Heuristic and Metaheuristic Algorithms for solving Scheduling Problems

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***Abstract*- The current work offers a comprehensive review of advancements in solving sequencing problems, which are critical optimization challenges across fields such as manufacturing, logistics, and healthcare. The evolution of solution methodologies starting with Johnson's method proposed in 1954, is examined, from classical dispatching rules and exact algorithms like Branch and Bound to advanced heuristic and metaheuristic approaches. These approaches address various sequencing environments, including single machine, flow shop, job shop, and mixed shop settings. The review categorizes existing literature by problem type and solution technique. Promising research directions are identified, including integrating machine learning techniques, and exploring ways to bridge the gap between theoretical models and real-world applications.**

***Key words*: Flow shop problem, Idle time, sequencing problems, total elapsed time**

I. INTRODUCTION

In the field of computational theory and practice, sequencing problems represent a class of challenges that are foundational to various fields like computer science, operations research, and bioinformatics. These problems involve determining the optimal arrangement of a set of elements under specific constraints and objectives, which can range from minimizing time and cost to maximizing efficiency and effectiveness. The complexity of sequencing problems is evident in their applications across diverse domains, such as scheduling tasks in manufacturing, organizing DNA sequences in genomics, and optimizing routing in logistics. This introduction aims to provide a comprehensive overview of sequencing problems, elucidating their significance and the underlying theoretical frameworks.

At their core, sequencing problems can be categorized into several types, each with unique characteristics and challenges. A prominent example of sequencing problems is the job-shop scheduling problem, which involves allocating machines to a set of jobs, each with specific processing times and precedence constraints. The goal is to minimize the overall makespan, which is the total time required to complete all jobs. This problem is particularly applicable in manufacturing environments where multiple jobs must be processed on limited resources. The complexity of job-shop scheduling can be exacerbated by factors such as machine breakdowns, setup times, and job priorities, making it a rich area for research and algorithm development.

The theoretical foundations of sequencing problems are deeply rooted in combinatorial optimization and graph theory. Many sequencing problems can be represented as graphs, where vertices correspond to elements to be sequenced and edges represent the relationships or constraints between them. This representation allows for the application of various algorithmic techniques, including dynamic programming, branch and bound, and greedy algorithms. The choice of algorithm often depends on the specific characteristics of the problem instance, such as the size of the input, the presence of constraints, and the desired solution quality.

Despite the significant progress made in developing algorithms for sequencing problems, many challenges remain. For instance, the scalability of algorithms to handle large datasets is a persistent issue. Researchers are increasingly exploring the integration of machine learning techniques with traditional optimization methods to enhance the performance and adaptability of algorithms in these contexts. Moreover, the

interdisciplinary nature of sequencing problems has led to collaborations across various fields. As we delve deeper into the study of sequencing problems, it is essential to recognize the broader implications of these challenges. The efficient resolution of sequencing problems has the potential to drive significant advancements in numerous sectors, including healthcare, manufacturing, and logistics. For instance, optimized scheduling in healthcare can lead to improved patient outcomes and resource utilization, while efficient routing in logistics can reduce costs and environmental impact. Thus, the pursuit of effective solutions to sequencing problems is not merely an academic exercise but a critical endeavor with far-reaching consequences.

In 1954, Johnson proposed an algorithm to solving the problems in production scheduling in which he had minimized the total idle time of machines and the total production times of the jobs. This algorithm is considered to be the best algorithm in the field of scheduling till date. This introduction sets the stage for a deeper exploration of the methodologies, applications, and future directions in the study of sequencing problems, highlighting their significance in contemporary research and practice. Some fundamental terms used in sequencing are given below:

A. Fundamental terms in Sequencing

Machines: A machine is a tool which is assigned to do some particular job. A machine need not be a mechanical device, but it can be human beings also. There should be sufficient number of machines to perform all the assigned jobs.

Job: The primary motives of sequencing are the items or jobs or orders or customers. There should be 'n' jobs to be processed or sequenced for the machines to carry out those jobs.

Processing time: Each machine requires a certain period of time to perform the assigned jobs. Such time period is termed as processing time.

Total Elapsed Time: The time between the start of the first job and the completion of the last job is known as the total elapsed time.

Idle time: Idle time is the time in which the machines remain idle during the processing of the jobs.

Technological order: It is the order in which the different jobs are to be completed by various machines.

No passing rule: It means no machines can be skipped from doing the assigned jobs. The same order of the jobs is carried out by all the machines.

B. Different kinds of sequencing problems

The following are few types of sequencing problems

i. Problem with 'n' jobs through one machine.

ii. Problem with 'n' jobs through two machines.

iii. Problem with 'n' jobs through three machines.

The main objectives of these problems are to find out the optimum sequence of the jobs to be processed, to minimize the total elapsed time and the idle time of all the machines which are assigned to perform certain jobs.

II. AN INSIGHT ON ALGORITHMS OF SOLVING SCHEDULING PROBLEMS

A review of the literature is crucial for identifying and analyzing novel methods in any field. This section undertakes a comprehensive review of existing research focused on developing efficient and streamlined techniques for optimal assignment of jobs to machines. The primary objective is to evaluate methodologies that aim to minimize both the total elapsed time of the production process and the idle time experienced by the machines involved. The review will systematically categorize and examine approaches, including classical dispatching rules and modern heuristic and metaheuristic algorithms, used to address various sequencing environments, such as single machine, flow shop, job shop, and mixed shop settings. Particular emphasis will be placed on identifying the strengths and weaknesses of these methods in different scenarios and for varying numbers of jobs and machines.

Ultimately, this review seeks to pinpoint areas where new and simpler methods for achieving optimal assignment with minimal idle time and total elapsed time can be further explored, potentially through the development of innovative heuristic techniques or through the refinement of existing algorithms like Johnson's algorithm. A review of earlier works is very significant and it helps to find and analyze new methods. Here, an attempt has been made to review

the past literature pertaining to establish new and simple methods to find optimal assignment of the jobs with minimum elapsed time and idle time of the machines.

Operations Research is relatively a new discipline that has its origin during World War II, and later became very popular throughout the world. It is widely used in almost all the fields and has become successful in all the fields. Operations Research helps us to make better decisions in complex scenarios. It also includes the application of scientific tools for finding the optimum solution to a problem involving the operations of a system. Sequencing problem is considered to be one of the most important applications of Operations research. A series, in which a few jobs or tasks are to be performed following an order, is called sequencing. The aim of sequencing problem is to find the optimal sequence of the jobs on machines so as to minimize the total amount of time required to complete the full process of all the jobs. The optimality of the sequencing problem can be known by the minimized costs, maximized profits, minimized elapsed time and meeting the due dates etc. Many researchers in the past have shown interest in sequencing problems because of its significance in the production field.

Interval computation was first suggested by Dwyer (1951)[1]. In 1954, an algorithm was proposed by Johnson (1954)[2] for scheduling jobs in two machines. Its primary objective is to find an optimal sequence of jobs and to reduce the total amount of time it takes to complete all the jobs. It also reduces the amount of idle time between the two machines. Furthermore, Johnson's method has been extended to 'm' machines problem with an objective to complete all the jobs in a minimum duration. This algorithm is considered to be the best algorithm in the field of scheduling till date. Generally, in sequencing problems, the processing times are valued precisely. But in reality, it is perceived that the processing times during the performance of the job are imprecise and uncertain. In order to handle these uncertainties, we use fuzzy interval and fuzzy numbers. Here, we consider intervals.

Johnson (1954)[2] and Bellman (1956)[3] considered a problem involving the scheduling of n jobs on two machines. Manufacturing requires use of various machine classes (or labor classes), we know by machine class what capacity has already been committed to production plans are known and, hence, what remaining capacity will be increasingly available as time goes on. The problem then is to determine shipping schedules that can be supported by this available capacity and will make good or best use of this capacity. Karush and Moody (1958)[4] determined the shipping schedules for commodities that are manufactured by job-shop production processes.

Mitten (1959(a), (b))[5, 6] and Johnson (1959)[7] treated a scheduling problem with arbitrary time lags. Sisson (1959)[8] reviewed the definition of the job shop, the models proposed to date, solutions based on these models, and surveys prospects for the future. Bowman (1959)[9] gave a tentative solution to the schedule-sequencing problems. Manne (1960)[10] proposed an application of discrete linear programming to the typical job shop scheduling problem - one that involves both sequencing restrictions and also non-interference constraints for individual pieces of equipment. In this work, no attempt has been made to establish the computational feasibility of the approach in the case of large-scale realistic problems, but this formulation considered fewer variables than two other previous findings.

Ackerman (1963)[11] described the Even-Flow system which maximizes the number of on-time completions by integrating the scheduling, loading, and dispatching functions in a manner which capitalizes on the "natural" movement of work through the shop. Giglio and Wagner (1964)[12] summarized experimental results from applying several computational methods for solving the classic three-machine scheduling model. They made a sampling study of the statistical characteristics of this model using 100 sets of data randomly generated and analysed the Frequency distributions of minimum and mean total processing time. Finally, they concluded that elementary probability theory can be applied to yield corresponding statements about the accuracy guaranteed by the approach.

Ignall and Schrage (1965)[13] applied the branch-and-bound technique and two flow-shop scheduling problems. They gave computational results for up to 9

jobs for the 2-machine problem when the objective is minimizing the mean completion time, which was previously untreated. They also developed results for up to 10 jobs, including comparisons with other techniques, are given for the 3-machine problem when the objective is minimizing the makespan. The concept of fuzzy sets was proposed by Zadeh (1965)[14]. Smith and Dudek (1967)[15] described an algorithm that yields the minimum make-span sequence for n-jobs requiring processing through M-machines when no passing is allowed. Theoretical development utilizing combinatorial analysis and proof of sequence optimality were given by them.

Campbell *et al.* (1970)[16] developed exact techniques, which usually require an electronic computer, to minimize some well defined criterion on problems involving a limited number of jobs. They described a simple algorithm for the solution of very large sequence problems without the use of a computer, which produced approximate solutions to the n job, m machine sequencing problem where no passing is considered and the criterion is minimum total elapsed time. Gupta (1971)[17] described a functional heuristic algorithm for seeking a quick and approximate solution to the n-job, M-machine flowshop scheduling problem under the assumption that all jobs are processed on all machines in the same order and no passing of jobs is permitted. He also proved that the proposed functional heuristic algorithm can be executed by hand for reasonably large size problems and yields solutions which are closer to optimal solutions than those obtained by Palmer's slope index algorithm.

Merten and Muller (1972)[18] analyzed the variance of flow-time and variance of waiting-time performance measures for the single machine sequencing problem. These measures are compared and contrasted to the performance measures of mean flow-time and mean waiting-time and showed that the sequence that minimizes the variance of flow-time is antithetical to the sequence that minimizes the variance of waiting-time, while the two mean performance measures attain their minimums at the same job sequence. And also they proved that the minimum values of the two variance measures are equal. Spencer and Dudek (1973)[19] developed a heuristic algorithm which yields an optimal or near optimal minimal makespan solution to a specific type of sequencing/scheduling problem defined as the dependent sequencing problem which includes the pure job-shop and flow-shop sequencing problems. There are some approaches to this class of problem which provide optimal solutions, but their time to solutions and/or theoretical approaches make them difficult to understand and apply. The dependent shop algorithm utilizes extensions of PERT/CPM techniques which are widely known and therefore it can be easily applied.

Szwarc (1973)[20] examined the elimination method of Szwarc (1971)[21] for solving the flow-shop scheduling problem. They examined the elimination techniques that reduce the set of solutions to a subset that must contain the optimal solution being sought and showed that how to construct a general counter example to any procedure that removes more sequences than this optimal method. Corwin and Esogbue (1974)[22] considered the two different flow shop scheduling problems that arise when, in a two machines problem, one machine is characterized by sequence dependent setup times and determined a schedule that minimizes makespan. After establishing the optimally of permutation schedules for both of these problems, an efficient dynamic programming formulation is developed for each of them and compared the formulations.

Schrage (1975)[23] gave a number of necessary conditions for an optimal sequencing of machines, when there are a finite number of jobs to be scheduled on a single machine where all the jobs are available from the start and the objective is to sequence the jobs so that the time-in-system or the completion time variance is minimized. Johnson (1954)[2] solved the two machine flow shop problem by giving an argument for a sufficient condition of optimality and by stating an efficient algorithm which produces a solution via satisfaction of the sufficient condition. Moreover, Johnson solved two special cases of the corresponding three machine flow shop problem. Since that time, six other special cases have been solved, two contributed by Arthanari and Mukhopadhyay (1971)[24], two by Smith *et al.* (1975)[25] and two of a different nature by Szwarc(1977)[26]. Burns and Rooker (1975)[27] gave

an extension to one of the classes given by Szwarc (1977)[26]. Baker (1975)[28] built a simple model in order to help provide some perspective on these computational results and described how elimination algorithms may behave in moderate-sized problems.

Bonney and Gundry (1976)[29] devised a slope matching method using geometrical relationships between the cumulative process times, which generally provides better results than Palmer's method and Gupta's algorithm. They showed that, it is possible to reformulate the flowshop sequencing problem in terms of the travelling salesman problem. This has enabled them to show that the performance of the slope matching, Palmer and Gupta methods to be compared against a non-heuristic solution. They also provided that expressing a job in terms of the slopes of the cumulative start and end times is satisfactory, this enables in n-job, M-machine flowshop to be converted into an equivalent n-job 2 machine flowshop to which the use of Johnson's method will provide optimal sequences.

Maggu and Das (1977)[30] established "equivalent - job for job-blocks theorem" for 2xn sequencing problem. In the '2-machine, n-job' makespan problem the concept of equivalent jobs for blocks in job sequencing was introduced by Maggu and Das. Further, Maggu and Das (1980)[31] considered 2xn flowshop problem wherein transportation times of jobs from one machine to another are assumed to occur. In all the preceding research, except Maggu and Das (1980)[31], the transportation times of jobs from one machine to another are neglected. Owing to theoretical and as well practical interest they provided a decomposition algorithm to obtain optimal schedule of jobs for the case of 2-machine n-job flow-shop problem wherein a block of ordered jobs is assumed to be an equivalent to a single job and, furthermore, gave the transportation times of jobs from one machine to another subsequent machine. First, they developed an algorithm producing an optimal schedule, for the two-machine n-job flow-shop problem, minimizing the total elapsed time. They prepared the basis for the same in the form of a theorem. They stated the theorem as, in a general flow-shop situation, where all the jobs must pass through all the machines in the same order, certain heuristic algorithms propose that the jobs with higher total process time should be given higher priority than the jobs with less total process time.

Maggu [1981][32] and Maggu *et al.* [1981][33] obtained optimal schedule principle for 2 x n sequencing problem wherein transportation times of jobs and equivalent-jobs for job-blocks are assumed to occur. Based on this premise, Nawaz *et al.* (1983)[34] proposed a simple algorithm, which produces very good sequences in comparison with existing heuristics and also showed that the proposed algorithm performs especially well on large flow-shop problems in both the static and dynamic sequencing environments. Sule and Huang(1983)[35] developed algorithms to determine an optimal sequence that minimizes the total elapsed time. Nabeshima and Maruyama (1984)[36] considered two- and three-machine flow-shop makespan scheduling problems with additional times separated from processing times. For two-machine case, they investigated the optimality of the passing of jobs and gave an optimal algorithm in permutation case with five independent times, that is, setup and removal times, start and stop lags and transportation times and generalized known algorithms. And for a three-machine case with independent setup times, they presented several sufficient optimality conditions for a specified sequence ω_1 and clarify not only their mutual relation but also the superiority of ω_1 as an approximate sequence.

Thompson and Zawack (1985)[37] provided a new zero-one integer programming model for the job shop scheduling problem with minimum makespan criterion. They considered the algorithm with a branch and bound parametric linear programming code for solving the job shop problem with fixed completion time and a problem expanding algorithm for finding the optimal completion time and presented the computational experience for problems having up to thirty-six operations. Ishii and Nishida (1986)[38] considered a scheduling problem and proposed an O(n log n) algorithm which finds an optimal speed of each machine and optimal schedule when processing order of two operations is not specified and the speeds of machines A, B are controllable. Cheng (1986)[39] proposed an analytical model for determining the optimal processing-time and number of operations multiples for the TWK and TWK + NOP due-date

assignment methods in a dynamic job shop subject to restrictive assumptions on queue discipline and processing time distribution and compared the analytical results with the experimental results obtained from simulation of a hypothetical job shop under various shop conditions. The close agreement of the results revealed the validity of the analytical model and he has proved that the TWK + NOP method is more effective in minimizing the missed due-date cost in a job shop.

Bagchi *et al*. (1987)[40] have worked on non-preemptive single machine scheduling problem where all jobs have a common due date and have zero ready time and developed a reasonably efficient procedure for solving the unconstrained Mean Square Deviation (MSD) problem. They have proved that a special case of the MSD problem, referred to as the unconstrained MSD problem, is equivalent to the completion time variance problem (CTV) studied by Merten and Muller (1972)[18] and Schrage (1975)[23]. They have also proposed a branching procedure for the constrained MSD problem and present computational results. Osman and Potts, (1989)[41] proposed Simulated annealing as a heuristic to obtain approximate solutions and showed that Extensive computational tests with problems having up to 20 machines and 100 jobs show simulated annealing to compare favourably with known constructive heuristics and with descent methods.

Matsuo *et al*. (1989)[42] developed a new controlled search simulated annealing method for addressing the single machine weighted tardiness problem. They used the proposed method to solve optimally 99% of fifteen job problems with less than 0.2 CPU seconds, and to solve one hundred job problems as accurately as any existing methods, but with far less computational effort. They achieved this by using controlled search strategies that employ a good initial solution, a small neighborhood for local search, and acceptance probabilities of inferior solutions that are independent of the change in the objective function value.

Taillard (1990)[43] compared the previously proposed best heuristic methods to solve the flow shop sequencing problem and improved the complexity of the best one and then applied this problem to taboo search, which is a new technique to solve combinatorial optimization problems and they also presented a parallel taboo search algorithm. Daniels and Chambers (1990)[44] considered the sequencing of jobs through a multimachine flow shop, where the quality of the resulting schedule is evaluated according to the associated levels of two scheduling criteria, schedule makespan (Cmax) and maximum job tardiness (Tmax) and presented constructive procedures that quantify the trade-off between Cmax and Tmax. The significance of this trade-off is that the optimal solution for any preference function involving only Cmax and Tmax must be contained among the set of efficient schedules that comprise the trade-off curve. For the special case of two-machine flow shops, they presented an algorithm that identifies the exact set of efficient schedules. Heruistic procedures for approximating the efficient set are also provided for problems involving many jobs or larger flow shops. They also showed that computational results for the procedures which indicate that both the number of efficient schedules and the error incurred by heuristically approximating the efficient set are quite small.

Hucthison and Chang (1990)[45] evaluated the heuristic job shop scheduling algorithm which is based on finding optimal nondelay schedules (an approach that had been overlooked by past researchers). They studied the makespan criterion and solved a wide range of job shop problems using an active and nondelay algorithm. They showed that the results define the quality of optimal nondelay schedules, as well as this nondelay algorithm can handle the size of problems in a reasonable amount of time.

Ho and Chang (1991)[46] gave a new improvement heuristic for solving the flow-shop scheduling problem. This new method minimizes gaps between successive operations in solutions generated by other heuristics. They compared the new heuristic with five well-known heuristics and the random case by using simulation with makespan, mean flow time and mean utilization as the performance measures. The simulation results showed that the new heuristic improves the best heuristic in the three measures by a significant percentage, while the increase in CPU time is modest. Bagga (1992)[47] gave a new procedure for finding the lower bonds in nx3 flow shop problem. Rajendran (1992)[48] developed a branch-and-bound

algorithm and two heuristic algorithms and discussed about the results of the experimental investigation of the effectiveness of the algorithms. Davis and Kanet (1993)[49] presented and demonstrated an efficient timetabling procedure which can be embedded in an enumerative algorithm allowing the search to be conducted over the domain of job permutations.

Vancheeswaran and Townsend (1993)[50] developed a heuristic for achieving a minimum makespan schedule for the shop. The heuristic they developed considers both jobs in a given queue and those yet to arrive at any machine, to sequence the operations to be processed on all machines. It is a singlepass procedure and creates a feasible, near-minimum makespan schedule. They improved the schedule by applying a modification (improvement) stage.

Ishibuchi *et al*. (1994)[51] examined two fuzzy flowshop scheduling problems with fuzzy due dates. First, they investigated the relations between the fuzzy scheduling problems and conventional scheduling problems and applied a genetic algorithm and neighborhood search algorithms (multi-start descent, taboo search and simulated annealing) to the fuzzy flowshop scheduling problems in order to examine the ability of these algorithms to find near optimal solutions. By computer simulations, they showed that the maximization problem of the total satisfaction grade is even more tractable by these algorithms than that of the minimum satisfaction grade and then showed how the performance of these algorithms can be improved for the latter problem by utilizing problem domain knowledge. Ishibuchi *et al*. (1995)[52] proposed two simulated annealing algorithms with a modified generation mechanism and applied to the m-machine and n-job flow shop sequencing problem with the objective of minimizing the makespan. They modified the generation mechanism in order to obtain robust performance with respect to the choice of a cooling schedule. By computer simulations on randomly generated test problems, they showed that the performance of the proposed algorithms is less sensitive to the choice of a cooling schedule than that of the standard simulated annealing algorithm.

Koulamas (1998)[53] presented a simple constructive heuristic (HFC) for the flowshop makespan problem which is capable of producing non-permutation schedules when it deems it appropriate. HFC determines the order of any two jobs in the final schedule based on their order in all two-machine problems embedded in the problem. He has discussed that Computational experiments indicate that HFC performs as well as NEH which is the currently best available constructive heuristic on problems where a permutation schedule is expected to be optimal. He showed that HFC outperforms NEH on problems where a non-permutation schedule may be optimal. Maggu (2002)[54] gave a technique to minimize the total ideal time of machines or the total production time of the jobs on the two machines production scheduling problems.

Zhang and Gen, (2005)[55] expanded Flexible Job-shop Scheduling Problem from the traditional Job-shop Scheduling Problem, which possesses wider availability of machines for all the operations. They offered two definitions (total and partial) of flexibility, considering the two states of the problem to separate the different availability information of machines. They have proposed a new multistage operation-based representation to make the chromosome simpler. By using this approach, they applied all the crossover and mutation methods to this optimal strategy and they proved that the efficiency has been improved after using the new representation, and also the objective values outperformed others. While Kalczynski and Kamburowski (2006)[56] dealt with the classical problem of minimizing the makespan in a two-machine flow shop with deterministic job processing times, the optimal job sequence was determined by applying Johnson‘s rule.

Ho *et al*. (2007)[57] proposed an architecture for learning and evolving of Flexible Job-Shop schedules called LEarnable Genetic Architecture (LEGA). LEGA provides an effective integration between evolution and learning within a random search process. Unlike the canonical evolution algorithm, where random elitist selection and mutational genetics are assumed; through LEGA, the knowledge extracted from previous generation by its schemata learning module is used to influence the diversity and quality of offsprings. The architecture specifies a population

generator module that generates the initial population of schedules and also trains the schemata learning module. They have taken a large range of benchmark data from literature and generated some data and they used the data to analyze the efficacy of LEGA.

For solving job sequencing problems, a new method was proposed by Sankara in 2008[58]. A heuristic algorithm for solving general sequencing or flow shop scheduling problem was given by Laha and Chakraborty (2009)[59] for minimizing elapsed time in no-wait flow-shop scheduling. Bozejko *et al*. (2010)[60] proposed two double-level parallel metaheuristic algorithms based on the new method of the neighborhood determination. In the proposed algorithms, they included two major modules: the machine selection module refers to executed sequentially, and the operation scheduling module executed in parallel. They used a metaheuristic algorithm on each level and hence called the method meta2heuristics.

Zhang *et al*. (2011)[61] proposed an effective genetic algorithm for solving the flexible job-shop scheduling problem (FJSP) to minimize makespan time. In the proposed algorithm, they designed Global Selection (GS) and Local Selection (LS) to generate high-quality initial population in the initialization stage and used an improved chromosome representation to conveniently represent a solution of the FJSP, and adopted different strategies for crossover and mutation operator. Rao *et al.* (2013)[62] and many others gave many techniques to minimize the total ideal time of machines or the total production time of the jobs on the two machines production scheduling problems. Chaudhary (2014)[63] has presented the heuristic approach for multistage flow shop weighted Scheduling problem in the reference of Maggu (1981)[32] study.

Pinedo *et al*. (2015)[64] considered five areas of scheduling in service industries, namely (i) project scheduling, (ii) workforce scheduling, (iii) timetabling, reservations, and appointments, (iv) transportation scheduling, and (v) scheduling in entertainment. The first two areas have applications in many different service industries and the other three areas are more related to some very specific service industries, like hospitality and health care industries, the transportation industries (of passengers as well as of cargo), and the entertainment industries. The similarities and the differences between the problem formulations and solution techniques used in the various different industries and also the design of the decision support systems that have been developed for scheduling in the service industries were discussed in a detailed manner.

Chaudhary (2015)[65] has given an algorithm to minimize total waiting time for obtaining on optimal schedule of jobs. The waiting time is to be important for scheduling job on the machines. Ahmad and Khan (2015)[66] gave an algorithm for the constrained flow-shop scheduling problem in which they considered the transportation time, weight of jobs and break down time with m-machines to obtained an optimal or near optimal solution. Also, in 2016, Gupta et. al. [67] developed a new algorithm for solving Job shop sequencing problem and compared the results with that of Johnson's method. They proposed SAI-Method for solving sequencing problems. They gave an easy procedure for solving the wide range of sequencing problems which involved the minimum numbers of iterations to obtain the sequence of jobs. Anita (2017)[68] proposed Deviation Technique to solve job sequencing problems of n jobs on two machines, sequencing of n jobs on three machines and sequencing of n jobs on m machines.

Aschauer *et al*. (2018)[69] dealt with the scheduling in a hot rolling mill which can be modeled as a flexible job shop with no-wait constraints. They extended the existing solutions for this type of problems to incorporate multiple complex constraints. Al-Rawi and Mukherjee (2019)[70] presented a constructive method for solving Labor Scheduling problem encountered in a construction company, suggesting an estimated labor cost over a week and the requirement of part-time labors in each shift, using linear programming techniques, thus, providing a logical way to organize these tasks and produce a new schedule each week, by the virtue of the changing demand for service while minimizing labor cost and maximizing labor preferences. Later, Geetha (2019)[71] developed Maximin – Minimax method to solve the job scheduling problems in such a way that the total elapsed time is minimized. Radhakrishnan and Saikeerthana (2020 (a), (b))[72, 73] solved the PERT and CPM methodology using intervals and

determined the critical path and project duration of the week.

Genetic algorithm (GA) is a popular heuristic algorithm investigated for solving different scheduling problems. Kalshetty *et al*. (2020)[74] presented feasibility preserving solution representation, initialization and operators for solving job shop scheduling problem. They proposed GA and obtained best known results with good success rate for Lawrence (1985)[75] datasets which showed fast convergence towards best solution. Radhakrishnan and Saikeerthana (2021)[76] considered the sequencing problem where the processing time, due dates, weights are taken as intervals and provided an algorithm for obtaining an optimal sequence and also for determining the minimum duration taken to complete all the jobs of sequencing problems for 'n' jobs on single machine, 'n' jobs on two machines, 'n' jobs on three machines and 'n' jobs on 'm' machines.

Hoffelner and Woschank (2022)[77] presented a paper in the 5th European International Conference on Industrial Engineering and Operations Management held in Rome, Italy. They have dealt with the topic of sequence planning in flow shop production. They have first discussed production planning and control as well as the areas of application of sequence planning. They have also explained the basic models and the currently used methods in more detail. The knowledge gained from this work provides a quick overview of sequence planning and its application in flow shop production systems. In 2024, Maurya and Antony [78] developed two optimal methods for solving the sequencing problems of 2 – machines, 3 – machines and *n* – machines. Bharathi and Sundar (2024)[79] considered the real-world application based on linear programming problem and solved one of its models based on open-source software. They have also discussed some of the models such as transportation, assignment, and traveling salesman in detail with its existing algorithms and by considering the sequencing model as the main part and studied the existing algorithm to find optimal sequence and total elapsed time in detail. They incorporated the sequencing algorithm using open-source software in Microsoft Excel which helped to solve the sequencing model. Using the open-source software "solver add-in" in Microsoft Excel, the sequencing model is solved within a short span of time. The advantage of using a solver add-in helps in time consumption to solve big problems. In our present work, a new and a simple method is proposed such that job scheduling problems can be solved easily and the results are compared with Johnson's method.

Schmid *et al*. (2024)[80] introduced an efficient encoding of job shop scheduling problems, which requires much fewer bit-strings for counting all possible schedules than previously employed encodings. For problems consisting of N jobs with N operations, the number of required bit-strings is at least reduced by a factor N/log2(N) as compared to time indexed encodings. This is particularly beneficial for solving job shop scheduling problems on quantum computers, since much fewer qubits are needed to represent the problem. This approach applies to the large class of flexible and usual job-shop scheduling problems, where operations can possibly be executed on multiple machines. The encoding introduced using variational quantum algorithms lead to significantly better performance of quantum algorithms than previously considered strategies. The encoding developed enabled significantly more compact classical representations which will be highly useful even beyond applicability on quantum hardware.

Though the Operations Research community showed interest in scheduling problems within flow shop systems, the Identical Parallel Flow Shop Scheduling Problem (IPFSSP) with multiple objective functions remained underexplored. Mansouri *et al*. (2024)[81] bridged this gap by developing a scheduling strategy for an Identical Parallel Flow Shop Scheduling Problem with a bicriteria objective, focusing on Makespan and the total Earliness and Tardiness of jobs while considering Waiting time constraint. Their study employed both a priori and a posteriori methods to address the problem. They designed the integration of algorithmic components to balance the loads across flow shop lines within the proposed algorithms.

Momenikorbekanadi and Kalganova (2025)[82] reviewed the industrial applications of AI-based intelligent system algorithms in the manufacturing sector to find the latest methods used for sustainability and optimisation. In contrast to previous review

articles that broadly summarised existing methods, this review specifically emphasises the most recent techniques, providing a systematic and structured evaluation of their practical applications within the sector such as intelligent system algorithms, including metaheuristics, evolutionary algorithms, and learning-based methods within the manufacturing sector, particularly through the lens of optimisation of workflow in the production lines, specifically Job Shop Scheduling Problems (JSSPs). This work critically evaluated various algorithms for solving JSSPs, with a particular focus on Flexible Job Shop Scheduling Problems (FJSPs), a more advanced form of JSSPs. The manufacturing process consists of several intricate operations that must be meticulously planned and scheduled to be executed effectively. In this regard, Production scheduling aims to find the best possible schedule to maximise one or more performance parameters. This study focusses on the integral part of production scheduling in both traditional and smart manufacturing and is mainly concerned with the aim of maximising operational efficiency by reducing production time and costs. Khyllemkharai and Antony (2025)[83] studied the existing methods and proposed two heuristic algorithms for solving the scheduling problems considering the minimum mean values calculated from the processing times of the given machines.

The reviewed literature highlights the multifaceted nature of scheduling, encompassing various problem types, objectives, and constraints. Notably, the field has seen significant evolution, driven by both theoretical advancements and the emergence of new technologies. Another crucial aspect is the growing need to incorporate more realistic constraints into scheduling models, moving beyond simplified assumptions. Factors such as machine breakdowns, resource availability constraints, setup times, and transportation delays are now routinely considered to bridge the gap between theoretical models and practical applications. future research directions will likely involve exploring further integration of approximate and exact methods, developing more principled analysis of scheduling search procedures, and leveraging emerging technologies like the Internet of Things (IoT), generative AI, and quantum computing to unlock new levels of scheduling optimization. The focus will continue to be on designing intelligent and adaptable scheduling systems that can thrive in dynamic and unpredictable environments, ultimately enhancing operational efficiency and customer satisfaction across industries.

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