Buckling Mode Shape Analysis of Symmetrically Laminated Rectangular Plates with General Boundary Conditions

Chandrashekharaiah S B [1], Mamatha A [2], Ananda C B [3]
[1] *Senior Scale Lecturer, Department of Machine Tool Technology, SMT L V Government, Polytechnic, Hassan, Karnataka, India*
[2,3] *Senior Scale Lecturer, Department of Mechanical Engineering, Government, Polytechnic, Channasandra, Karnataka, India*

Abstract- The purpose of this paper is to offer a semi-analytical extended Kantorovich technique for the buckling analysis of symmetrically laminated rectangular plates with general boundary conditions. The solution is generated as a multi-function expansion that enables the analysis of laminated plates that are characterized by a solution that cannot be separated. Shear buckling of any form of plate and buckling of angle-ply laminates under inplane compression are the two types of buckling that occur the most often among these. Both the iterative extended Kantorovich approach and the variational principle of total energy reduction serve as the foundation for the formulation. In order to solve the differential eigenvalue issue that was produced as a consequence, the exact element technique was chosen as the approach to use. The numerical demonstration of the capabilities of the suggested methodology and its application to the buckling analysis of composite laminated plates, which cannot be examined using the traditional single-term extended Kantorovich method, is shown here. There is a comparison made between the findings and precise answers (in cases where they are available) as well as approximate results from other numerical approaches. Discussion is also held over the correctness and convergence of the technique that has been suggested.

Key words: Buckling analysis, Boundary conditions, Rectangular plates, Mode shape.

1. INTRODUCTION

Composite materials find extensive application across various contemporary engineering fields. The mechanical behavior of rectangular laminated plates has garnered significant interest among the various aspects of structural performance in composite material structures.

Specifically, examining the buckling phenomena in these plates is crucial for effective and dependable design, as well as for ensuring the safe application of the structural element. The analysis of composite laminated plates is typically more complex than that of homogeneous isotropic plates, owing to the anisotropic and coupled behavior of the materials involved.

Specifically, exact closed-form solutions for the buckling problem of rectangular composite plates exist only for a restricted set of boundary conditions and lamination configurations. The study encompasses cross-ply symmetric and angle-ply antisymmetric rectangular laminates featuring at least two opposite edges that are simply supported. It also includes similar plates where two opposite edges are clamped yet allowed to deflect (guided clamp), or configurations where one edge is simply supported while the opposite edge is equipped with a guided clamp. The majority of the precise solutions are examined in the works of Whitney [1] and Reddy [2]. For all other configurations, where only approximate results are accessible, various semi-analytical and numerical techniques have been established. The Rayleigh–Ritz method [3], the finite strip method (FSM) [4,5], the element-free Galerkin method (EFG) [6], the differential quadrature technique [7], and the widely utilized finite element method (FEM) [8] represent the most prevalent approaches.

The Kantorovich method [9] is a distinct and often beneficial semi-analytical technique that integrates a variational approach, closed-form solutions, and an iterative procedure. The approach posits a solution represented as a summation of products of functions oriented in one direction alongside functions oriented in the opposite direction. By assuming the functions in one direction, the variational problem of the plate simplifies to a set of ordinary differential equations.

In the context of buckling analysis, the variational problem simplifies to a standard differential eigenvalue and eigenfunction problem. The outcome of the resulting issue is an approximate one, with its precision reliant on the functions assumed in the initial direction. The extended Kantorovich method (EKM) introduced by Kerr [10] utilizes this approximation as the foundation for an iterative process, where the solution derived in one direction serves as the assumed functions in the subsequent direction. Upon executing this procedure multiple times, convergence is achieved. A mathematical proof demonstrating the convergence of the extended Kantorovich method for free vibration problems of rectangular thin plates is presented by Chang et al. [11], ensuring that the accuracy of the solution is manageable. The capability to transform the plate problem, which inherently presents as a partial differential equation, into the resolution of ordinary differential equations stands out as one of the key benefits of the extended Kantorovich method.

This method also has the advantage of not relying on the initial guess. The iterative procedure may result in a suboptimal initial guess, which could fail to meet any of the boundary conditions. The extended Kantorovich method was utilized for a buckling analysis of rectangular plates by various investigators. Eisenberger and Alexandrov [12] employed the method for conducting a buckling analysis of isotropic plates that exhibit variable thickness. Shufrin and Eisenberger [13,14] expanded the solution to thick plates with both constant and variable thickness by employing the first and higher order shear deformation theories. Ungbhakorn and Singhatanadgid [15] expanded the solution to the buckling of symmetrically cross-ply laminated rectangular plates. Throughout these investigations, a singular term was employed in the proposed solution. Conversely, the single-term formulation does not adequately address several key issues related to the buckling of rectangular plates. For example, the diagonal symmetric and antisymmetric buckling modes cannot be represented by the single-term separation. As a result, the buckling analysis of rectangular isotropic plates subjected to in-plane shear loading, as well as the buckling analysis of anisotropic plates under various load conditions, exceeds the limitations of the single-term extended Kantorovich method.

The multi-term formulation of the extended Kantorovich approach to the simplest examples of rectangular isotropic plates was introduced by Yuan and Jin [16]. This study demonstrated that the supplementary terms in the expansion can be utilized to enhance the solution. Furthermore, it was shown that a singular term solution does not exist for the shear buckling of rectangular plates. In mathematical terms, the standard eigenvalue problem arising from the single-term formulation of the shear buckling analysis produces solely complex solutions.

The stability analysis of generally laminated rectangular composite plates subjected to various combinations of normal and shear loads is defined by an interconnected solution.

Consequently, the single-term solution [15] is inadequate or yields a suboptimal approximation. The coupled material behavior of the laminated plate necessitates a more comprehensive description for the unknown functions. This description can be attained via the multi-term expansion.

This paper presents a solution strategy utilizing a multi-function expansion of the extended Kantorovich method. The multi-term approach is utilized to address the shortcomings of the classical extended Kantorovich method, which is inadequate for rectangular plates with diverse lay-up configurations, arbitrary boundary conditions, and various combinations of external loading. The governing equations and boundary conditions are established based on the principle of minimizing potential energy.

The resulting ordinary differential eigenvalue problem is addressed using the exact element method for stability analysis [17], which is applied here to the multi-function formulation. The method's versatility and capabilities are analyzed through a numerical study of various rectangular laminated plates, considering different combinations of in-plane loads and boundary conditions. The convergence of the solution is examined, and the findings are juxtaposed with the exact solution (where relevant) as well as with numerical solutions documented in existing literature.

2. MODE SHAPES

In this study, the finite element method is used to guess the bending loads and ways that bonded rectangle plates will move. Figure 1 depicts the dimensionless buckling loads and modes for unidirectional Rectangular CFCF plates when subjected to a variety of different combinations of axial compression. According to the comparison between Fig. 1a (in which the plate is uniaxially loaded in the fiber direction) and Fig. 1b (in which the compression is perpendicular to the fiber direction) illustrates the differences attributed to the orthotropic performance of the plate. Fig. 1c further demonstrates that the buckling mode of the orthotropic plate is not symmetric when it is subjected to symmetric compression in both directions. The stiffening impact of the tensile stresses on the material is shown in Figure 1d, which demonstrates that the buckling load is significantly enhanced when tensile loads are applied to a single direction.

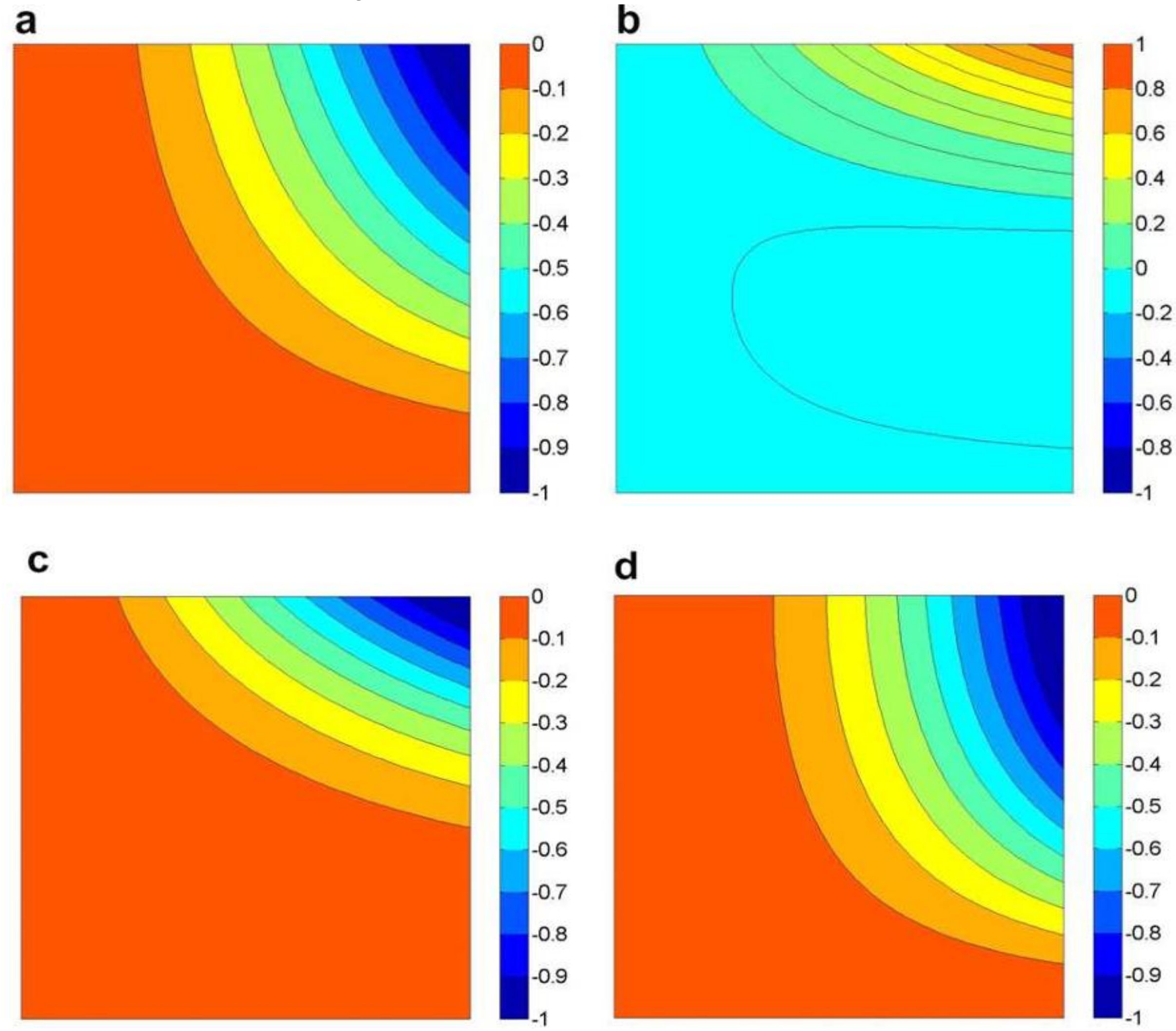


Fig.1. Buckling modes and load parameters.

The buckling modes for single-ply self-supporting composite plates As seen in Figure 2, the modes undergo considerable variations as a result of the alterations in the plying orientation. The buckled condition is h=0. two halfwaves in the longitudinal direction and one halfwave in the transverse direction are characteristic of mode. As the value of h increases to 30, the number of longitudinal halfwaves decreases to three, but the normalized buckling load decreases. The nodal lines are not parallel to the margins of the plate, as seen in Figure 2b of the previous figure. It is not possible to notice this impact by utilizing a single word expansion. In the case when h equals sixty, the number of halfwaves is raised even higher, yet at the same time, the normalized buckling load remains unchanged. There is a correlation between this behaviour and the increase in the short direction of the plate.

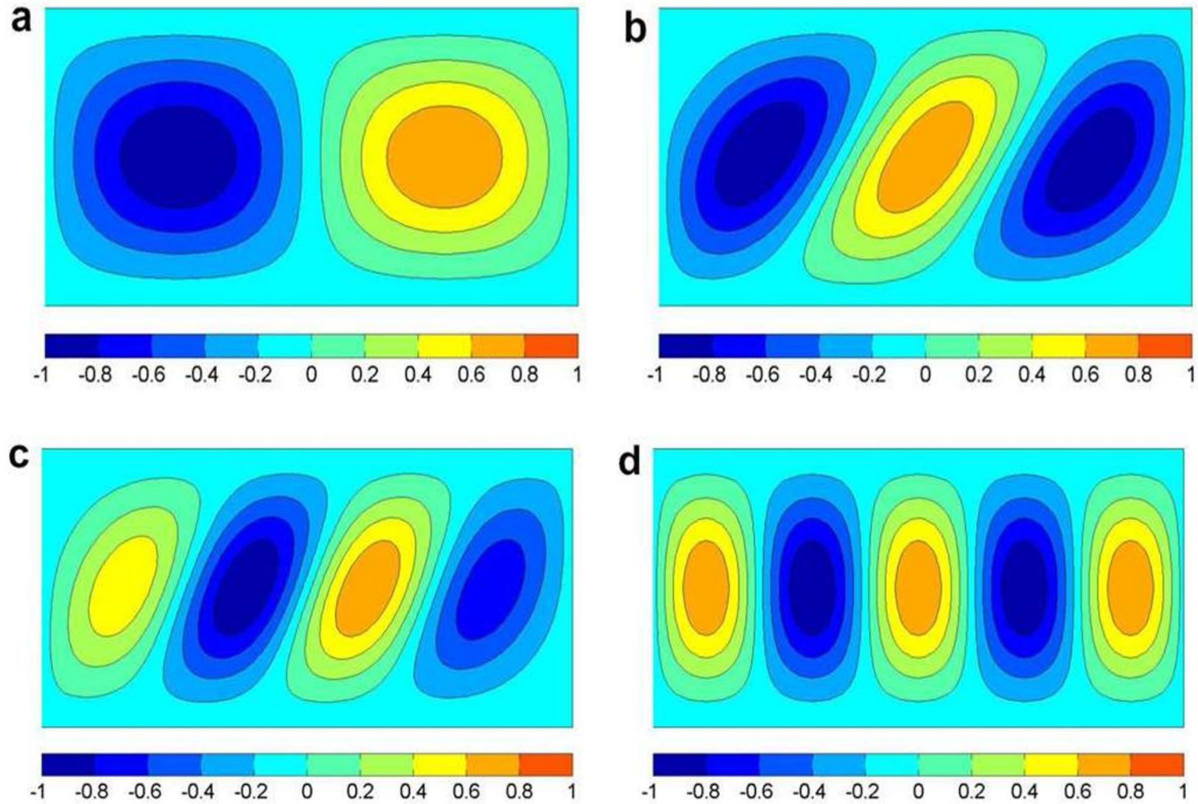
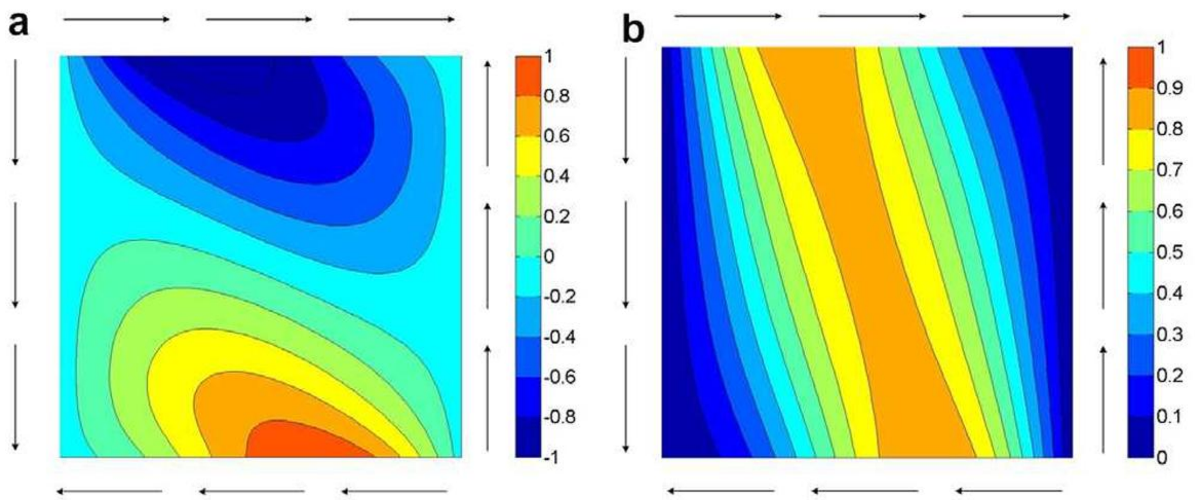


Fig.2. Buckling modes and load parameters

Figure 3 is an illustration of the shear buckling modes for angle-ply square SSFF plates. The comparison between Fig. 3a and Fig. 3b, in which the plying direction has been altered from 0 to 90 degrees, demonstrates that the modes and shear buckling loads have changed significantly as a result of the non-symmetric boundary conditions. Plates with the identical fiber orientations (h = 30) exhibit radically distinct buckling modes and critical loads when subjected to reverse shear loading, as illustrated in Figure 4c and d. This is another tendency that may be seen. It is clear from the comparison between examples (e) and (f) (h = 60) that the direction of the shear load has a substantial influence on the buckling load and the buckling mode. This is shown by the fact that the behaviour of both cases is comparable. In spite of the fact that these effects are well defined by the model that was created here, the physical technique is not capable of detecting them.



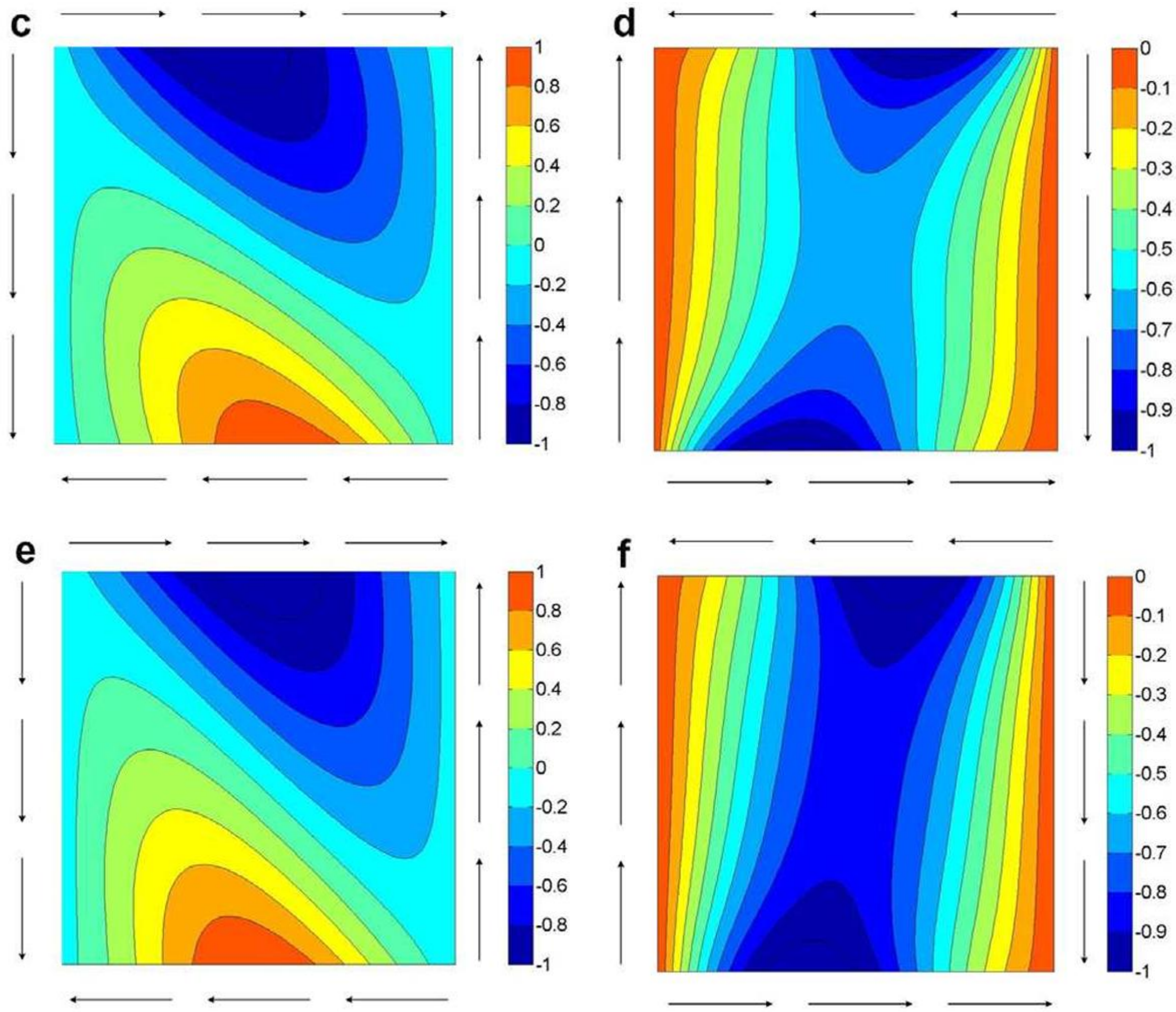


Fig. 3. Buckling modes and load parameters.

3. NUMERICAL EXAMPLES AND DISCUSSION

Buckling loads under various BC's

Using a six-term expansion, the calculations have been carried out based on the outcomes of the prior examples. Table 5 presents the results of an investigation into the normalized buckling load for square plates under a variety of in-plane compression combinations. By using equation, the dimensionless buckling load factors are defined. Plates having guiding edges, clamped edges, free edges, and simply supported edges are being examined. The findings indicate that the buckling loads decrease as the angle of plying orientation increases. Furthermore, in the circumstances of uniaxial compression, laminates with the same fiber orientation have the lowest buckling load.

Table5: Buckling load parameters for symmetric angle- ply laminated plates under various combinations of in-plane compression.

Boundary conditions	Laminated scheme	Buckling parameter		
		λ_x	$\lambda_x=\lambda_y$	λ_y
CCFF	$[30/0]_s$	15.366	8.936	10.971
	[30/−30$_s$	14.828	10.713	13.011
	$[30/30]_s$	12.872	8.084	10.304
	$[45/0]_s$	9.594	8.484	11.479
	$[45/-45]_s$	7.661	9.106	14.249
	$[45/45]_s$	6.536	6.412	10.399
CGSS	$[30/0]_s$	12.362	4.354	5.639
	$[30/-30]_s$	14.009	4.896	6.336
	$[30/30]_s$	12.024	4.299	5.577
	$[45/0]_s$	12.385	4.754	6.210
	$[45/-45]_s$	14.696	5.758	7.458
	$[45/45]_s$	11.141	4.666	6.133
SFSF	$[30/0]_s$	0.748	0.528	1.018
	$[30/-30]_s$	1.069	0.754	1.425
	$[30/30]_s$	0.679	0.464	0.862
	$[45/0]_s$	0.799	0.587	1.197
	$[45/-45]_s$	1.253	0.905	1.772
	$[45/45]_s$	0.764	0.546	1.081

4. CONCLUSIONS

For the purpose of conducting a buckling analysis of symmetrically laminated rectangular plates with generic boundary conditions, a semi-analytical technique has been devised. Using the multi-term extended Kantorovich approach, the solution of the

partial differential buckling equations has been reduced to a solution of a set of ordinary differential equations. This was accomplished by reducing the scope of the equations. By using a multi-term expansion, one is able to circumvent the constraints that are associated with the traditional extended Kantorovich technique. This, in turn, makes it possible to analyze a far larger variety of buckling issues.
In particular, it may be used for the buckling analysis of composite laminated plates, which, in many instances, include a non-separable solution. This is because the laminated construction of the plates is laminated. By solving a number of laminated plate buckling problems, the approach's applicability, accuracy, and convergence characteristics have been shown numerically. This was accomplished by the use of the method. It is possible to further expand the semi-analytical technique that has been presented here for symmetrically laminated plates in order to perform buckling analysis on plates that have non-uniform characteristics, either in the planer directions or in the plate thickness direction. As a result, the basic technique that is described in this article serves as the foundation for the semi-analytical solution of a wide variety of structural issues.

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