

# BUCKLING LOAD FOR CROSS PLY SIMPLY SUPPORTED COMPOSITE PLATE FOR DIFFERENT ASPECT AND MODULI RATIOS UNDER BIAXIAL COMPRESSION

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**Abstract-** In order to investigate the buckling load in biaxially loaded rectangular laminated plates, a numerical technique is used. For the purpose of conducting buckling analysis on thin rectangular laminated plates within the framework of classical laminated plate theory (CLPT), a Fortran software that is based on finite elements (FE) has been created. An investigation has been conducted on the issue of buckling stresses of composite plates that are typically stacked. Analysis and resolution of the issue are accomplished by the use of the energy strategy, which is described by a finite element model. A model with four nodes is used in this approach, which involves the application of quadrilateral parts. At each node, there are three degrees of freedom for each individual constituent. As far as degrees of freedom are concerned, they are as follows: lateral displacement, rotation, and around the axes and correspondingly. Buckling loads are analyzed and verified with other works that are accessible in the literature in order to verify the correctness of the approach that is currently being performed. This indicates that the finite element approach that was employed is reliable since it has a good agreement with the data that is accessible.

**Index Terms-** Finite element, Fortran program, Buckling load, Simply supported, Composite plate.

## I. INTRODUCTION

In solid mechanics, the deformation of a plate under transverse and/or in-plane pressure has two components: flexural deformation resulting from the rotation of cross-sections and shear deformation arising from the sliding of sections or layers. The resultant deformation is contingent upon two parameters: the thickness-to-length ratio and the ratio of elastic to shear moduli. When the thickness-to-length ratio is little, the plate is classified as thin and mostly deforms by flexure; conversely, when both the thickness-to-length and modulus ratios are substantial, the plate predominantly deforms via

shear. The elevated ratio of in-plane modulus to transverse shear modulus results in more significant shear deformation effects in composite laminates under transverse and/or in-plane stresses compared to isotropic plates under analogous loading circumstances. The three-dimensional laminate theories, where each layer is considered a homogeneous anisotropic media (Reddy), are unsolvable. The anisotropy in laminated composite structures often results in intricate reactions under various loading circumstances, generating complex couplings among extension, bending, and shear deformation modes. Except in certain instances, it is impractical to completely resolve an issue in three dimensions owing to complexity, computational demands, and the generation of superfluous data, particularly for composite structures. Numerous theories addressing transverse shear and normal stresses are present in the literature, such as those proposed by Mindlin in 1951. These are too many to examine here. Only select classical works and those that provide foundational context for the current argument will be examined. According to Phan and Reddy, these theories are categorized into two main types based on the presumed fields: (1) stress-based theories and (2) displacement-based theories. Stress-based theories originate from stress fields presumed to change linearly over the plate's thickness, whereas displacement-based theories are founded on an assumed displacement field. This research is referenced in the works of Yang, Norris, and Stavsky; Whitney and Pagano; and Phan and Reddy. Various theories for plate analysis have been developed based on distinct assumptions on displacement fields. Theories may be classified into three primary categories: individual layer theories (IL), equivalent single layer (ESL) theories, and

three-dimensional elasticity solution methods. These categories are subdivided into sub-theories by the adoption of certain assumptions. The second group encompasses the classical laminated plate theory (CLPT), first-order shear deformation theory (FSDT), and higher-order shear deformation theory (HSDT), as referenced in Seloodeh and Karami Noor Omurtag (2000) and Huang and Thambiratnum.

Dorgruoglu and In this study, the analysis used classical laminated plate theory (CLPT), which excludes transverse shear deformations. This hypothesis posits that the laminate experiences plane stress, each lamina exhibits linear elasticity, and there is complete adhesion between the layers. The traditional laminated plate hypothesis posits that the line normal to the mid-surface stays straight and perpendicular to the mid-surface both before and after deformation.

Consequently, this theory is suitable for the buckling analysis of thin laminates. A Fortran program has been created, and the convergence and correctness of the finite element solutions for biaxial buckling of thin laminated rectangular plates are validated by comparison with diverse theoretical and experimental solutions, alongside the generation of fresh numerical data.

## II. RESEARCH OBJECTIVE

This research includes a thorough examination of the following objectives: An examination of several plate theories and methodologies used to forecast the behavior of laminated plates subjected to buckling stresses.

The formulation of a theoretical model designed to forecast buckling loads in a thin laminated plate is a novel and unparalleled methodology.

The advancement and utilization of the finite element method for the examination of rectangular laminated plates under buckling loads.

Additional research on the impact of coupling between bending and extension and/or twisting on the behavior of laminated plates may be conducted.

## III. METHODS

The finite element approach is used in this work to forecast the buckling loads and deformation modes of laminated rectangular plates. This analytical approach employs four-noded components. These components are the four-noded bilinear rectangular

elements of a plate. Every element has three degrees of freedom at each node. The degrees of freedom consist of the lateral displacement ( $w$ ) and the rotations ( $\phi_i$ ) and ( $\psi_i$ ) around the ( $x$ ) and ( $y$ ) axes, respectively. The finite element approach is derived from the energy method. The numerical technique may be encapsulated in the following procedures:

- The element's choice and shape play a role.
- In order to create both element stiffness and differential matrices, a finite element model was made using the energy method.
- Using the rules of non-dimensionality to change the element matrices into their non-dimensional forms.
- Putting together the element stiffness and differential matrices to get the global matrices that go with them.
- Adding border conditions that are needed for the edges of the plates.
- The problem can be fixed with the right tools. In this case, FORTRAN and ANSYS were used.

## IV. CONVERGENCE STUDY

A convergence analysis may be performed to determine the optimal number of plate elements in any direction (i.e. mesh size or discretization) that should be employed in order to calculate the buckling loads with a level of precision that is acceptable. In order to calculate the appropriate number of finite elements, a number of parameters are taken into consideration. These considerations include the qualities of the material, the size of the plate, the lamination scheme, the boundary conditions, and the storage capacity of the computer RAM. One might make the observation that the number of finite components that are needed rises in proportion to the number of modes that are present. Because of this, it is reasonable to anticipate that the higher modes will need a greater quantity of components. All of the studies that are discussed in this thesis have been carried out under the assumption that the plate has been exposed to support conditions that are either identical or equal to one another on each of the four sides of the plate. A clamped – clamped (CC) edge condition, a simply – simply supported (SS) edge condition, and a clamped – simply supported (CS) edge condition are the three sets of edge conditions that are used in this context. See the table below for further information.

**Table 1: Boundary conditions.**

Boundary Conditions	Plate dimensions in y – coordinate $x = 0, x = a$	Plate dimensions in x – coordinate $y = 0, y = b$
CC	$w = \phi = \psi = 0$	$w = \phi = \psi = 0$
SS	$w = \psi = 0$	$w = \phi = 0$
CS	$w = \phi = \psi = 0$	$w = \phi = 0$

A study of the convergence of the non-dimensional buckling load of a simply supported square isotropic plate made of stainless steel with a length to thickness ratio of twenty ( $a/h=20$ ) is shown in the table 2 below. The material parameters that are being considered are as follows: material 1:

$$E_y/E_x = 1.0, G_{xy}/E_x = G_{yz}/E_x = G_{xz}/E_x = 0.4, \nu$$

Based on the data shown in Table, it is possible to notice that the values of the buckling parameter  $\bar{P} = Pb^2/E_2h^3$  tend to converge as the number of

elements in the mesh increases (that is, as the mesh size gradually decreases). Based on these findings, it seems that a mesh that is placed over the plate is sufficient for the work that is being done (i.e., there is a difference of less than 1.32% when compared to the finest mesh result that is currently available). After careful consideration, it has been shown that a mesh size of is enough for accurately predicting the first seven types of buckling stress. The first three forms of buckling are the only ones that are adequate in practice.

**Table 2 : Convergence study of non – dimensional modes of buckling of simply supported (SS) isotropic square plate with  $a/h=20$ .**

Mesh Size	Mode Sequence Number						
	1	2	3	4	5	6	7
2 × 2	30.69	76.89	83.18	83.49	94.71	94.95	101.78
3 × 3	32.64	79.12	79.18	117.58	179.04	189.78	191.05
4 × 4	33.60	82.38	82.44	123.22	165.70	166.35	192.53
5 × 5	34.10	84.08	84.14	127.71	168.69	168.92	202.10
6 × 6	34.39	85.10	85.15	130.85	170.41	170.52	208.35
7 × 7	34.58	85.75	85.79	133.03	171.55	171.61	212.50
8 × 8	34.70	86.19	86.23	134.57	172.34	172.39	215.79
9 × 9	34.78	86.50	86.53	135.68	172.92	172.97	218.07
10 × 10	34.84	86.72	86.75	136.52	173.35	173.40	219.78

Many comparisons were carried out in order to determine whether or not the current FE approach is accurate, legitimate, and applicable to the situation at hand. The theoretical, ANSYS simulation, and experimental findings are all included in the comparison and analysis.

## V. RESULTS AND DISCUSSION

In Table3 the non – dimensional critical buckling load is shown in order to compare with (Yu and

Wang Mohammadi et al., 2009) and (Moktar for an isotropic plate of material 1 with varied aspect ratios. According to the table, the current findings are in excellent agreement with the findings of Mohammadi et al. (2009) and Yu and Wang Mohammadi et al. (2009).

**Table 3 : Comparison of the non – dimensional critical buckling load for an isotropic plate.**

Aspect Ratio a/b	References Value	References Value	References Value	Present Study
0.5	12.33	12.3370	12.3370	12.3
1.0	19.74	19.7392	19.7392	19.7

An examination of the impact of plate aspect ratio and modulus ratio on non-dimensional critical loads is shown in Table 4, which may be seen below. As the modular ratio grows, it has been shown that the non-dimensional buckling force for symmetric laminates increases as well. Both (Osman and

Elmardi, 2017) and (Reddy 2004) were discussed in relation to the findings that were presented here. The procedure of verification revealed a high degree of agreement, particularly when the aspect ratio was increased and the modulus ratio was gradually decreased.

**Table 4 : Buckling load for (0/ 90/ 90/ 0) simply supported (SS) plate for different aspect and moduli ratios under biaxial compression.**

Aspect Ratio a/b	Modular Ratio	Biaxial Compression				
	$E_1/E_2$	5	10	20	25	40
0.5	Present	10.864	12.122	13.215	13.726	14.000
	Ref.	-	12.307	-	13.689	-
	Ref.	11.120	12.694	13.922	14.248	14.766
1.0	Present	2.790	3.130	3.430	3.510	3.645
	Ref.	-	3.137	-	3.502	-
	Ref.	2.825	3.174	3.481	3.562	3.702
1.5	Present	1.591	1.602	1.611	1.613	1.617
	Ref.	-	1.605	-	1.606	-
	Ref.	1.610	1.624	1.634	1.636	1.641

Under biaxial compression, the influence of plate aspect ratio and modulus ratio on non-dimensional critical buckling loads of simply supported (SS) antisymmetric cross-ply rectangular laminates is shown in Table 5. This table highlights the relationship between these two factors. It was decided to employ the characteristics of material. As the modulus ratio grows, it has been shown that the

non-dimensional buckling load for antisymmetric laminates reduces. This is the case. For comparison, the current findings were compared with (Reddy 2004)

It was found that the validation method produced a satisfactory level of agreement, particularly when the aspect ratio was increased and the modulus ratio was decreased.

**Table 5 : Buckling load for (0/ 90/ 90/ 0) simply supported (SS) plate for different aspect and moduli ratios under biaxial compression.**

Aspect Ratio a/b	Modular Ratio	Biaxial Compression				
	$E_1/E_2$	5	10	20	25	40
0.5	Present	4.000	3.706	3.535	3.498	3.442
	Ref.	3.764	3.325	3.062	3.005	2.917
1.0	Present	1.395	1.209	1.102	1.079	1.045
	Ref.	1.322	1.095	0.962	0.933	0.889
1.5	Present	1.069	0.954	0.889	0.875	0.853
	Ref.	1.000	0.860	0.773	0.754	0.725

The buckling loads for symmetrically laminated composite plates with a layer orientation of (0/90/90/0) have been obtained for three alternative aspect ratios ranging from 0.5 to 1.5 and two modulus ratios of material 2 (40 and 5). These results are shown in Tables 6 and 7. As a result of this observation, the buckling load becomes more significant as the aspect ratio grows for biaxial compression loading. In the case of

clamped – clamped (CC) and clamped – simply supported (CS) boundary conditions, the buckling load is at its highest, while it is at its lowest for simply – simply supported (SS) boundary conditions. It can be observed from the tables and that the values of buckling loads that were determined by the current research are much closer to the values that were found in (Osman and Elmardi, 2017).

**Table 6 : Buckling load for (0/ 90/ 90/ 0) plate with different boundary conditions and aspect ratios under biaxial compression.**

Aspect Ratio a/b	Comparisons of Results	Boundary Conditions		
		CC	SS	CS
0.5	Present	1.0742	0.4143	0.9679
	Ref.	1.0827	0.4213	1.0022
1.0	Present	1.3795	0.4409	1.0723
	Ref.	1.3795	0.4411	1.0741
1.5	Present	1.6402	0.4400	1.2543
	Ref.	1.6367	0.4391	1.2466

**Table 7 : Buckling load for (0/ 90/ 90/ 0) plate with different boundary conditions and aspect ratios.**

Aspect Ratio a/b	Comparisons of Results	Boundary Conditions		
		CC	SS	CS
0.5	Present	1.7786	0.6787	1.6325
	Ref.	1.8172	0.6877	1.6838
1.0	Present	2.1994	0.6972	1.8225
	Ref.	2.2064	0.6985	1.8328
1.5	Present	2.7961	0.8943	1.7643
	Ref.	2.8059	0.8962	1.7618

## VI. CONCLUSIONS

In order to calculate the buckling loads of laminated plates that have a rectangular cross-section, a finite element model has been developed. In order to ensure that the current method is accurate, buckling loads are analyzed and verified in comparison to other studies that are already published in the relevant literature. Additional comparisons were carried out, and the results provided by the ANSYS program and the experimental findings were compared with all of the other findings. This indicates that the finite element approach that was employed is reliable since it has a good agreement with the data that is accessible.

## REFERENCES

- [1]. Osama Mohammed Elmardi, 'Verification of dynamic relaxation (DR) method in isotropic, orthotropic and laminated plates using small deflection theory', International Journal of Advanced Science and Technology, volume 72; (2014), pp. 37 – 48
- [2]. Osama Mohammed Elmardi Suleiman 'Nonlinear analysis of rectangular laminated plates using large deflection theory' International Journal of Emerging Technology and Research, volume 2, issue 5; (October2015), pp. (26 –48)
- [3]. Osama Mohammed Elmardi Suleiman 'Validation of Dynamic Relaxation DR Method in Rectangular Laminates using Large Deflection Theory' International Journal of Advanced Research in Computer Science and Software Engineering, volume 5, issue 9; (September2015), pp. (137 –1440).
- [4]. Osama Mohammed Elmardi Suleiman 'Nonlinear analysis of rectangular laminated plates' Lap Lambert Academic Publishing, Germany, August 2015
- [5]. Osama Mohammed Elmardi Suleiman 'Text Book on Dynamic Relaxation Method' Lap Lambert Academic Publishing, Germany, September 2016
- [6]. Phan N.D. and Reddy J.N., 'Analysis of laminated composite plate using higher – order shear deformation theory', International Journal of numerical methods in engineering, vol.21; (1985): pp. (2201– 2219).
- [7]. Putcha N.S. and Reddy J.N., 'A refined mixed shear flexible finite element for the non – linear analysis of laminated plates', computers and structures, vol.22, No.4 ;( 1986): PP. (529 –538).
- [8]. Reddy J.N., ' A simple higher – order theory for laminated composite plates', Journal of applied mechanics, vol. 51, No. 745; (1984): pp. (13– 19).
- [9]. Reddy J. N., 'Mechanics of laminated composite plates and shells, theory and analysis', second edition, CRC press, Washington; (2004).
- [10]. Rushton K.R., ' large deflection of variable thickness plates', International Journal of mechanical sciences, vol.10; (1968): PP. (723 – 735).
- [11]. Seloodeh A.R., Karami G., ' Static, free vibration and buckling analysis of anisotropic thick laminated composite plates on distributed and point elastic supports using a 3–D layer wise FEM', Engineering structures (26); (2004): pp. (211– 220).
- [12]. Turvey G.J. and Osman M.Y., ' Elastic large deflection analysis of isotropic rectangular Mindlin plates', International Journal of mechanical sciences, vol.22; (1990): PP. (1 – 14).
- [13]. Turvey G.J. and Osman M.Y., 'Large deflection analysis of orthotropic Mindlin plates', proceedings of the 12th Energy resource technology conference and exhibition, Houston, Texas; (1989): PP. (163– 172).
- [14]. Turvey G.J. and Osman M.Y., ' Large deflection effects in anti-symmetric cross-ply laminated strips and plates', I.H. Marshall, composite structures, vol.6, Paisley College, Scotland, Elsevier Science publishers; (1991): PP. (397– 413).
- [15]. Vernon B. John, ' introduction to engineering materials', second edition; (1972).
- [16]. Whitney J.M. and Pagano N.J., ' Shear deformation in heterogeneous anisotropic plates', Journal of applied mechanics, vol.4; (1970): PP. (1031 – 1036).
- [17]. Yu L. H., and Wang C. Y., 'Buckling of rectangular plates on an elastic foundation using the levy solution', American Institute of Aeronautics and Astronautics, 46; (2008): PP. (3136 – 3167).