents an expansion of the binary set theory represented by the values of TRUE and FALSE (1 and 0), and the latter is an expansion of the traditional logic in the calculation of system outputs [71]. The fuzzy relation is mostly founded on the measure of similarity or the degree thereof in between data sets and Fuzzy reasoning gets cast in the IF THEN format and commonly known methods of reasoning are Mamdani method [72] and Takagi Sugeno method [73]. Fuzzy logic-based EMSs have been investigated in the literature over the past years [74-77]. The object of these EMSs is to disseminate the power through fuzzy rules. Usually, the design of fuzzy logic rules is made depending on driver power demand and SOC. The fuzzy logic controller consists of a group of linguistic rules; this group of linguistic rules include a linguistic antecedent and two linguistic consequents. Having considered a hybrid system as a time varying and nonlinear plant, there can be adaption of fuzzy logic controllers to real-time application and sub-optimal control using a set of fuzzy logical rules. Also it is imperative to formulate a membership function which will maximize the distribution of power. Thus, to optimize the membership function the GA is used as described in reference [78]. The other variations of modified fuzzy logic-based EMSs are presented in [79-81]. In [82], the development of a fuzzy logic controller (FLC) on parallel Hybrid Electric Vehicles is carried out. A fuzzy logic model of a multi-input controller of a power-split hybrid electric car is presented in [83] and bench marks the rule-based EMSs in terms of fuel economy and emissions. Considering the intended driver torque, using FLC technique, power allocation is controlled regarding speed and battery SOC, which makes it ameliorate fuel consumption and adaptability to variations compared to the traditional rule-based EMS. Lee et al. [84] proposed a fuzzy logic based energy management strategy that combines reduction of emissions of NOx and meeting the power requirement of the drivers.

The control considerations of this fuzzy logic controller would be the speed of the electric motor and the acceleration pedal stroke. The controller is said to have the capability of helping reduce NOx by up to 20 percent of the conventional cars. The downside of this approach however is that, it cannot guarantee the SOC charge-sustainability of battery. In order to address this problem, Lee et al. [85] came up with a more detailed fuzzy logic controller which is a combination of both; a power balance controller and a driver intention predictor. Baumann et al. [86] developed an extensive fuzzy logic controller on the basis of road load estimation in order to offset the difference between actual engine torque and that of required torque. In order to enhance flexibility of fuzzy based EMS, Tian et al. [87] developed an EMS of a plug-in hybrid electric bus, with an optimal SOC reference provided by a neural network and then a fuzzy logic controller. Such fuzzy rule-based EMSs may control relatively fine control parameter change through predetermining a set of fuzzy rules. The approach however provides an inferior level of flexibility as a result of the difficulty of choosing a proper membership function based on varied inputs.

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