

# Transient Analysis of Delaminated Composite Plates

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**Abstract-** The present article is investigated the transient analysis of delaminated composite plate integrated with Active Fiber Composite (AFC). The deformation in both the healthy and delaminated regions is analyzed using the First Order Shear Deformation Theory (FSDT). An active velocity feedback gain ( $G_v$ ) control algorithm is adopted here to control the undesirable responses. The formulation of the governing equation is based on the minimum total potential energy approach. A parametric study on effect of boundary condition, velocity feedback gain ( $G_v$ ) and AFC patch location, in presence of small delamination (e.g 6%, 8% and 10% of total area of plate) on the laminated plate. Key observations from the parametric study are: the dynamic displacement responses are increased in presence of delamination, which are attenuated under active fiber patches and in velocity feedback control algorithm.

**Keywords:** Delamination, Active fiber composite (AFC), First order shear deformation theory (FSDT), Velocity feedback gain ( $G_v$ ).

## 1. INTRODUCTION

Composite materials are extensively used in aircraft industries, marine engineering, sports industries and automobile sectors in various applications due to its well known property such as light weight and high strength. Composite laminate usually go through the repeated cyclic stress and other external impact during service, which results delamination or debonding between plies of laminates. Delamination degrades the stiffness and strength of composite structures and simultaneously high deformation along longitudinal and transverse direction is occurred because of increased residual stress in between lamina. A smart structural model is required to compensate the undesirable response. An Active fiber composite (AFC) is kind of piezoelectric fiber which have high actuation and sensing capability in active structure. Plenty of research articles are available on laminated composite plate modelling. They can be broadly

classified into equivalent single layer (ESL) theories [1] and layerwise theory (LW) [2]. ESL theories can further be classified into classical plate theory (CPT) [3], first order shear deformation theory (FSDT) [4], higher order shear deformation theory (HSDT) [5], and zigzag theory [6]. A detailed review on the type of plate theories available and their applicability is presented by Maji and Mahato [7]. Delamination refers to a defect associated with the laminated structures. The cause of delamination may be either due to imperfect fabrication techniques (air entrapment or residual stresses), or due to fatigue loads during the service period of the structure. Due to delamination in composite structures, structural stiffness degrades. The study of delamination is mainly categorized into two approaches: (i) Region wise approach: In this approach laminate is divided into sublaminates i.e. healthy laminated region and delaminated region. The continuity condition is applied at the interface of healthy and delaminated sublaminates [8]

Several research works are available on dynamics analysis of delaminated composite plates by Parhi et al. [9]. Dynamic study of multiple delaminations in laminate is discussed by Jinho et al. [10]; author applied the Lagrange multiplier method for time integration analysis to avoid the penetration problem in delaminated region. Ganesh et al [11] developed a theory based on transient analysis and control mechanism of delaminated composite plates with hazardous environment conditions by using actuator and sensor for the study of dynamic and frequency response of plate composite. Swain et al [12] have discussed the effect of delamination characteristic of smart delaminated plate and control of flutter velocity by using a 3D degenerated element through a direct matrix abstraction program method.

In context of dynamic control of smart delaminated composite structures few researches are available. Chattopadhyay et al. [13] have discussed the

deflection of delaminated composite beam in time history. Piezoelectric sensor are used to delamination detection in beam with finite element formulation is done by Perel et al. [14]. Effect of delamination in dynamic response is carried by Ghoshal et al. [15].

In this study AFC patches are used as actuator and sensor to control the undesirable vibratory responses in time history. The Electro-mechanical formulation is based on first order shear deformation theory. The finite element analysis of AFC patches and delaminated plate is coded in MATLAB. The dynamic response is extracting by the help of Newmark’s time integration scheme. A parametric study on effect of different boundary conditions, velocity feedback gain ( $G_v$ ) and AFC patch location, in presence of small delamination on the laminated plate is studied.

2. MATHEMATICAL FORMULATION

The constitutive equations of Electro-Elastic relationship are given in Equation (1) and Equation (2). Equation (1) corresponds to the in-plane stress-strain and shear stress-strain relationship and Equation (2) corresponds to the electro-mechanical displacement due to AFC (active fiber composite) is given as following.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} - \begin{bmatrix} e_{11} & 0 & 0 \\ e_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (1)$$

$$\begin{Bmatrix} \sigma_{23} \\ \sigma_{13} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{23} \\ \varepsilon_{13} \end{Bmatrix} - \begin{bmatrix} 0 & e_{21} & 0 \\ 0 & 0 & e_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} e_{11} & e_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{43} & 0 \\ 0 & 0 & 0 & 0 & e_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \\ \varepsilon_{23} \\ \varepsilon_{13} \end{Bmatrix} + \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} \quad (2)$$

Where,  $\{\sigma_{ij}\}$  is the stress vector  $[Q_{ij}]$  is the constitutive matrix,  $\{\varepsilon_{ij}\}$  is the strain vector due to mechanical loading,  $\{D_i\}$  is the electric displacement,  $[e_{ij}]$  is the piezoelectric stress coefficient matrix,  $[\kappa]$  is the dielectric constant,  $\{E_i\}$  is the electric field vector.

If  $V$  is the electric potential difference between the two electrodes and  $h_{et}$  is the distance between two electrodes and we assuming the electric field is acting along the X-direction so the electric field vector can be written,

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} -1/h_{et} \\ 0 \\ 0 \end{Bmatrix} V \quad (3)$$

In above equations (1 - 3), the constitutive relations are in material co-ordinate systems. Each lamina may have different orientations with respect to global or structural co-ordinate system; hence the co-ordinate transformation from the material co-ordinate system to the structural co-ordinate system is required. If the lamina is oriented at an angle  $\theta$  with respect to the structural/global reference frame then the in-plane strain transformation matrix is given by,

$$\begin{Bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_{XY} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{Bmatrix} \quad (4)$$

Similarly shear strain transformation matrix is given by,

$$\begin{Bmatrix} \varepsilon_{YZ} \\ \varepsilon_{XZ} \end{Bmatrix} = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \begin{Bmatrix} \varepsilon_{23} \\ \varepsilon_{13} \end{Bmatrix} \quad (5)$$

Where  $m = \cos \theta$  and  $n = \sin \theta$

If the piezoelectric layers are oriented at angle  $\theta_p$  with X-axis then piezoelectric stress coefficient matrix  $[e]$  is transformed into  $[e]_{xy}$  and can be written as

$$[e]_{xy} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} [e] \begin{bmatrix} m & -n & 0 \\ -n & m & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

Where  $m = \cos \theta_p$  and  $n = \sin \theta_p$ .

3. Delamination modelling

Laminated plate is divided into sub-laminate i.e. delaminated and healthy regions and analyzed separately. In Figure 1, ‘abcd’ is delaminated area of laminated plate, so the laminate is also split into upper delaminated sub-laminate and lower delaminated sub-laminate.

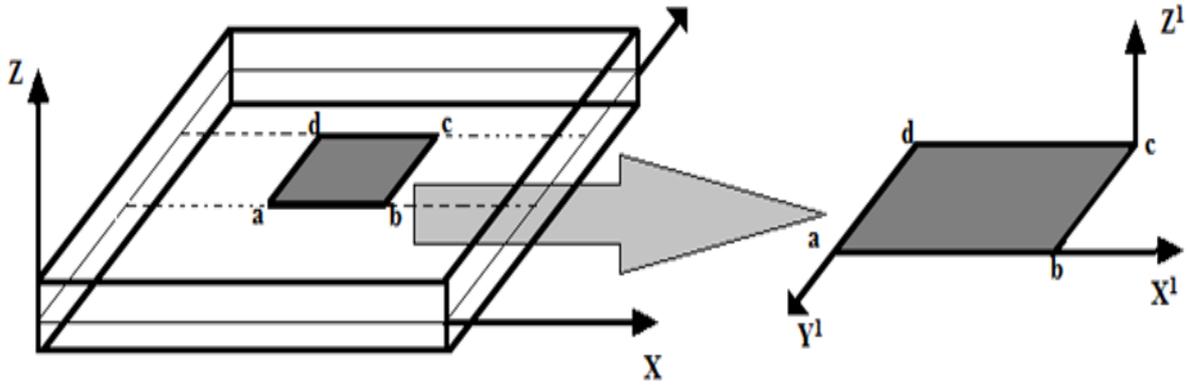


Fig. 1. Isometric view and coordinate system of delaminated plate

If a single delamination is present at the middle of the plate, then the upper and lower segment of the delaminated region is meshed separately. The coordinate system of delaminated segment is  $x^1, y^1, z^1$ . The  $z$  co-ordinate remains the same for the integral laminate as well as the delaminated sub-laminates, this way, we account for the eccentricities of the sub-laminate mid-planes with respect to the mid-plane of the integral laminate. So, according to the first order shear deformation theory, displacement field variable in the delaminated region is given below, similarly, for the sub-laminate displacement field for sub-laminate is obtained by changing the subscript.

$$\begin{aligned}
 u_B(x^1, y^1) &= u_B^0 + z\theta_{x_B} \\
 v_B(x^1, y^1) &= v_B^0 + z\theta_{y_B} \\
 w_B(x^1, y^1) &= w_B^0
 \end{aligned}
 \tag{7}$$

#### 4. RESULTS AND DISCUSSION

Numerical results are presented in this section based on finite element code developed in MATLAB. The AFC patches are integrated at the top and bottom of the plate. Electro-Mechanics is also incorporated into the finite element code developed for the AFC patches. A parametric study on the effect of velocity feedback gain ( $G_v$ ), AFC patch location, different size of small midplane delamination and various boundary conditions is carried out. The material properties of graphite/epoxy and AFC (piezoelectric) layer are listed in Table 1.

Table 1. Material properties of graphite/epoxy and AFC -50% fiber volume fraction.		
Elastic module	graphite/epoxy	AFC layer
$E_{11}$ (Gpa)	128	119.7
$E_{22}$ (Gpa)	6.12	129.1
$\mu_{12}$	0.3	0.35
$\mu_{13}$	---	0.38
$G_{12}$ (Gpa)	5.0	39.14
$G_{13}$ (Gpa)	5.0	32.35
$G_{23}$ (Gpa)	2.5	32.35
$e_{11}$ (c/m <sup>2</sup> )	---	14.14
$e_{21}$ (c/m <sup>2</sup> )	---	-3.34
$e_{24}$ (c/m <sup>2</sup> )	---	10.79
$\kappa_{11}$ (F/m)	---	$8.599 \times 10^{-9}$
$\kappa_{33}$ (F/m)	---	$6.485 \times 10^{-9}$
Density (Kg/m <sup>3</sup> )	1600	6700

At first an example is presented to validate the current approach. A rectangular cantilever plate of ply orientation  $[0/90]_{4s}$  and geometry

$0.311m \times 0.051m \times 0.00218m$  is considered. A PZT sensor is bounded at a distance of  $0.0207m$  from fixed end. The size of the piezoelectric sensor is

$0.0415m \times 0.0255m \times 0.00025m$ . A delamination of size  $0.104m \times 0.051m$  is considered at a distance of  $0.104m$  from the fixed end and located at the second interface from the mid-surface. A constant load  $10N$  is acting on free end of plate. The material properties of PZT sensor and composite laminae are given in Ref. (Ghoshal et al. 2005) The linear

deflection of smart healthy and delaminated plate in time history is obtained from current approach, and compared with the data available in article (Ghoshal et al. 2005) in which layer wise approach is used; a good correlation is obtained while comparing the results is shown in Fig.2 (healthy plate) and Fig 3 (delaminated plate).

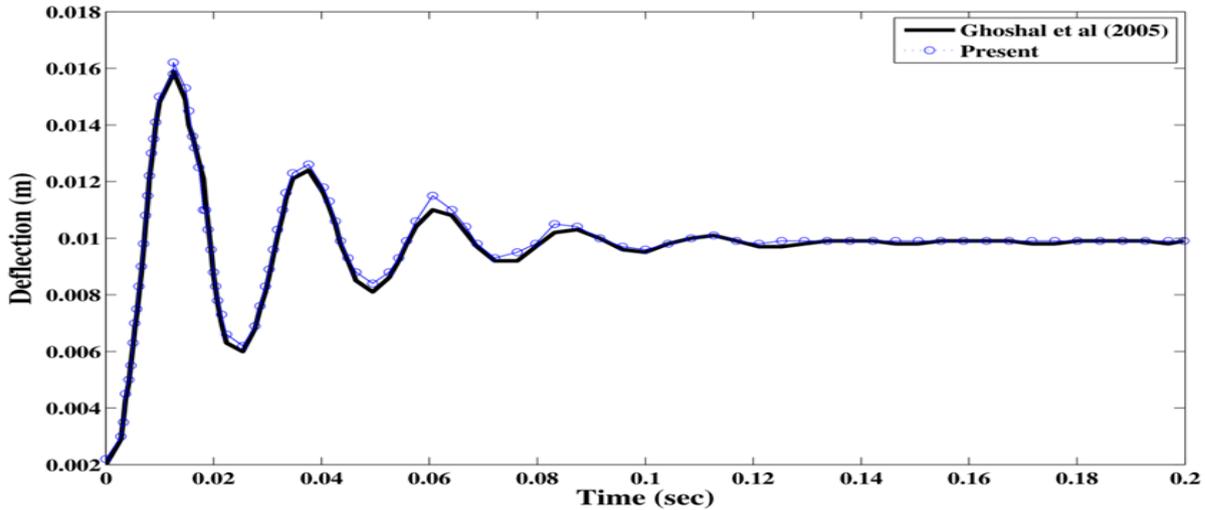


Fig.2 Linear transient response of healthy plate.

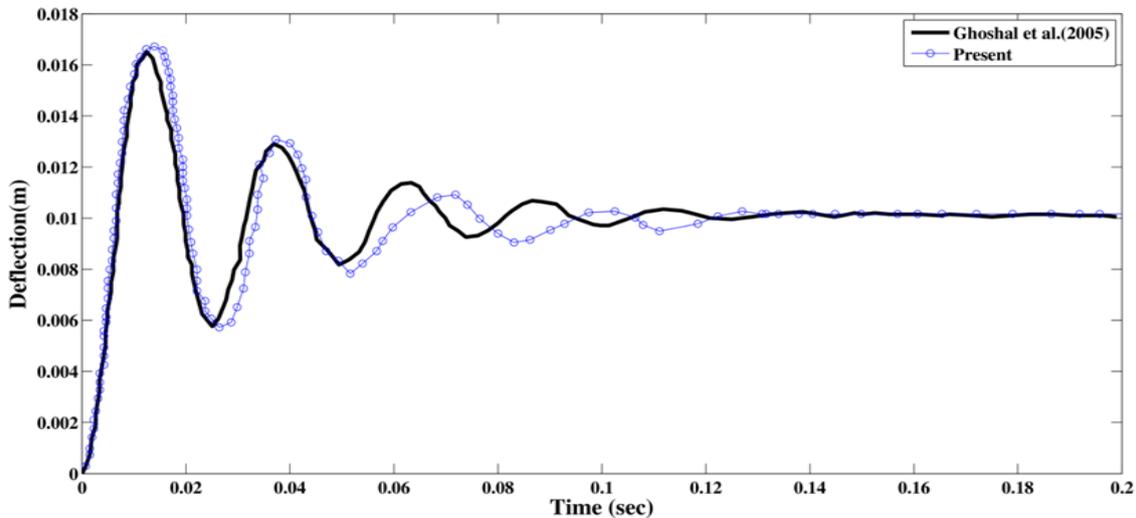


Fig.3 Linear transient response of delaminated plate.

An asymmetric laminated square plate of dimension  $0.6m \times 0.6m \times 0.006m$  and orientation of each ply is (90/0/90/0) is taken for the analysis. A square delamination of very small sizes  $0.14697m \times 0.14697m$  (6% of the total area),  $0.15874m \times 0.15874m$  (7% of the total area),  $0.16970m \times 0.16970m$  (8% of the total area),  $0.18m \times 0.18m$  (9% of the total area) and  $0.18974m \times 0.18974m$  (10% of the total area), are present in middle of the laminate. Space between two electrodes is  $0.00025m$ . The plate is divided into  $8 \times 8$  mesh

element. The analysis is carried out for three different boundary conditions i.e. all edges are simply supported (S-S-S-S), cantilever boundary condition (C-F-F-F) and clamped-clamped boundary condition (C-C-C-C), here 'C' stand for clamped edge 'F' for free edge and 'S' for simply support edge. A point load of  $1000N$  is applied at the middle of the plate (middle node) in case of S-S-S-S and C-C-C-C boundary condition and at midpoint of free end in C-F-F-F boundary condition. Figure 4 shows the geometry of the plate and delamination

location within the plate. Here 'a' and 'b' is the length and breadth of plate and 'a<sub>d</sub>' and 'b<sub>d</sub>' is the length and breadth of delaminated region.

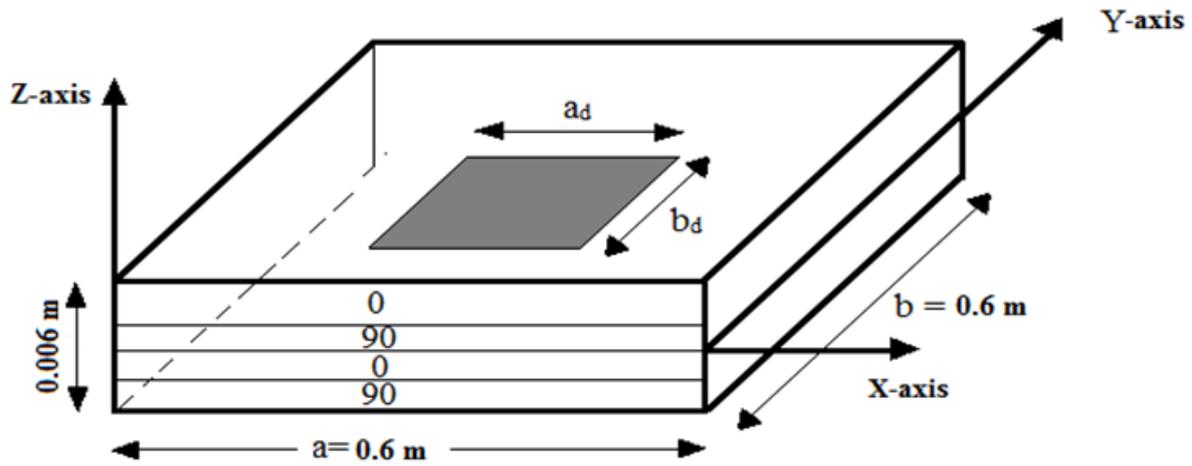


Fig. 4 Plate geometry and delamination location

Figure 3 shows the AFC patches position at different location on both side of plate. Here, two different location of patches position is discussed for the dynamics analysis of delaminated composite plate. Dimension of AFC patches which is shown in Figure 5 (a), 5 (b), are discussed in Table 2. Figure 5(a)

shows the AFC patch position 1 which is away from the delaminated region and Figure 5(b) shows the AFC patch position 2 which is likely same dimension as the size of delamination details are given in Table 2.

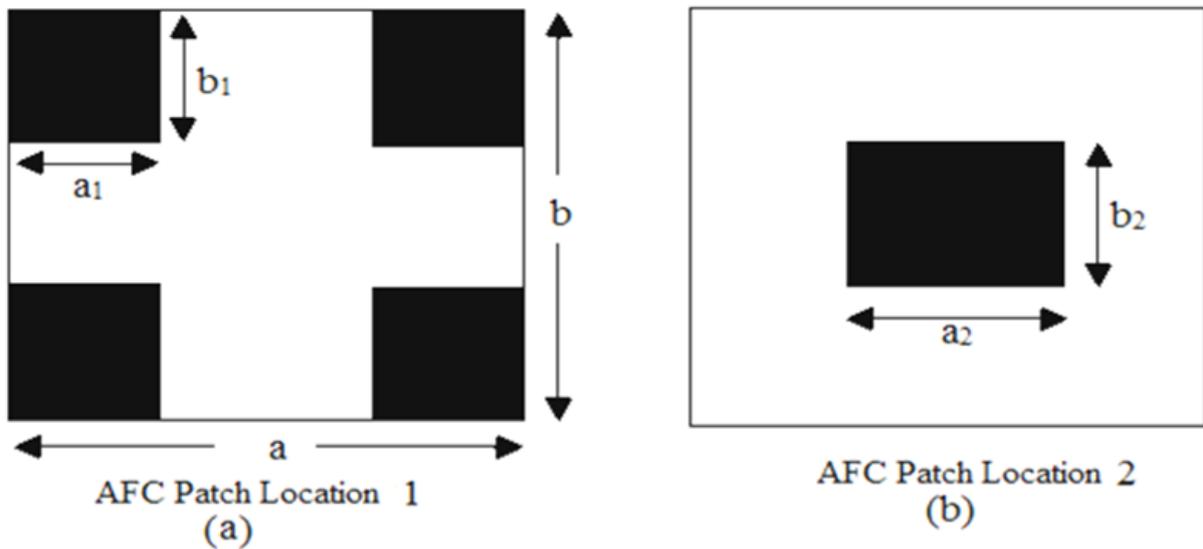


Fig. 5 AFC patches location at different places on plate

Table 2. Dimensions of delaminated region and AFC patches.			
Delamination along midplane (% of total area)	Size of delamination a <sub>d</sub> × b <sub>d</sub> (m) Refer figure 2	AFC patch location 1 a <sub>1</sub> × b <sub>1</sub> (m) Refer figure 3(a)	AFC patch location 2 a <sub>2</sub> × b <sub>2</sub> (m) Refer figure 3(b)
6% delamination	0.14697 × 0.14697	0.2265 × 0.2265	0.14697 × 0.14697
7% delamination	0.15874 × 0.15874	0.2206 × 0.2206	0.15874 × 0.15874
8% delamination	0.16970 × 0.16970	0.2151 × 0.2151	0.16970 × 0.16970
9% delamination	0.18 × 0.18	0.21 × 0.21	0.18 × 0.18
10% delamination	0.18974 × 0.18974	0.2051 × 0.2051	0.18974 × 0.18974

## CONCLUSION

A matlab based finite element code is developed for the delaminated composite plates including piezoelectric (AFC) patches as actuators/sensors and feedback control loop. The numerical analysis of delaminated plate including control analysis is carried out in S-S-S-S, C-F-F-F and C-C-C-C boundary conditions. The transverse displacements have increased in delaminated plate which is negated by activated AFC patches. The dynamics response is reduces in time domain with varying  $G_v$ . Some extensive conclusions are as following.

- Dynamic Amplitude of plate increases under increasing size of delamination due to decrease in stiffness of plate.
- In C-F-F-F boundary condition the dynamic displacement response is not much affect under delamination.
- The effective damping is induced due to velocity feedback loop, which attenuate the dynamics response. As the velocity feedback gain is increases the response come to static state.

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