A Comprehensive Review of Deterministic and Probabilistic Approaches for Retaining Wall Analysis

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Abstract—**The stability assessment of retaining walls has traditionally relied on deterministic approaches, where soil and structural parameters are considered fixed and safety factors are employed to account for uncertainties. While these methods offer simplicity and ease of application, they often fail to capture the inherent variability in geotechnical and seismic parameters, leading to either over-conservative or under conservative designs. To address this limitation, probabilistic approaches have emerged as a powerful alternative, enabling the explicit treatment of uncertainties associated with soil properties, seismic loading, and structural performance. This review provides a comprehensive synthesis of deterministic and probabilistic methods applied to retaining wall analysis, with a focus on seismic conditions and reliability-based design. Deterministic frameworks, including limit equilibrium and numerical techniques such as finite element analysis, are discussed in relation to their applicability and limitations. In contrast, probabilistic approaches—including Monte Carlo simulation, First-Order and Second-Order Reliability Methods (FORM/SORM), and hybrid machine learning-based techniques such as ANFIS coupled with metaheuristic optimization—are highlighted for their capacity to quantify failure probabilities and improve risk-informed decision making. The review also identifies current research trends, such as the integration of artificial intelligence in probabilistic analysis, and emphasizes the importance of balancing computational efficiency with accuracy. Finally, critical knowledge gaps and potential directions for future research are outlined, particularly in the context of developing practical design frameworks for earthquake-resistant retaining walls. This work aims to serve as a valuable resource for researchers and practitioners seeking a deeper understanding of deterministic and probabilistic approaches in retaining wall engineering.**

Index Terms—**Retaining wall, Deterministic Analysis, Probabilistic Analysis, Monte Carlo simulation, Reliability-based design, Seismic reliability, Machine learning (ANFIS / metaheuristic optimization).**

I. INTRODUCTION

Retaining walls are essential geotechnical structures designed to resist lateral pressures exerted by soil, surcharge loads, and, in many cases, seismic forces. They are widely used in infrastructure such as highway embankments, basement excavations, waterfront developments, and hilly terrains where maintaining the stability of soil masses is critical. The performance and safety of these structures have always been of prime concern to engineers, particularly in seismically active regions where lateral loads increase significantly during earthquakes. Over the years, methods for analyzing and designing retaining walls have advanced considerably, moving from simplified classical solutions to modern probabilistic and artificial intelligence-based frameworks. This evolution reflects the growing recognition that soil parameters and loading conditions are highly uncertain, and that design methods must adequately capture this variability to ensure rational safety.

Historically, the design of retaining walls has been dominated by deterministic methods, in which soil strength parameters, surcharge loads, and seismic coefficients are treated as fixed values. Classical theories by Coulomb (1776) and Rankine (1857) provided the basis for active and passive earth pressure determination, and their formulations are still widely applied in modern codes due to their simplicity and practicality. Later developments, such as the Mononobe–Okabe method introduced in the early 20th century, extended these approaches to account for seismic conditions by incorporating pseudo-static accelerations into the pressure calculations. Deterministic refinements, including the work of Seed and Whitman (1970), provided valuable experimental validation and guidelines for seismic coefficients. More recently, numerical methods such as finite element analysis have enabled the simulation of nonlinear soil–wall interaction, as demonstrated by Cai and Bathurst (1995). Their finite element study of geosynthetic-reinforced retaining walls under seismic loading showed that dynamic wall behavior can be realistically captured, addressing the shortcomings of traditional analytical methods. offering more realistic predictions of wall performance under static and dynamic conditions. While deterministic approaches have proven effective for preliminary design and remain indispensable for routine engineering, they are inherently limited by their assumption of certainty in material properties and loading conditions.

The inherent variability of geotechnical parameters has motivated the adoption of probabilistic methods. Soils are natural materials with spatial variability, and seismic loading is unpredictable in both magnitude and frequency. Deterministic designs based solely on safety factors may appear adequate, yet they often fail to provide consistent reliability across projects. Probabilistic approaches explicitly model soil properties and loads as random variables with probability distributions, enabling the evaluation of the likelihood of failure rather than relying on a single margin of safety. Studies such as those by Basha and Babu (2011) and Zevgolis (2010) demonstrated that walls designed using conventional methods can exhibit widely varying probabilities of failure once uncertainty is considered. Similarly, probabilistic tools such as Monte Carlo Simulation (MCS), First-Order Reliability Method (FORM), and random field models have been applied to retaining wall analysis, allowing engineers to quantify reliability indices and capture the influence of soil variability more realistically. These frameworks mark a significant shift from factor-of-safety based design to reliability-based design, providing a more rational measure of performance and risk.

Although probabilistic methods are theoretically robust, their practical application has been constrained by the high computational effort required for large-scale analyses and by the difficulty of obtaining sufficient statistical data on soil properties. This has led to the growing interest in artificial intelligence (AI) and hybrid approaches, which combine probabilistic concepts with machine learning to achieve greater efficiency. AI models such as Artificial Neural Networks (ANN), Adaptive Neuro-Fuzzy Inference Systems (ANFIS), and Support Vector Machines (SVM) have been successfully applied to approximate complex soil–structure interaction functions. For instance, Mishra, Samui, and Mahmoudi (2021) showed that machine learning algorithms can provide faster and more accurate reliability predictions compared to conventional simulations. Hybrid frameworks, where AI models are coupled with optimization techniques like Particle Swarm Optimization (PSO) or Genetic Algorithms (GA), have further improved efficiency and accuracy. The work Mustafa, Samui & Kumari (2022) highlighted the potential of such models for seismic reliability assessment of retaining walls, demonstrating that they can achieve realistic predictions with reduced computational cost. They evaluated four models—ANFIS–PSO, ANFIS–GA, ANFIS–FFA, and ANFIS–GWO—comparing their predictive performance and reliability metrics. The results highlighted that ANFIS–PSO offered the highest reliability index and lowest failure probability, validating hybrid AI–probabilistic approaches for practical retaining wall analysis.

Despite these advancements, challenges remain. Probabilistic and AI-based methods require reliable statistical data, yet site investigations are often limited by cost and time constraints. Additionally, while deterministic approaches are embedded in design codes and widely accepted by practitioners, probabilistic and AI-based frameworks have not yet achieved the same level of standardization. This creates a gap between academic research and practical implementation. Nevertheless, with the increasing availability of computational resources and the expansion of geotechnical databases, these methods are gaining attention and are likely to play an important role in future design codes.

The present review aims to synthesize the developments in both deterministic and probabilistic frameworks for retaining wall analysis, with special emphasis on seismic performance. Deterministic methods are revisited to highlight their fundamental assumptions and limitations, while probabilistic approaches are critically examined to demonstrate their ability to quantify risk and provide reliability-based design solutions. Finally, the potential of AI-based hybrid models is discussed as a means to enhance computational efficiency and integrate uncertainty into routine practice. By bringing together these perspectives, this review seeks to contribute to the ongoing discourse on how retaining wall design can evolve from traditional deterministic models toward more robust, reliability-oriented frameworks.

II. AIMS AND OBJECTIVES

The present review article has been undertaken with the following objectives:

1. To critically review the deterministic approaches for retaining wall analysis, highlighting their assumptions, strengths, and limitations in both static and seismic conditions.
2. To examine probabilistic frameworks that incorporate soil and seismic uncertainties, and to discuss their effectiveness in reliability-based design of retaining walls.
3. To explore the role of Artificial Intelligence (AI) and hybrid approaches in enhancing efficiency and accuracy of reliability analysis for retaining walls.
4. To identify research gaps and future directions for developing integrated deterministic–probabilistic–AI frameworks for practical and reliable retaining wall design

III. LITERATURE REVIEW

3.1 Deterministic Approaches

Rankine (1857) proposed a stress-based theory to calculate active and passive earth pressures, eliminating the need to assume a failure surface. While his method simplified the analysis and is still widely taught and used in design codes, it neglected wall friction and the effect of wall movement type, often leading to conservative estimates.

Mononobe (1929) and Okabe (1926) extended the classical Coulomb theory by incorporating pseudo-static seismic forces. The Mononobe–Okabe method provided one of the first rational frameworks for seismic earth pressure calculation. Despite its assumptions of uniform acceleration and neglect of soil–structure interaction, it continues to serve as the basis for seismic retaining wall design in many codes worldwide.

Seed and Whitman (1970) critically examined pseudo-static seismic coefficients and emphasized the role of soil and wall characteristics in defining realistic design loads. Their study included experimental insights, making it one of the most influential works on seismic design of retaining walls.

Richards and Elms (1979) investigated the seismic behavior of gravity retaining walls using analytical and experimental methods. They highlighted that wall inertia, not just soil pressures, plays a critical role in seismic performance. This work significantly advanced the understanding of dynamic soil–wall interaction.

Choudhury and Subba Rao (2005) refined seismic passive earth pressure calculations by modifying the Mononobe–Okabe approach. They proposed expressions that considered both cohesion and soil friction under seismic conditions. Their contribution improved the accuracy of seismic design for retaining walls in cohesive backfills.

3.2 Probabilistic Approaches

Ang and Tang (1975, 1984) pioneered reliability-based engineering by formalizing the concepts of reliability index and probability of failure. Their framework provided the theoretical foundation for incorporating uncertainty into structural and geotechnical analysis.

Phoon and Kulhawy (1999) compiled a landmark database on geotechnical property variability, covering parameters such as shear strength, stiffness, and unit weight. Their work provided practical probability distributions that engineers could adopt in reliability-based retaining wall design.

Fenton and Griffiths (2008) advanced probabilistic geotechnics by applying random field theory to model spatial variability of soils. Their methods allowed engineers to go beyond simple random variable models, making probabilistic analyses of retaining walls more realistic and powerful.

Zevgolis (2010) employed Monte Carlo Simulation (MCS) to analyze cantilever walls under variable soil conditions. His findings demonstrated that small changes in soil strength parameters could cause large variations in probability of failure, underscoring the importance of probabilistic design.

Basha and Babu (2011) introduced a reliability-based framework for earthquake-resistant design of reinforced soil retaining structures. Their study incorporated uncertainties in seismic loading and soil parameters, showing that deterministic design approaches may either underestimate or overestimate safety. This work emphasized the role of reliability analysis in achieving balanced and cost-effective designs.

Javankhoshdel and Bathurst (2017) performed both deterministic and probabilistic analyses of geosynthetic-reinforced soil slopes using Monte Carlo simulations. Their study illustrated the impact of various failure mechanisms and highlighted the significance of parameter variability in slope reliability.

Li, Zhang & Sheng (2025) conducted a probabilistic stability study on axes of non-linear soil backfills for retaining structures. Using probabilistic optimization, they identified optimal wall configurations that balance safety and cost effectively, building a compelling case for reliability-based design in realistic soil scenarios

3.3 AI-Based and Hybrid Approaches

Goh & Kulhawy (2003) demonstrated the application of neural networks for modeling limit-state surfaces in reliability analysis. Their approach offered a practical surrogate modeling strategy for complex geotechnical systems and laid the groundwork for AI-based reliability methods.

Shahin (2005) reviewed the application of Artificial Neural Networks (ANNs) in geotechnical engineering, highlighting their ability to approximate nonlinear soil–structure relationships. He argued that ANNs can act as surrogate models for probabilistic simulations, thereby reducing computational effort.

Mishra, Samui & Mahmoudi (2021) presented a machine-learning-based probabilistic design for cantilever retaining walls. Using AI models such as Emotional Neural Networks, Multivariate Adaptive Regression Splines, and SOS–LSSVM coupled with first-order reliability calculations, they demonstrated improved predictive accuracy and computational efficiency for reliability evaluation.

Mustafa, Samui & Kumari (2022) conducted a reliability analysis of gravity retaining walls using hybrid ANFIS models optimized by metaheuristics. They evaluated four models—ANFIS–PSO, ANFIS–GA, ANFIS–FFA, and ANFIS–GWO—comparing their predictive performance and reliability metrics. The results highlighted that ANFIS–PSO offered the highest reliability index and lowest failure probability, validating hybrid AI–probabilistic approaches for practical retaining wall analysis.

IV. CONCLUSIONS

This review was carried out with the objective of examining the different approaches adopted for the analysis of retaining walls, with particular attention to deterministic, probabilistic, and AI-based methods. The findings reaffirm the importance of traditional frameworks while emphasizing the growing need for advanced techniques that account for uncertainty in soil and seismic conditions.

Deterministic methods, as established by Coulomb, Rankine, and later extended by Mononobe–Okabe, remain the most widely used due to their simplicity and acceptance in design codes. These approaches provide clear formulations for evaluating earth pressures and are essential for preliminary design. However, their assumption of fixed soil and load parameters is a limitation, as it cannot capture the natural variability encountered in practice. Thus, while they remain practical tools, deterministic frameworks alone cannot ensure uniform reliability.

Probabilistic approaches were reviewed to address this shortcoming. By representing soil properties and seismic loads as random variables, they allow the probability of failure and reliability indices to be calculated. Studies consistently show that such methods provide a more rational measure of safety compared to conventional factors of safety. The main challenge, however, lies in the availability of statistical data and the computational intensity of methods such as Monte Carlo simulations. Despite these hurdles, probabilistic analysis offers valuable insights into the sensitivity of retaining walls to parameter variability and therefore contributes significantly to reliability-based design.

The review also explored AI-based and hybrid approaches, which have emerged as promising tools for overcoming computational challenges. Neural networks, support vector machines, and neuro-fuzzy systems, often integrated with optimization algorithms, have demonstrated the ability to approximate complex soil–structure interactions with greater efficiency. These models can complement probabilistic frameworks, enabling faster and more accurate reliability predictions, thereby making advanced design methods more practical.

Finally, the study identified gaps and future directions. A major requirement is the development of integrated frameworks that combine the practicality of deterministic methods, the rigor of probabilistic concepts, and the efficiency of AI-based models. Equally important is the need for richer geotechnical databases and standardized guidelines to facilitate the wider adoption of reliability-based retaining wall design in engineering practice.

In summary, the review concludes that while deterministic methods will continue to be indispensable, the integration of probabilistic and AI-based approaches represents the way forward for achieving safer, more resilient, and cost-effective retaining wall designs.

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