# Mathematical Modeling and Optimization of Inventory Control Systems: Insights from Reliance Smart, Sikar

Sumitra Bagria[1], Sapna Shrimali[2]

[1]*Research Scholar, Department of Mathematics,Janardan Rai Nagar Rajasthan Vidyapeeth (Deemed to be University), Udaipur, Rajasthan, India*

[2]*Associate Professor, Department of Mathematics, Janardan Rai Nagar Rajasthan Vidyapeeth (Deemed to be University), Udaipur, Rajasthan, India*

*Abstract*—**Efficient inventory management is crucial in the retail sector, particularly in semi-urban areas like Sikar. This paper proposes a compartmental mathematical model inspired by epidemiological modeling (SIR-type) to understand and optimize inventory control at Reliance Smart, Sikar. The model divides the system into demand (D), stock (S), and consumption (C) compartments. A system of differential equations is developed to analyze inventory flow, including order placement, fulfillment, and shrinkage. Numerical simulations are conducted, and strategies such as EOQ and ROP are incorporated. The findings demonstrate that mathematical modeling significantly improves inventory accuracy, reduces costs, and enhances service levels. This work contributes to the literature by adapting nonlinear modeling techniques to a real-world retail setting and offers a framework for data-driven decision-making in inventory control.**

## 1. INTRODUCTION

Inventory control is a vital pillar of retail operations, exerting a direct influence on both cost efficiency and customer satisfaction (Chopra &Meindl, 2016). In semi-urban retail outlets such as Reliance Smart, Sikar, maintaining an optimal balance between stock availability and overstocking presents a persistent challenge. When inventory decisions rely heavily on managerial intuition or fragmented historical data, retailers are prone to inefficiencies such as stockouts, excess inventory, shrinkage, and dead stock (Nahmias& Olsen, 2015). These inefficiencies not only increase operational costs but also reduce service quality and erode customer trust. Given these challenges, there is a growing emphasis on data-driven, quantitative methods to improve forecasting accuracy and standardize replenishment practices.

Mathematical modeling provides a systematic and replicable framework for inventory optimization. Inspired by compartmental models in epidemiology, particularly the Susceptible-Infected-Recovered (SIR) structure, this study develops a similar three-compartment model to represent the flow of goods through demand (D), stock (S), and consumption (C) stages (Hethcote, 2000). By translating inventory dynamics into a system of differential equations, the model offers insights into real-time inventory behavior and allows managers to simulate the effects of various control strategies. Furthermore, classical models such as Economic Order Quantity (EOQ) and Reorder Point (ROP) are incorporated to examine cost minimization and replenishment timing (Tersine, 1994). This integration of system-based modeling and traditional inventory techniques enables the development of robust, context-specific solutions for retail settings in semi-urban regions.

## 2. COMPARTMENTAL INVENTORY MODEL

Inspired by the SIR model used in epidemiology, this study defines three compartments for inventory control: D(t) - Demand, S(t) - Stock, and C(t) - Consumption. Demand flows into stock upon order placement, and stock flows into consumption upon sales. The system incorporates ordering rate, fulfillment rate, shrinkage, and natural loss. The schematic diagram (Fig. 1) and parameter table (Table 1) summarize the model.

Figure 1: Diagram of the Inventory Flow Model (D → S → C)

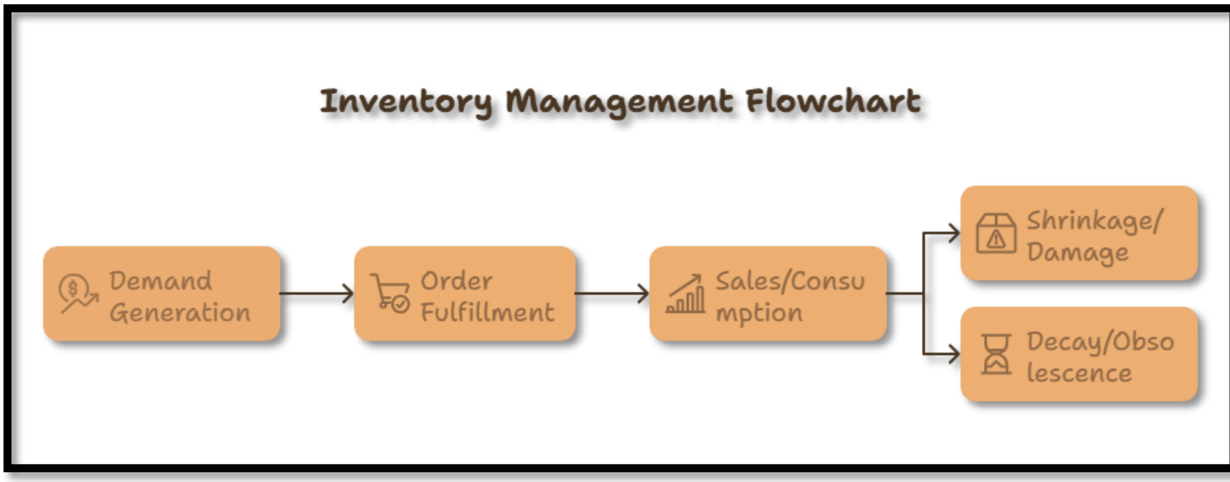


Figure 1 represents the compartmental flow of inventory across three stages—Demand (D), Stock (S), and Consumption (C). The flow from D to S is governed by the order fulfillment rate (α), while the transition from S to C is driven by the sales/consumption rate (β). This structure enables modeling of real-time inventory dynamics in a retail environment using differential equations

Table 1: Model Parameters and Descriptions

| Parameter | Description |
|---|---|
| Y | Demand generation rate |
| α | Order fulfillment (order placement) rate |
| β | Sales/consumption rate |
| δ | Shrinkage/damage rate |
| d | Decay/obsolescence rate |

Table 1 outlines the key parameters used in the compartmental inventory model developed for this study. The parameter Y denotes the *demand generation rate*, representing the number of units requested or required by customers over a given time period. This rate serves as the input into the inventory system and determines how quickly stock is needed. The symbol α corresponds to the *order fulfillment rate*, indicating the proportion of demand that is successfully converted into stock through procurement or replenishment actions. It reflects the efficiency and responsiveness of the supply system in meeting demand.

The parameter β represents the *sales or consumption rate*, i.e., the rate at which stocked inventory is purchased by customers and exits the system. A higher β implies quicker inventory turnover. The parameter δ captures the *shrinkage or damage rate*, accounting for units lost due to theft, spoilage, or handling errors before reaching the consumer. Finally, d indicates the *decay or obsolescence rate*, referring to the rate at which inventory becomes unsellable due to expiration, trend shifts, or technological redundancy. Together, these parameters form the structural foundation of the dynamic inventory model, allowing for real-time analysis of stock flows and losses within the retail system.

## 3. MATHEMATICAL MODEL

Let D(t), S(t), and C(t) represent the number of units in demand, stock, and consumed categories respectively. The inventory dynamics can be described by the following system of ordinary differential equations:

$dD/dt = Y - dD - \alpha D$

dS/dt = αD - (δ + d + β)S
dC/dt = βS – dC

## 4. ASSUMPTIONS OF THE MODEL

- Demand arrives at a constant rate (Y)
- Order placement is proportional to demand (αD)
- Stock loss includes shrinkage, damage, and expiry (δ + d)
- Sales are proportional to available stock (βS)
- Consumed items leave the system at rate d

## 5. SCALING THE MODEL

To simplify and generalize the model, we apply scaling. Let total inventory system population be normalized to 1, such that:
d(t) + s(t) + c(t) = 1, where d(t) = D(t)/N, s(t) = S(t)/N, c(t) = C(t)/N.

We define scaled parameters and rewrite the differential equations accordingly.
dd/dt = y - d·d - α·d
ds/dt = α·d - (δ + d + β)·s
dc/dt = β·s - d·c

## 6. NUMERICAL ILLUSTRATION AND SIMULATION

We simulate the model using assumed realistic values:
Y = 100 units/day, α = 0.5, β = 0.7, δ = 0.05, d = 0.02.
Initial conditions: D(0) = 300, S(0) = 500, C(0) = 200.

Figure 2: Simulation of the inventory model over 30 days.

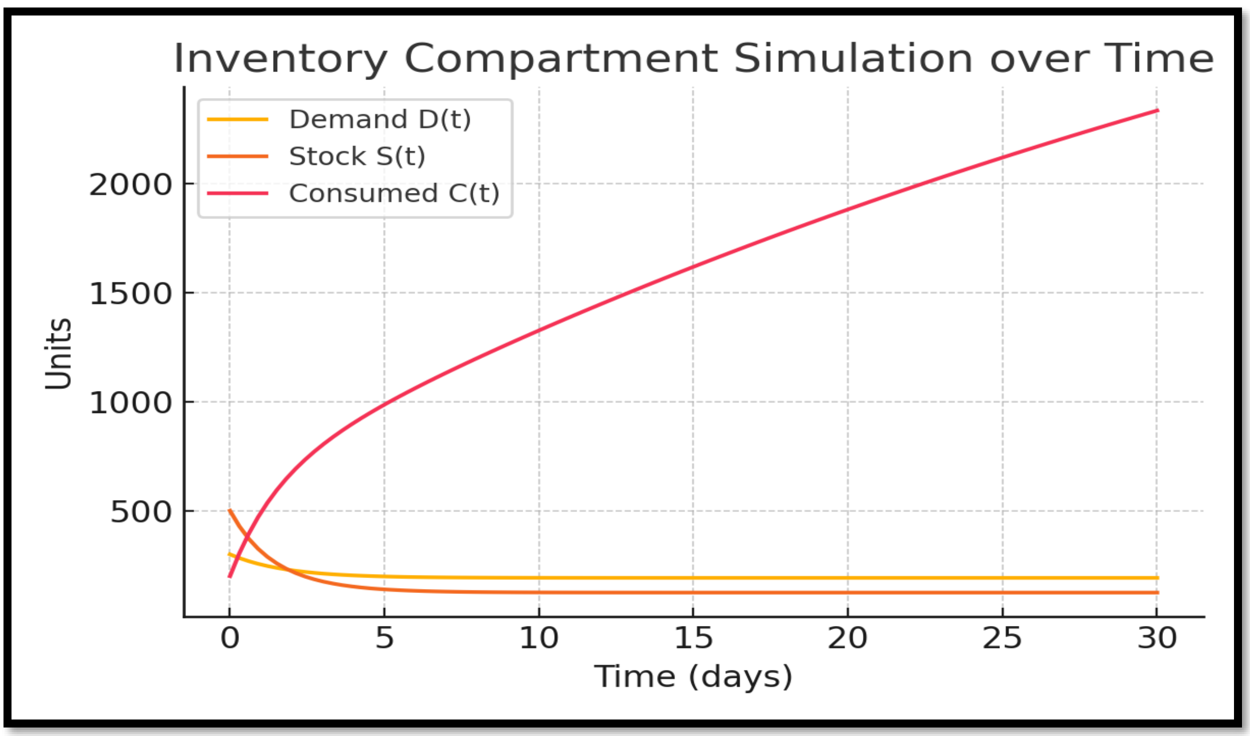


Figure 2 depicts a simulation of inventory compartments over a 30-day period using the proposed differential equation model. It demonstrates that demand (D) stabilizes quickly, stock (S) decreases initially and then stabilizes at a lower level, while consumption (C) increases steadily, indicating effective stock depletion and order fulfillment. The simulation validates the model's capacity to capture dynamic inventory behavior over time.

## 7. EOQ AND ROP INTEGRATION INTO THE MODEL

To complement the dynamic model, classical inventory formulas are incorporated. Economic Order Quantity (EOQ) minimizes the total cost of ordering and holding inventory, defined as:

$EOQ = \sqrt{2DS / H}$

Where D is annual demand (units), S is ordering cost per order, and H is holding cost per unit per year. For example, for D = 10,000 units/year, S = ₹500/order, and H = ₹25/unit/year:

$EOQ = \sqrt{2 \times 10000 \times 500 / 25} = 632$ units

The Reorder Point (ROP) is the inventory level at which a new order is placed, considering lead time (L):

$ROP = d \times L + Z \times \sigma L$

Assuming d = 30 units/day, L = 5 days, Z = 1.645 (95% service level), $\sigma L = 12$:

$ROP = 30 \times 5 + 1.645 \times 12 \approx 150 + 19.74 = 169.74 \approx 170$ units

Table 2: EOQ and ROP Example Calculations

| Metric | Formula | Result |
|---|---|---|
| EOQ | $\sqrt{2DS / H}$ | 632 units |
| ROP | $d \times L + Z \times \sigma L$ | 170 units |

## 8. OPTIMIZATION SCENARIO: COST MINIMIZATION

We define the total inventory cost (TIC) as the sum of ordering and holding costs:

$TIC(Q) = (D/Q) \times S + (Q/2) \times H$

Where Q is the order quantity. The objective is to find Q such that TIC is minimized. For illustration, we calculate TIC over a range of Q values and identify the minimum.

Figure 3: Total Inventory Cost curve showing EOQ as cost minimum.


Figure 3 illustrates the relationship between total inventory cost (TIC) and order quantity (Q), highlighting a typical cost optimization curve. The graph shows that TIC decreases with increasing order quantity until reaching an optimal point (around Q = 632 units), beyond which the cost begins to rise again. This demonstrates the effectiveness of Economic Order Quantity (EOQ) in minimizing inventory-related expenses.

## 9. CONCLUSION

This study presents a mathematical modeling framework combining differential equations and classical optimization methods to manage inventory at Reliance Smart, Sikar. The D-S-C model effectively visualizes inventory flow, while EOQ and ROP enhance operational efficiency. Simulation and cost analysis validate the model's utility in minimizing inventory costs and meeting customer demand. The approach offers a scalable solution for inventory planning in semi-urban retail sectors.

## REFERENCES

[1] Chopra, S., & Meindl, P. (2016). Supply Chain Management: Strategy, Planning, and Operation. Pearson.
[2] Nahmias, S., & Olsen, T. L. (2015). Production and Operations Analysis. Waveland Press.
[3] Zipkin, P. (2000). Foundations of Inventory Management. McGraw-Hill. - Snyder, L. V., & Shen, Z. J. M. (2019). Fundamentals of Supply Chain Theory. Wiley.
[4] Tersine, R. J. (1994). Principles of Inventory and Materials Management. Prentice Hall.
[5] Hethcote, H. W. (2000). The mathematics of infectious diseases. *SIAM Review*, *42*(4), 599–653. https://doi.org/10.1137/S0036144500371907
[6] Ballou, R. H. (2004). Business logistics management: Planning, organizing, and controlling the supply chain.
[7] Lee, H. L., & Billington, C. (1992). Material requirements planning and supplier development. Journal of Operations Management, 11(2), 147-163.
[8] Silver, E. A. (2008). Inventory management: An overview. International Journal of Production Economics, 114(2), 475-486.