Performance Evaluation of Smc Strategies Along with Dob for Third-Order Uncertain Plants

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***Abstract*—This study proposes a systematic design and rigorous analysis of a Disturbance Observer (DOB) tailored for second and third-order interval plants. The developed framework focuses on real-time estimation and active compensation of external disturbances and inherent model uncertainties, ensuring robust performance across a range of system variations. The DOB was constructed using a nominal third-order model while accounting for parameter variations within defined intervals. By feeding the estimated disturbances back into the system, the method significantly enhances the robustness and stability even under substantial external disruptions. A conventional PID controller is designed in conjunction with the DOB to regulate plant dynamics, thereby improving performance under both nominal and disturbed conditions. To validate the efficacy of the proposed control strategy, comprehensive MATLAB simulations were performed under diverse disturbance conditions. These included standard profiles such as step, sinusoidal, square, sawtooth, and stair-generator signals. The simulation outcomes demonstrate the robustness and adaptability of the controller in accurately compensating for a wide spectrum of dynamic disturbances. Furthermore, a comprehensive comparative analysis was conducted involving four advanced control methodologies: PID control, SMC, Modified-SMC, and PID-Sur-SMC. These strategies were systematically evaluated on both second and third-order interval systems, as well as DC motor models, to assess their performance in terms of disturbance rejection, tracking fidelity, and robustness under parametric uncertainties. Stability analyses of the linear and nonlinear components are performed using Lyapunov's direct method, whereas the Kharitonov polynomial approach is employed to guarantee robust stabilization across interval uncertainties. The findings of this study reveal that integrating the proposed Disturbance Observer (DOB) with PID control and advanced Sliding Mode Control (SMC) variants significantly enhances system performance. This hybrid approach delivers superior disturbance attenuation, faster transient response, and improved quiescent state accuracy contrasted with existing control techniques. Extensive simulation results affirm the robustness and reliability of the proposed methodologies, establishing a resilient and high-precision control framework well-suited for systems operating under substantial external disturbances and structural uncertainties.**

***Index Terms*—Low Pass Filter (LPF), Disturbance Observer (DOB), Sliding Mode Control (SMC), Modified Sliding Mode Control (M-SMC), Proportional-Integral-Derivative surface Sliding Mode Control (PID sur-SMC), Surface-Mounted Permanent Magnet Synchronous Motor (SPMSM)**

I. INTRODUCTION

Systems subject to external disturbances and internal uncertainties, including environmental variations, sensor noise, and parameter fluctuations, present significant challenges in terms of control system stability and performance. Disturbance observers (DOBs) are crucial for addressing these issues, particularly in dynamic systems characterized by multiple energy storage elements, such as mass, capacitance, and inductance, and are widely applied across mechanical, electrical, and aerospace engineering domains. The sources of uncertainty often include manufacturing tolerances, component degradation, and changes in operational conditions. Robust control strategies, particularly those incorporating DOBs, are essential for ensuring system reliability across these variations. DOBs estimate disturbances in real time and compensate for their effects by appropriately modifying the control input, thereby enhancing the system's robustness against unknown and time-varying disturbances and certain model uncertainties. By efficiently compensating for external disturbances and internal uncertainties, Disturbance Observers (DOBs) empower the controller to uphold the desired dynamic performance of the system. This capability renders DOBs especially advantageous in the robust control of interval plants, where system parameters are subject to variation and uncertainty.

Sliding Surface–Based Control is acknowledged as a reliable control strategy to handling system uncertainties, external disturbances, and nonlinear dynamics, making it well-suited for complex and time-varying environments. Despite its robustness, conventional SMC is hindered by the inherent issue of chattering—undesirable high-frequency oscillations arising from the discontinuous switching action of the signum function on the sliding surface. This phenomenon can lead to wear in mechanical systems and degrade control performance, highlighting the need for refined SMC variants. To mitigate this, smooth nonlinear functions, such as the sigmoid, are used, providing continuous control with a tunable gain parameter to minimize oscillations without compromising system responsiveness. An advanced control approach, PID Surface-Based Sliding Mode Control (PID-Sur-SMC), enhances system performance by incorporating position, velocity, and integral error components into the sliding surface formulation. This integration significantly refines tracking precision and dynamic responsiveness. To ensure robustness, the stability of both second and third-order systems governed by this method was rigorously verified using the Kharitonov polynomial framework, which provides a systematic means of addressing uncertainty in interval system parameters. This study compares four control techniques—PID, standard SMC, Modified SMC (M-SMC), and PID sur-SMC—on DC motors and higher-order systems. Although PID control ensures simplicity, SMC variants offer superior robustness against disturbances and modelling uncertainties.

LITERATURE REVIEW:

A Disturbance Observer (DOB) is an essential element in control systems, designed to estimate and counteract external disturbances to enhance disturbance rejection, improve system stability, and minimize sensitivity to modelling errors. For linear systems, DOBs are typically developed using state-space methods, pole placement, and frequency-domain techniques. In contrast, nonlinear systems require strategies such as sliding mode observers or extended nonlinear adaptive observers. Applications include mechanical systems (e.g., servo motors and actuators), aerospace (for flight stability against atmospheric disturbances), robotics, and process control systems, where they mitigate the effects of external forces and varying conditions. Although traditional PID controllers can handle minor disturbances, their effectiveness decreases when disturbances exceed certain thresholds, necessitating DOB integration. Here, the disturbance is estimated and subtracted from the system input to neutralize its effect. In interval systems, the Kharitonov polynomial method, introduced by Vladimir L. Kharitonov, ensures stability by verifying the controller performance across eight extreme plants, providing a rigorous foundation for robust control.

DC motors play a pivotal role in industrial automation, robotics, and CNC machinery due to their excellent dynamic response, broad operational speed range, and high starting torque. To optimize their performance, PID controllers are commonly employed, with design parameters typically tuned via tools like the PID Tuner GUI. However, the effectiveness of these controllers is often challenged by their sensitivity to parameter fluctuations, necessitating frequent retuning to maintain optimal control under varying operating conditions. Sliding Mode Controllers (SMC) has evolved into a reliable solution, providing fast response, higher fidelity, and simple implementation; however, they are prone to chattering, which is an undesirable high-frequency oscillation. To mitigate this, a State Feedback-based Sliding Mode Controller (SMSFC) was developed, achieving a 40–55% reduction in chattering through gain optimization and disturbance-sensitive sliding surface design. Moreover, Disturbance Observer (DOB)-based SMC schemes are employed to address unmodeled dynamics, utilizing a modified signum function to further suppress chattering. Integrating sliding mode control with PID-based surfaces additionally enhances disturbance rejection and robustness, significantly outperforming traditional PID and conventional SMC methods under parameter variations.

OBJECTIVES OF STUDY (ADDRESSING GAPS)

Effectively controlling third-order interval plants poses a substantial challenge, as the presence of parameter uncertainties and external disturbances can critically impair system stability and performance. These complexities demand advanced control strategies capable of maintaining robustness and precision under highly variable and uncertain operating conditions of the system. While robust and adaptive control techniques have seen considerable advancement, conventional disturbance observers—such as those introduced by Ohnishi (1987) and later refined by Kim and Chung (2002)—were originally tailored for lower-order systems with fixed parameters. As a result, their applicability is constrained in highly dynamic and uncertain environments, underscoring the need for more adaptable and resilient observer-based control strategies. Interval plant theory, explored by Hollot, Yang, and Barmish, underscores the need for robust strategies; however, real-time disturbance estimation in uncertain high-order systems remains underdeveloped. Advanced methods, such as adaptive backstepping and sliding mode control, offer potential solutions but often introduce complexity and scalability issues. This study proposes a novel adaptive disturbance observer specifically designed for third-order-interval plants. By integrating classical observer principles with adaptive and fuzzy control techniques, the proposed approach achieves accurate real-time disturbance estimation while maintaining robustness against parameter variations, offering a practical and scalable solution for higher-order uncertain systems.

This study presents a robust control strategy employing PID-based surface Sliding Mode Control (SMC) for second and third-order interval plants. Traditional SMC methods [1], [2] offer robustness against system uncertainties and disturbances but often suffer from high-frequency chattering and complex implementations in higher-order systems. Although conventional PID controllers [5], [6] are effective under nominal conditions, they lack robustness when faced with parameter variations or external disturbances. Robust control solutions [7] address some of these challenges, but often do not scale well to interval or higher-order systems. Recent research has explored hybrid approaches integrating PID with SMC [9], offering improved tracking performance and disturbance rejection. However, these solutions still face challenges in reducing chattering [10] and managing mismatched uncertainties [8]. This study bridges these gaps by designing a PID surface within the SMC framework, enhancing control precision, ensuring system stability, and significantly reducing chattering. Additionally, it addresses the estimation of states and disturbances [11], thereby enhancing the robustness and reliability of the control.

ORGANIZATION OF PAPER:

Section II discusses the Conceptual Framework of Disturbance Observer Design**.** Section III discusses the Design Methodology of the filtering element Q(s). Section IV discusses Robustness Analysis Using Kharitonov Family of Interval Plants. Section V discusses the Disturbance Observer Design for Third-Order Interval Systems. Section VI discusses the Dynamic Modelling of a Direct Current Drive System. Section VII discusses Mathematical modelling of second and third-order dynamics. Section VIII discusses the Development and Tuning of the PID Controller. Section IX discusses the Implementation of Conventional SMC. Section X discusses the development of Modified-SMC. Section XI discusses Advanced Design of PID Surface-Based Sliding Mode Control. Section XII discusses the Enhancement of PID Control Using Low-Pass Filtering. Section XIII discusses the Conventional filtered SMC Integration. Section XIV discusses the Modified filtered SMC Enhancement. Section XV discusses the Integrated PID Surface-Based SMC with Low-Pass Filtering. Each section is explained using a detailed block diagram in the MATLAB environment. Section XVI discusses the Simulation Results and Comparative Performance Analysis. Section XVII discusses the Conclusion and Summary of Contributions. Section XVIII, References

II. CONCEPT OF DISTURBANCE DETECTION FILTER

The DOB, originally developed by Ohnishi in 1987, remains a fundamental control strategy for disturbance rejection in dynamic systems. In its standard form, the DOB employs a nominal system dynamics pn(s) and a Q-filter Q(s) to detect and mitigate the effects of external deviations, with such a desired behavior driven by a reference input ur. System performance is significantly affected by external deviations d, measurement noise σ, and the real-time system output y. To facilitate causal and practical implementation, an equivalent structure based on signal flow graph theory is often used, and the output is derived as a function of the reference, disturbance, and noise signals. The closed-loop characteristics, including the input and output sensitivity responses, capture the uncertainty observer's capacity to reject slow dynamics disturbances while reducing high-bandwidth noise. Such a filter Q(s) is central to this behavior: when Q(s)≈1, the system achieves near-perfect disturbance rejection, whereas when Q(s)≈0, it effectively filters out the measurement noise. Consequently, the filter is commonly formulated as a Q-filter that is carefully tuned to maintain a balance between robustness, disturbance rejection, and noise immunity across a wide range of operating conditions.

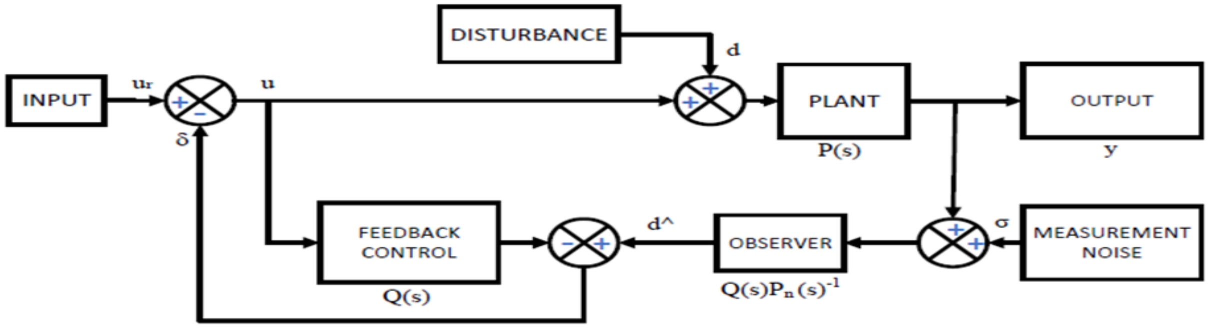


Fig 1. DISTURBANCE REJECTION LOOP

$$y = \frac{P(s)P_n(s)}{P_n(s)+[P(s)-P_n(s)]Q(s)} u_r + \frac{P(s)P_n(s)[1-Q(s)]}{P_n(s)+[P(s)-P_n(s)]Q(s)} d - \frac{P(s)Q(s)}{P_n(s)+[P(s)-P_n(s)]Q(s)} \sigma \rightarrow (1)$$

$$S_{DOB} = \frac{P(s)P_n(s)[1-Q(s)]}{P_n(s)+[P(s)-P_n(s)]Q(s)} \quad \rightarrow (2)$$

$$T_{DOB} = \frac{P(s)Q(s)}{P_n(s)+[P(s)-P_n(s)]Q(s)} \quad \rightarrow (3)$$

III. DESIGN OF THE FILTERING ELEMENT Q(s)

In the design of DOB, the LPF, also known as the Q-filter, serves as a pivotal component for achieving robust disturbance estimation and ensuring system stability. By effectively attenuating high-rate oscillatory disturbances while retaining the essential low-frequency dynamics where disturbances predominantly reside, the Q-filter facilitates accurate estimation without compromising system responsiveness. Its judicious design directly influences the observer's ability to reject uncertainties and external perturbations, making it integral to the overall performance of DOB-based control architectures. Ideally, the Q-filter should approximate an identity function at low frequencies to accurately reconstruct disturbances while exhibiting strong attenuation characteristics at high frequencies to suppress measurement noise and attenuate the impact of model uncertainties. The design of the Q-filter involves selecting an appropriate cutoff frequency and filter order, balancing the trade-off between responsiveness and robustness of the filter. A higher bandwidth enables faster disturbance rejection but increases sensitivity to noise and modelling errors, whereas a lower bandwidth enhances noise immunity at the expense of slower disturbance compensation. Typically, first- or second-order low-pass filters are utilized because of their simplicity and favourable phase characteristics, with the cutoff frequency typically chosen based on the dominant disturbance frequencies and noise spectrum. Furthermore, the Q-filter must be a stable and appropriate transfer function to secure the overall robustness formulated by the DOB-based feedback system. Therefore, the careful design and tuning of the low-pass filter are essential to achieve high-performance disturbance rejection without compromising robustness and noise resistance.

$$P(s) = \frac{(s+z_1)\ldots\ldots(s+z_m)}{s^l(s+p_1)\ldots\ldots(s+p_n)} \rightarrow (4)$$, wher Pi>0, Zi>0, l+n>m.

Umeno and Hori [16] (Umeno et al., 1993) proposed a widely used regular form of Q(s) for systems modeled using Equation (4). $Q(s) = \frac{1+\sum_{k=1}^{N-r} a_k(\tau s)^k}{1+\sum_{k=1}^{N} a_k(\tau s)^k} \rightarrow (5)$, $Q(s) = \frac{1+\sum_{k=1}^{L} c_k(\tau s)^k}{1+\sum_{k=1}^{N} c_k(\tau s)^k} \rightarrow (6)$

IV. DESIGN OF A THIRD-ORDER INTERVAL PLANT BY USING KHARITONOV POLYNOMIAL METHOD

To address parametric uncertainties in disturbance observer (DOB) design, a third-order interval plant model is often adopted, where in each polynomial coefficient varies independently within the prescribed bounds. The structured dynamic robustness of the interval system integrity is maintained using the Kharitonov polynomial method, which states that verifying the stability of four specific Kharitonov polynomials suffices to guarantee the stability of the entire family of polynomials. In this framework, the nominal plant model is represented by constructing four third-order Kharitonov polynomials and systematically selecting combinations of lower and upper coefficient bounds. By designing the DOB to maintain stability across these representative polynomials, robust disturbance estimation and compensation can be achieved despite the model uncertainties. The application of the Kharitonov method in third-order plant modeling ensures that the designed DOB remains effective over the entire uncertainty range, thus significantly enhancing the system reliability and performance under real-world operating conditions.

$$G(s) = \frac{N(s)}{D(s)} = \frac{q_3 s^3 + q_2 s^2 + q_1 s + q_0}{p_3 s^3 + p_2 s^2 + p_1 s + p_o} \rightarrow (7)$$

$$N_1(s) = q_3^u s^3 + q_2^u s^2 + q_1^l s + q_0^l. \qquad D_1(s) = p_3^u s^3 + p_2^u s^2 + p_1^l s + p_0^l.$$

$$N_2(s) = q_3^l s^3 + q_2^u s^2 + q_1^u s + q_0^l. \qquad D_2(s) = p_3^l s^3 + p_2^u s^2 + p_1^u s + p_0^l.$$

$$N_3(s) = q_3^u s^3 + q_2^l s^2 + q_1^l s + q_0^u. \qquad D_3(s) = p_3^u s^3 + p_2^l s^2 + p_1^l s + p_0^u. \rightarrow (8)$$

$$N_4(s) = q_3^l s^3 + q_2^l s^2 + q_1^u s + q_0^u. \qquad D_4(s) = p_3^l s^3 + p_2^l s^2 + p_1^u s + p_0^u.$$

Considering i, and j, = 1,2,3,4 , $G_k(s) = G_{ij}(s) = \frac{N_i(s)}{D_j(s)} \rightarrow (9)$

V. DESIGN AND ANALYSIS OF DOB FOR THIRD-ORDER DYNAMICS

The design of a Disturbance Observer (DOB) for a third-order interval plant is aimed at achieving robust regulation of system dynamics exposed to considerable external interferences and inherent model uncertainties. This approach enhances the system's resilience by actively estimating and compensating for unknown inputs, thereby sustaining robust and accurate performance throughout varied operating states. The third-order linear interval plant with coefficients varying within known bounds is considered, and Kharitonov polynomials are used to systematically represent the uncertainty space. Initially, a conventional PID controller is designed and tuned to satisfy the desired transient performance specifications, such as system settling duration and peak response overshoot. If the tuned PID system alone fails to maintain regulation in the presence of large disturbances, a DOB is implemented to identify and counteract external disturbances. Real-time disturbance estimation is carried out by the observer and then appropriately subtracted at the disturbance location to neutralize its effect on the plant. The DOB design process involves iterative tuning to ensure minimal deviation between the estimated and actual disturbance signals and to optimize the filter structure if necessary. The integration of the DOB ensures that disturbances are effectively cancelled at the plant input, enabling the PID controller to regulate the plant reliably under uncertainty. The combined effectiveness of the PID controller and DOB was subsequently validated through detailed MATLAB simulation studies, confirming the robust regulation of third-order plants across a range of disturbance scenarios.

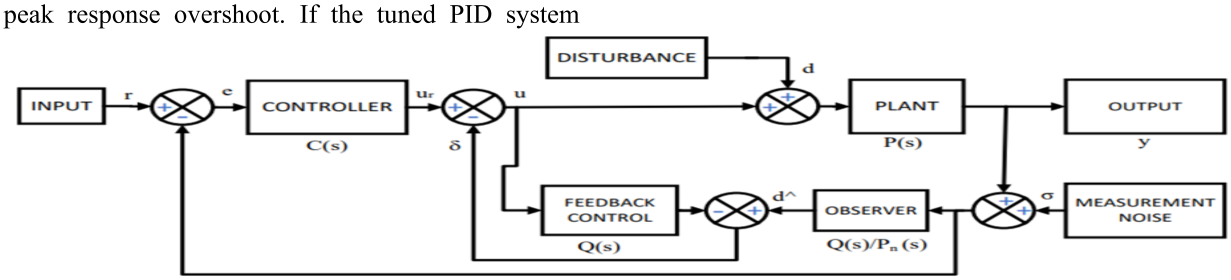


Fig 2. PID CONTROLLER ALONG WITH THE DISTURBANCE OBSERVER

VI. DYNAMIC MODELLING OF DC MOTOR

State-space modeling of the DC drive system forms the foundation for robust controller design in interval systems, particularly in high-precision applications such as electric drives [3], [4]. Motor dynamics are expressed using fundamental electromagnetic principles through system equations incorporating electrical and mechanical subsystems [2]. The standard transfer function relates the armature voltage to the angular velocity, offering a simplified linear model essential for control synthesis [5]. To enhance the dynamic performance under parameter variations and disturbances, state-space modelling is employed, capturing multivariable interactions and providing a framework for advanced controllers such as Sliding Mode Control (SMC) [1], [8]. Fractional-order PID tuning techniques [6] and robust designs [7] have demonstrated significant improvements in the transient response and stability margins. In this study, a PID Surface-based SMC approach was adopted to regulate second and third-order DC motor interval plants, ensuring robustness against uncertainties. The modelling and simulations were validated using MATLAB, with stability analysis reinforced via Lyapunov methods and Kharitonov polynomial theory [11].

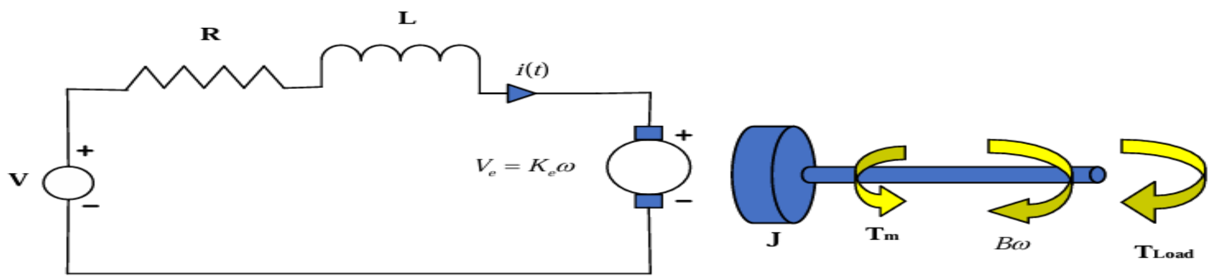


Fig 3. EQUIVALENT CIRCUIT OF DC MOTOR

SYSTEM EQUATIONS:
In SI units, it is assumed that the motor torque constant Kt and the back electromotive force (EMF) constant Ke are equal, i.e., Kt=Ke. The fundamental dynamics of the DC motor are derived by applying Kirchhoff's Voltage Law to the electrical circuit and Newton's Second Law to the mechanical system. These governing equations collectively describe the electromechanical behavior of the motor and form the basis for control-oriented modeling.

$$Ri + L\frac{di}{dt} = v - K_e w$$

$$J\dot{w} + bw = Ki$$

By eliminating the armature current I(s) from the previously derived equations, the open-loop transfer function of the DC motor is obtained. In this formulation, the armature voltage serves as the input, while the rotational speed is treated as the system output. This transfer function characterizes the motor's dynamic response and serves as a foundational element for controller design and analysis.

$$P(s) = \frac{w(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2}\left[\frac{rad/sec}{V}\right]$$

State-space representation:
The governing equations can be reformulated in state-space representation by selecting the rotational speed and armature current as the system's state variables. In this framework, the armature voltage is defined as the control input, while the rotational speed is designated as the system output. This state-space formulation provides a structured and compact model suitable for advanced control design and stability analysis.

$$\dot{X}_1 = \dot{w} = \frac{d\theta}{dx} = -\frac{b}{J}w + \frac{K_t}{J}i$$

$$\dot{X}_2 = \dot{i} = \frac{di}{dt} = -\frac{K_e}{L}w - \frac{R}{L}i + \frac{1}{L}u$$

$$m_x = [\frac{X_1}{X_2}] \quad A = \begin{bmatrix} -\frac{b}{J} & \frac{K_t}{J} \\ -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \quad B = [\begin{matrix} 0 \\ \frac{1}{L} \end{matrix}]\, u \quad C = [1 \quad 0]$$

$$y = [1\ 0]x$$

$$P(s) = \frac{w(s)}{V(s)} = \frac{0.6}{0.00676S^2+0.1622S+0.36}[\frac{rad/sec}{V}]$$

$$\frac{w(s)}{u(s)} = \frac{\frac{K}{JL}}{S^2+\left(\frac{R}{L}+\frac{b}{J}\right)S+\frac{Rb+K^2}{JL}}$$

VII. MODELLING OF SECOND AND THIRD-ORDER PLANT

Accurate modeling of second and third-order dynamic systems is essential for the development of robust control strategies, especially in environments characterized by parameter variability and external disturbances. Such models provide a foundational framework for designing controllers that ensure stability and performance under uncertain operating conditions. Second-order systems are typically characterized by mass-spring-damper dynamics, whereas third-order systems introduce additional energy storage elements, increasing complexity and requiring refined control strategies [1], [2]. Transfer function and state-space models are formulated by applying Laplace transforms to the system equations derived from Newton's laws or Kirchhoff's laws [3], [4]. For DC motor-driven systems, parameters such as inertia, damping, and back-EMF constants are critical for accurate model development [5], [6]. Incorporating interval analysis addresses parameter uncertainties and enhances reliability [7]. Sliding Mode Control (SMC) techniques are highly effective for managing mismatched uncertainties [8], whereas hybrid PID-SMC approaches further improve the transient and steady-state performance [9]. Mitigating chattering in SMC, which is essential for practical deployment, is achieved through smooth approximations [10]. Furthermore, state and disturbance estimation are integral to achieving high precision in real-world interval systems [11].

To evaluate the performance of various control strategies, a second-order system was selected for simulation studies. The system dynamics are represented in state-space form as follows, providing a structured framework for controller design and comparative analysis. This formulation, expressed by the following equation, facilitates frequency-domain analysis and serves as a basis for controller design and performance evaluation. $G(s) = \frac{8}{S^2+4S+8}$

$$\dot{x} = \begin{bmatrix} -4 & -8 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad , \quad y = [0 \quad 8]x$$

Consider a third-order dynamic system and describe it using a transfer function. $H(s) = \frac{Y(s)}{U(s)} = \frac{24}{S^3+9S^2+26S+24}$

$$\text{state equations: } \dot{q} = Aq + Bu$$

$$y = Cq+Du$$

$$\text{State} - \text{space equations } q_1 = z, \quad q_2 = \dot{z}, \quad q_3 = \ddot{z}$$

$$\dot{q_1} = \dot{z} = q_2 \ , \quad \dot{q_2} = \ddot{z} = q_3 \ , \quad \dot{q_3} = \dddot{Z} = u - 9\ddot{Z} - 26\dot{Z} - 24$$

$$\dot{q} = \begin{bmatrix} -9 & -26 & -24 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} q + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$\dot{q} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} q + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [0 \quad 0 \quad 24]$$

VIII. CONTROL METHODS – PID

The PID controller continues to serve as a cornerstone in the regulation of dynamic systems, owing to its structural simplicity and proven effectiveness across diverse application domains. In the context of second- and third-order interval plants, accurate PID tuning becomes essential to maintain robustness against parameter uncertainties and external disturbances [5], [6]. Recent advancements—such as the application of particle swarm optimization for PID parameter tuning [6], as well as hybrid frameworks that combine PID with sliding mode control techniques [9]—have shown marked improvements in both tracking accuracy and disturbance rejection, reinforcing the controller's relevance in modern robust control design. Studies have shown that traditional PID strategies often struggle under mismatched uncertainties [8], leading to the adoption of more adaptive PID-SMC methods [1], [2]. Furthermore, integrating PID within a sliding surface framework reduces chattering effects while preserving a fast dynamic response [10], [11]. By embedding disturbance estimation mechanisms along side PID control, as demonstrated in recent research [7], the regulation of interval plants becomes highly robust and efficient, making them ideal for real-world control applications.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$$

$$u(t) = K_p (e(t) + \frac{1}{T_i} \int_0^t e(\tau)\, d\tau + T_d \frac{de(t)}{dt}$$

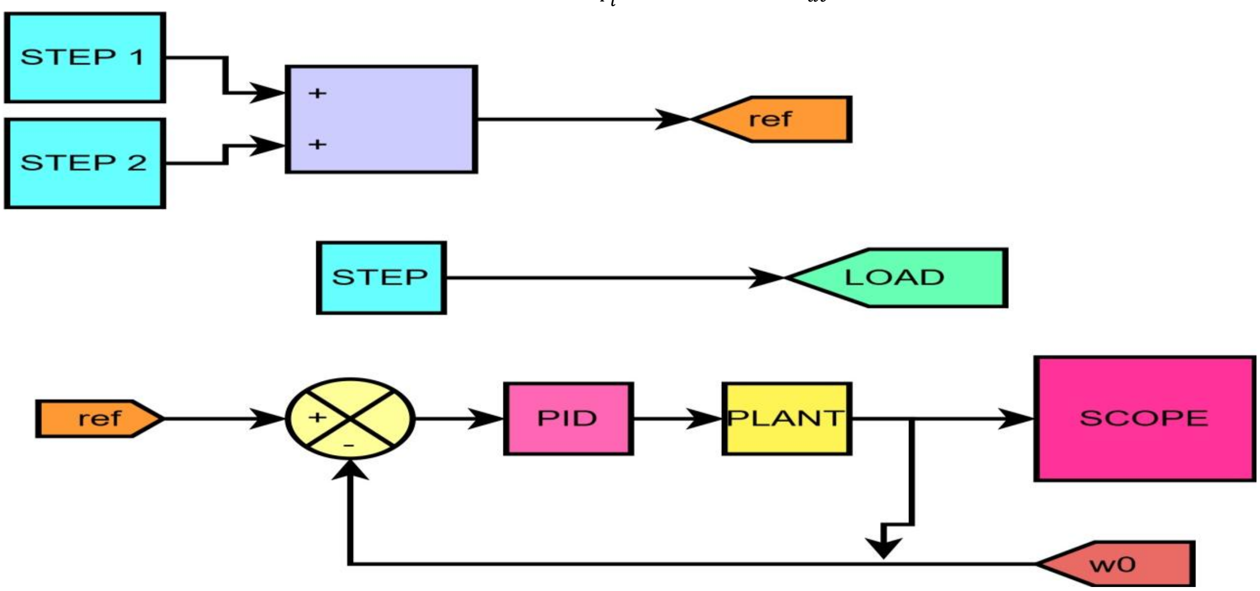


Fig 4. PID CONTROLLER

IX. CONVENTIONAL SLIDING MODE CONTROL

SMC methodology exhibits inherent robustness in handling system uncertainties and external disturbances, particularly for second and third-order systems. Traditional SMC designs focus on enforcing system states to reach and maintain motion along predefined sliding surfaces. However, challenges, such as chattering, remain prevalent. B. A. Reddy and P. V. Krishna [1] highlighted effective SMC strategies for coupled systems that enhance system resilience. Similarly, Anusha et al. [2] demonstrated SMC's superior robustness of SMC against parameter variations in DC motors. To mitigate drawbacks such as chattering, researchers such as A. A. Al Rawi et al. [10] proposed chatter-less modifications, achieving smoother control. Moreover, the integration of disturbance observers [8] further strengthens the SMC performance under mismatched uncertainties. These conventional approaches provide a solid foundation for integrating advanced PID-based sliding surfaces, enabling precise control, even for complex interval plants.

(i) Defining the Sliding Surface using the equation

$S = \{x\epsilon R^n : s = 0\}$

Let $x\epsilon R^n$ denote the state vector of the system, and in sliding mode, the dynamics evolve on the surface characterized by s=0, they are constrained to evolve along this surface, ensuring desired dynamic behavior and robust system performance.

(ii)Control Law: $u = u_{eq} + u_d$, $u_d = -k.sign(s)$

System transfer function is considered for modelling the system as shown in Eq

$$w(s) = \frac{88.76}{S^2+24S+53.25}u(s)$$

$$\dot{x} = Ax+Bu \quad , \quad y = Cx$$

$$\dot{x} = \begin{bmatrix} -24.00 & -53.25 \\ 1.00 & 0 \end{bmatrix}x + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u \,, \quad y = [0 \quad 88.76]x$$

The system can be arranged in the time domain as given below using the equation

$$\frac{d^2w(t)}{dt^2} = -24\frac{dw(t)}{dt} - 53.25w(t) + 88.76u_{eq}(t)$$

The switching function and its derivative are designed and described below.

$$S(t) = Ce + \frac{de}{dt} \quad , \quad e = w_r(t) - w(t)$$

$$\frac{dS(t)}{dt} = C\frac{d}{dt}[w_r(t) - w(t)] + \frac{d^2}{dt^2}[w_r(t)\text{-}w(t)]$$

Here, ω(t) denotes the actual angular velocity, $w_r(t)$ represents the reference angular velocity, and C is a performance parameter introduced to ensure system stability. Under steady-state conditions, the sliding surface conditions s(t)= 0 hold, leading to a simplified expression that characterizes the equilibrium behavior of the control system.

$$\frac{dS(t)}{dt} = 0 = C\frac{d}{dt}[w_r(t) - w(t)] + \frac{d^2}{dt^2}w_r(t) + 24\frac{dw(t)}{dt} + 53.25w(t) - 88.76u_{eq}(t)$$

$$\frac{d^2w_r(t)}{dt^2} = \frac{dw_r(t)}{dt} = 0, \text{ here reference speed is constant}$$

$$u_{eq}(t) = \frac{1}{88.76}\left[53.25w(t) + \frac{dw(t)}{dt}(24 - C)\right]$$

$$u(t) = \frac{1}{88.76}\left[53.25w(t) + \frac{dw(t)}{dt}(24 - C)\right] + Ksgn(S(t))$$

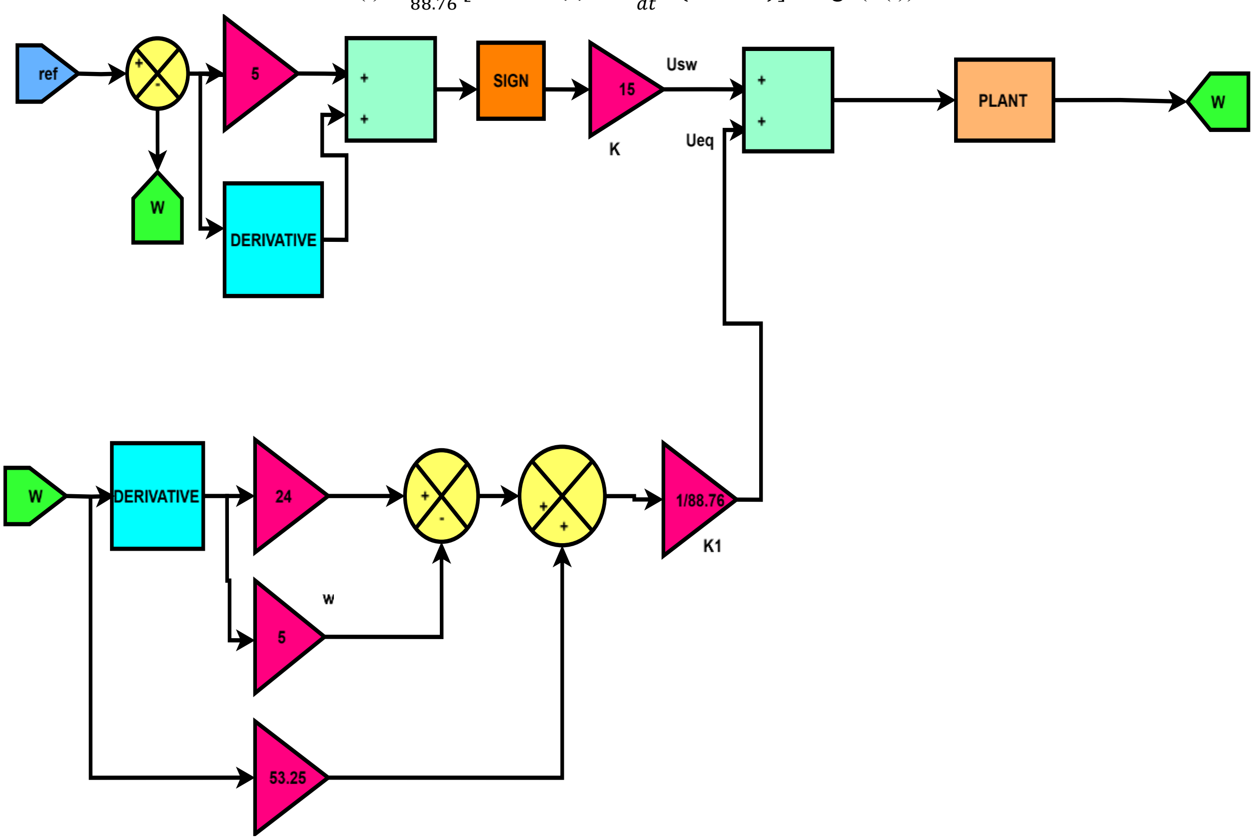


Fig 5. CONVENTIONAL SLIDING MODE CONTROL

X. MODIFIED SMC

Modified Sliding Mode Control (SMC) strategies have evolved to address the limitations of conventional SMC, particularly in mitigating chattering and enhancing robustness in nonlinear systems. Integrating proportional–integral–derivative (PID) elements into the sliding surface design has shown significant improvements in control performance. For instance, employing a PID-based sliding surface in electrohydraulic positioning systems has demonstrated superior tracking accuracy and reduced steady-state errors compared with traditional PID controllers. Further advancements include the development of second-order sliding mode controllers with PI sliding surfaces, which have been experimentally validated to offer better disturbance rejection and faster convergence in electromechanical systems. Furthermore, the incorporation of nonlinear fractional-order PID sliding surfaces has been explored for SPMSM Control systems, leading to notable enhancements in transient performance and a significant reduction in chattering phenomena. This advanced control formulation offers improved responsiveness and robustness, particularly suited for high-precision motion control applications. These modified SMC approaches, which enhance the sliding surface design and incorporate adaptive mechanisms, provide robust and efficient control solutions for complex dynamic systems, making them highly suitable for applications involving second and third-order interval plants

$$u_{sw}(t) = K\frac{S}{|S|+\delta}$$

$$u(t) = \frac{1}{88.76}[53.25w(t) + \frac{dw(t)}{dt}(24 - C) + K\frac{S}{|S|+\delta}]$$

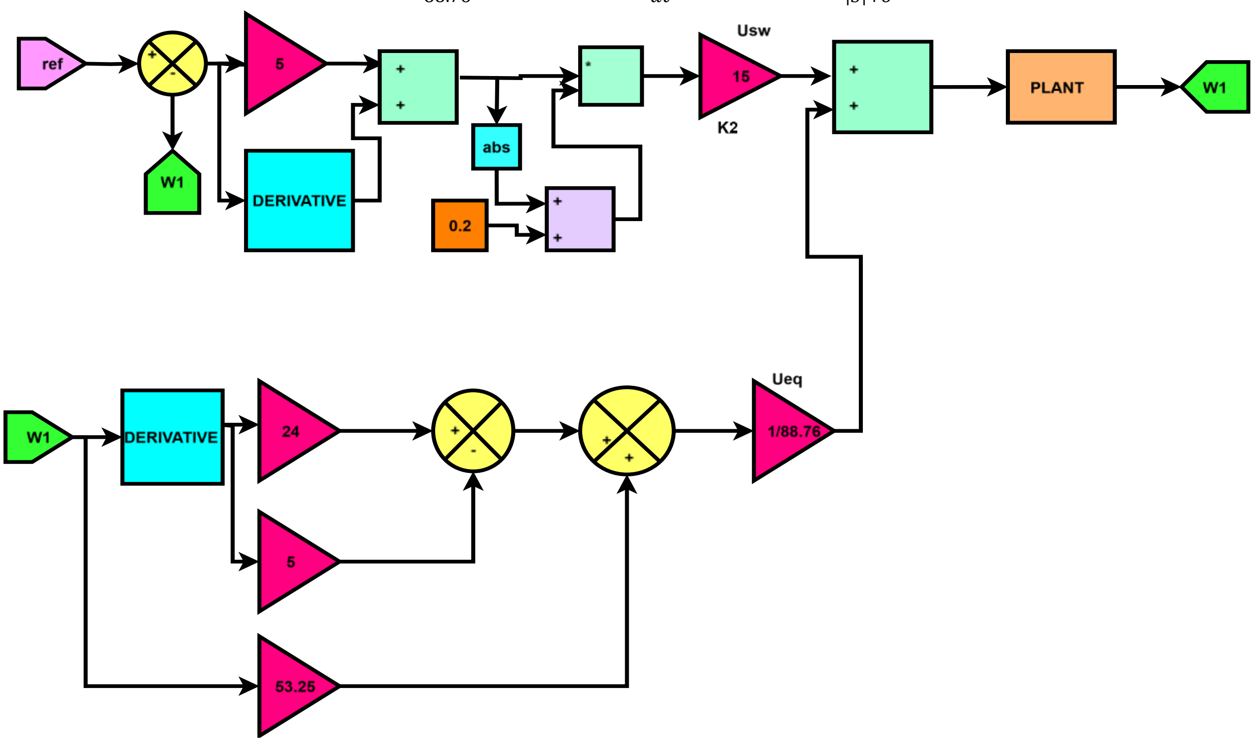


Fig 6. MODIFIED SLIDING MODE CONTROL

XI. PID SURFACE SMC

PID-SMC is a Resilient Hybrid Control Scheme that enhances traditional sliding mode control (SMC) by integrating PID characteristics into the sliding surface to achieve smoother dynamics and reduced chattering effects. B. A. Reddy and P. V. Krishna [1] demonstrated that combining PID surfaces within SMC frameworks significantly improves the transient response and steady-state accuracy, particularly in coupled systems. Furthermore, Anusha et al. [2] emphasized that higher-order sliding modes further improve performance under parameter uncertainties. Integrating PID dynamics into the sliding manifold, as explored by Zhou et al. [9], provides superior control robustness and finite-time convergence for nonlinear systems, such as missile actuators. Additionally, the disturbance observer strategies presented by Yang et al. [8] enhance disturbance rejection when PID-based sliding surfaces are used. These advancements make the PID Surface SMC a promising technique for the precise regulation of second- and third-order interval plants, balancing robustness and smoothness in the control action.

$S = \lambda_1 e + \lambda_2 \int e + \lambda_3 \dot{e}$

$u_{eq} = \frac{1}{\lambda_3 c}[\lambda_1 \dot{e} + \lambda_2 e + \lambda_3 \ddot{w}_d + \lambda_3 A\dot{w}]$

$u(t) = u_{eq}(t) + u_{sw}(t) \quad , \quad u_{sw}(t) = K_{sw}.sat(\frac{s}{\varphi})$

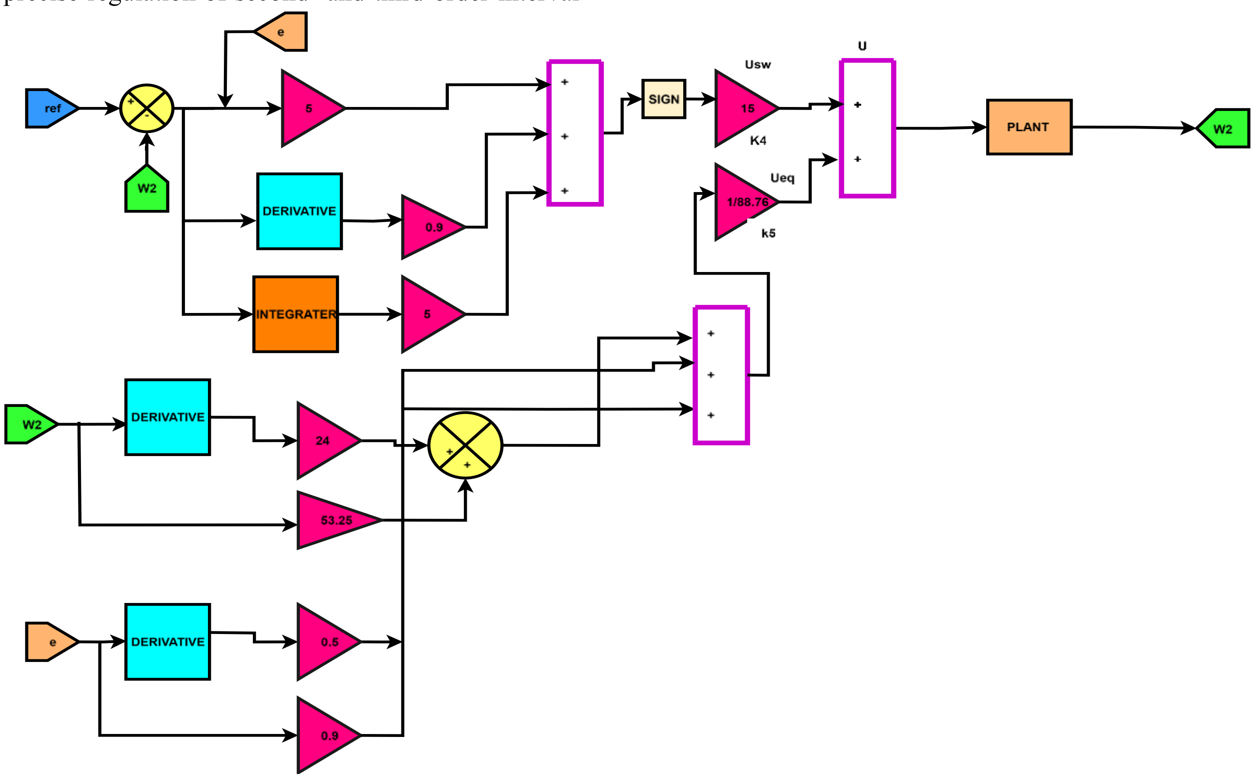


Fig 7. PID SURFACE SLIDING MODE CONTROL

INTERVAL PLANT STABILIZATION

$G(s) = \frac{N(s)}{D(s)} = \frac{q_m s^m + q_{m-1} s^{m-1} + \cdots + q_0}{p_n s^n + p_{n-1} s^{n-1} + \cdots + p_0}$

Two possible plants in a family of sixteen plants of an interval second-order plant are given below in equations

$G(s) = \frac{\eta_{max}}{d_{2max} S^2 + d_{1max} S + d_{0min}}$

$G(s) = \frac{\eta_{min}}{d_{2min} S^2 + d_{1min} S + d_{0max}}$

Two possible plants in a family of sixteen plants of an interval third-order plant are given below in equations

$G(s) = \frac{\eta_{min}}{d_3 S^3 + d_{2max} S^2 + d_{1min} S + d_{0max}}$

$G(s) = \frac{\eta_{max}}{d_3 S^3 + d_{2min} S^2 + d_{1max} S + d_{0min}}$

XII. PID CONTROLLER WITH A LOW-PASS FILTER

In high-precision control systems, the classical PID controller remains fundamental because of its simplicity and effective regulation. However, the derivative action is highly sensitive to noise. To address this, integrating a low-pass filter with the derivative term is crucial, as it suppresses high-frequency noise while retaining system responsiveness [5]. V. Yadav and V. K. Tayal [5] highlighted improved performance in DC motor control using optimized PID parameters, while Reddy and Krishna [1] noted enhanced stability when traditional control approaches are refined through noise mitigation. Furthermore, Jain et al. [6] demonstrated that refined PID structures, including filtering strategies, ensure better transient and steady-state behaviours under uncertain plant dynamics. By incorporating a low-pass filter, the PID controller becomes more robust against measurement noise, particularly in systems with high-frequency disturbances [7]. This approach is particularly advantageous in second and third-order interval systems, offering smooth regulation and improved control fidelity.

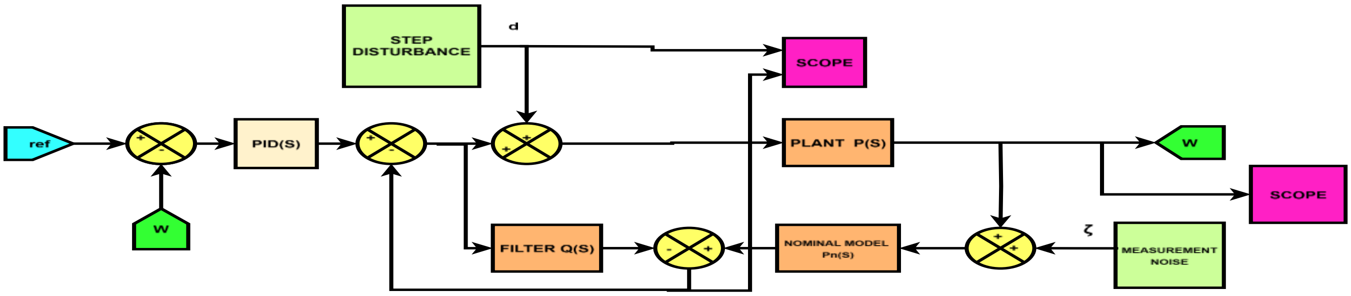


Fig 8. PID CONTROLLER WITH A LOW-PASS FILTER

XIII. CONVENTIONAL SMC WITH LOW-PASS FILTER

In conventional Sliding Mode Control (SMC), robustness and finite-time convergence are well established; however, high-frequency chattering remains a critical limitation. Incorporating a low-pass filter (LPF) into the control law effectively suppresses this phenomenon while maintaining performance integrity. The LPF attenuates rapid switching, thereby enhancing system smoothness and actuator longevity, which is particularly important in second- and third-order interval systems. Reddy and Krishna [1] demonstrated the effectiveness of such control in nonlinear coupled tank systems. Anusha et al. [2] validated higher-order SMC's resilience under parameter variations in motor control. Zhou et al. [9] proposed a hybrid PID-SMC scheme, incorporating filtering for improved missile actuator control, highlighting LPF's utility. Furthermore, Al Rawi et al. [10] introduced a chatter-less SMC using filtering techniques for DC motors. These advancements underscore the role of LPF in refining conventional SMC for high-precision applications like interval plant regulation, aligning with robust, low-noise control strategies.

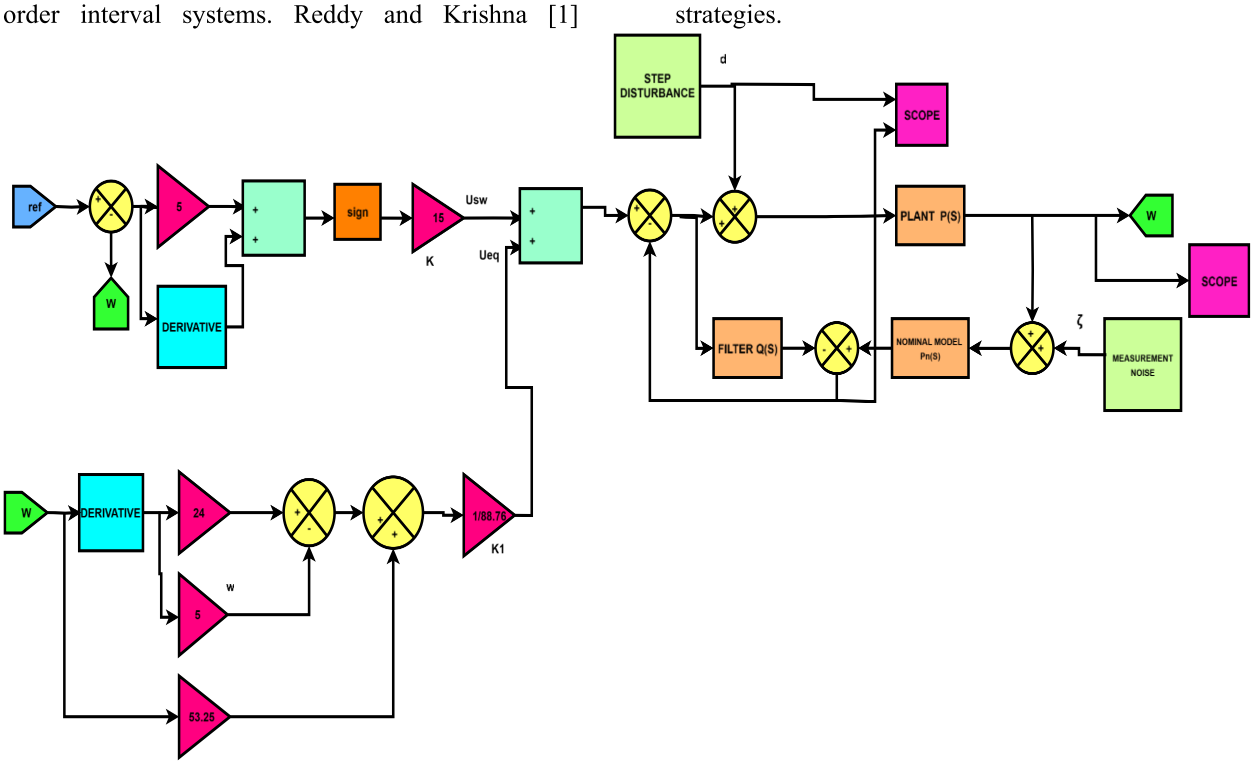


Fig 9. CONVENTIONAL SMC WITH LOW-PASS FILTER

XIV. MODIFIED SMC WITH LOW-PASS FILTER

The Modified SMC, integrated with an LPF, effectively overcomes the classical chattering issue while preserving robustness against model uncertainties and external disturbances. In traditional SMC strategies, such as those analysed by Reddy and Krishna [1], system trajectories exhibit high-frequency switching. Incorporating LPF smoothens the discontinuous control action, improving actuator lifespan and system performance. Anusha et al. [2] demonstrated the effectiveness of higher-order SMC techniques in mitigating parameter variations in DC motors, highlighting the need for improved control smoothness. Zhou et al. [9] presented a hybrid PID-SMC controller for missile actuator systems, where in filtering enhanced stability under rapid dynamic conditions. Al Rawi et al. [10] also proposed chatter-less controllers using modified SMC frameworks. By integrating LPF into SMC designs, the control scheme becomes highly suitable for second and third-order interval plants, ensuring robust regulation with minimal oscillations, making it ideal for advanced engineering applications.

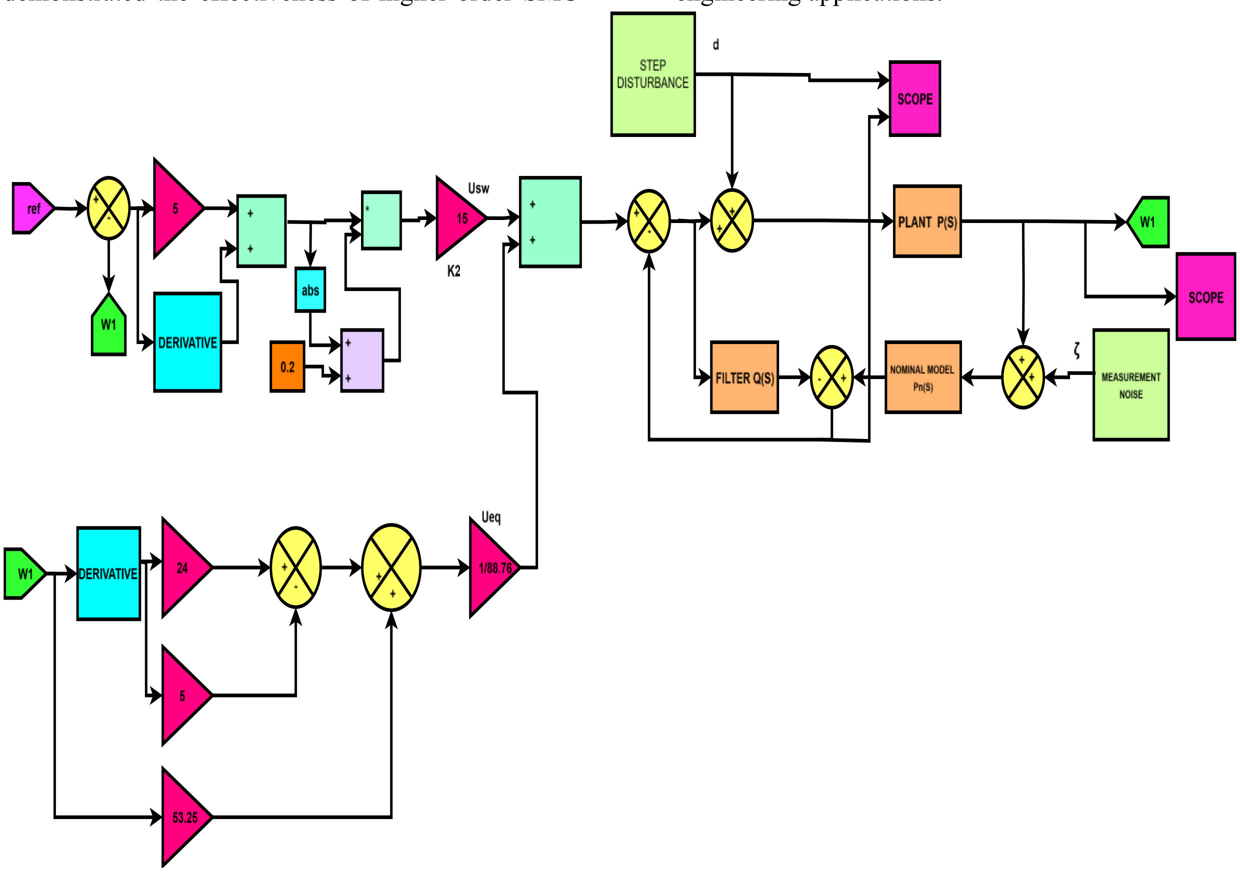


Fig 10. MODIFIED SMC WITH LOW-PASS FILTER

XV. PID SURFACE SMC WITH LOW-PASS FILTER

The integration of a PID-based sliding surface with SMC, complemented by an LPF, significantly enhances control robustness while effectively suppressing chattering effects. This synergistic configuration is particularly well-suited for the regulation of second- and third-order interval plants, ensuring stable and precise performance even under conditions of uncertainty and external disturbances. The PID surface ensures smooth error convergence, while the sliding control law provides resilience against disturbances. As demonstrated by Zhou et al. [9], hybrid PID-SMC schemes significantly improve actuator response in complex dynamics. The addition of an LPF, as emphasized by Al Rawi et al. [10], attenuates high-frequency switching, enhancing system stability. Reddy and Krishna [1] showcased the flexibility of SMC in coupled systems, while Anusha et al. [2] reinforced its reliability under parameter uncertainties. Yang et al. [8] also introduced disturbance observers for handling mismatched uncertainties, making the control framework highly adaptable. This PID surface SMC-LPF hybrid thus emerges as a superior solution for precise and robust control of higher-order interval plants in real-time applications.

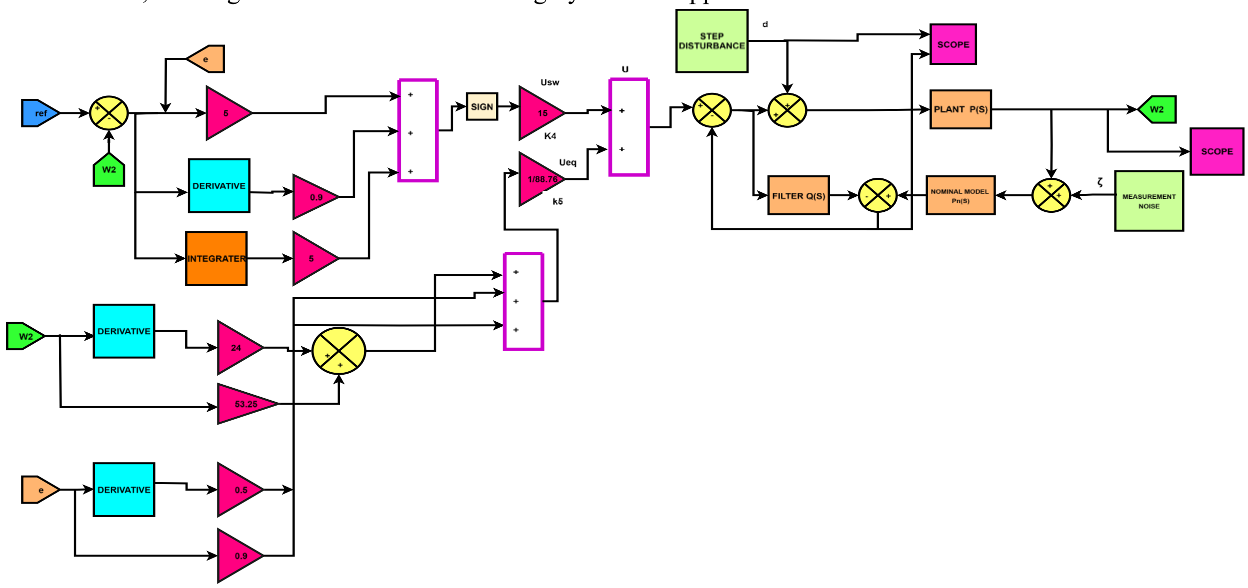


Fig 11. PID SURFACE SMC WITH LOW-PASS FILTER

XVI. SIMULATION RESULTS

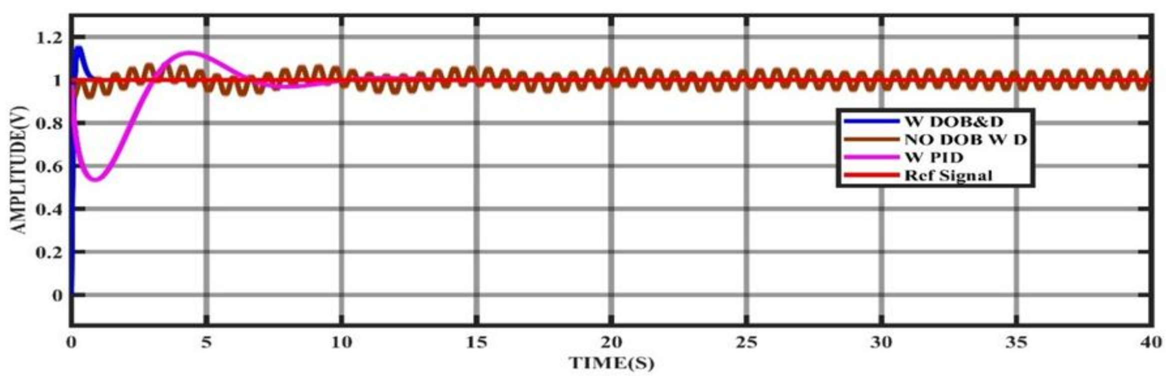


FIG 12. THE STEP RESPONSE WITH SINE DISTURBANCE

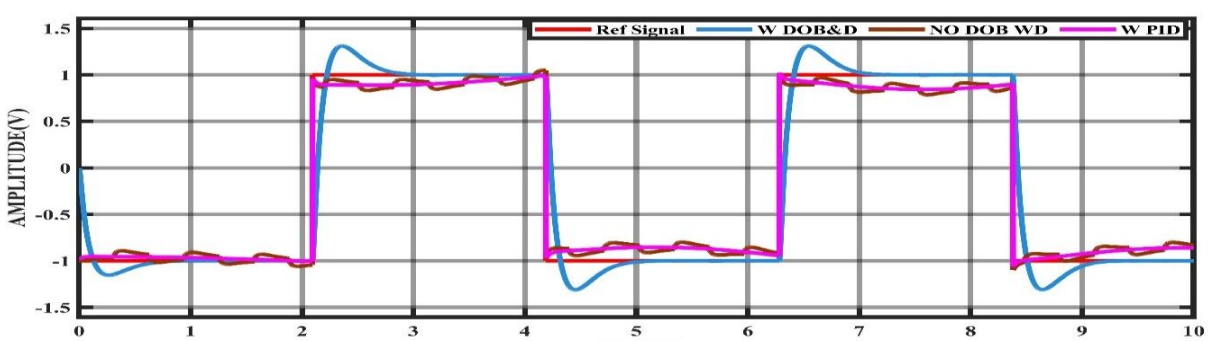


FIG 13. THE SQUARE RESPONSE WITH SQUARE DISTURBANCE

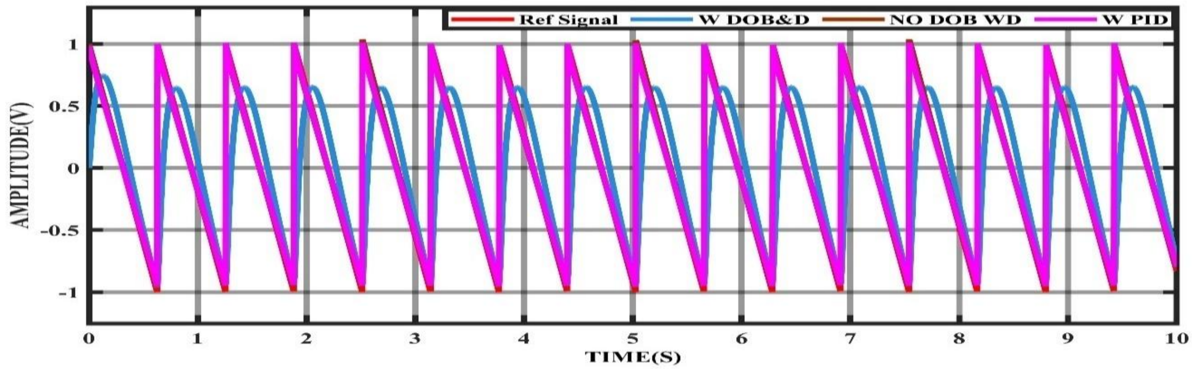


FIG 14. THE SAWTOOTH RESPONSE WITH SAWTOOTH DISTURBANCE

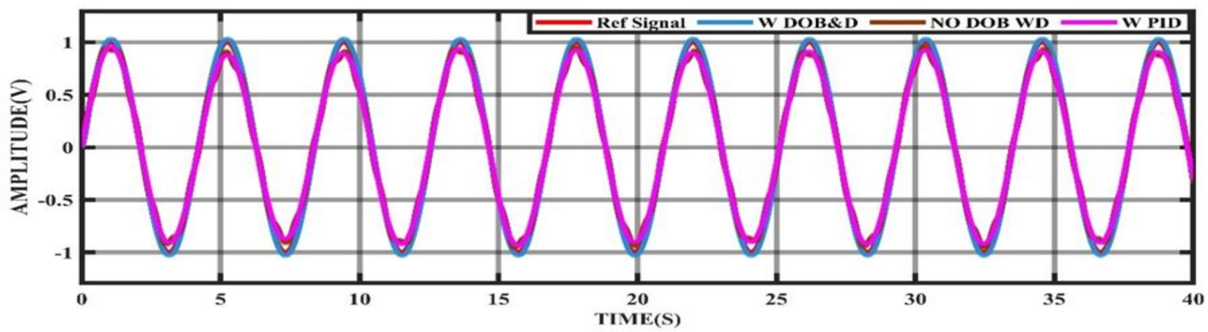
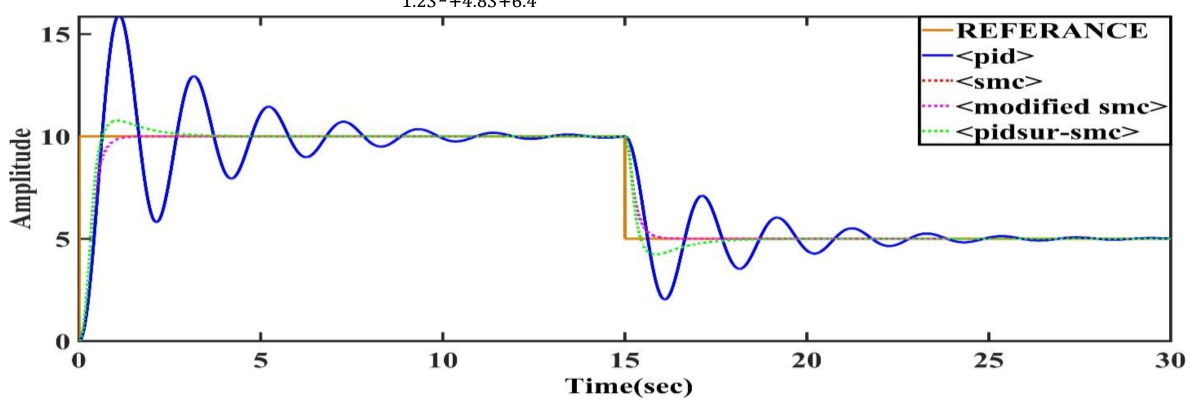


FIG 15. THE SINE RESPONSE WITH SINE DISTURBANCE

$G(s) = \frac{9.6}{1.2S^2+4.8S+6.4}$ FIG. 16



$G(s) = \frac{28.8}{1.2S^3+7.2S^2+20.8S+28.8}$ FIG. 17

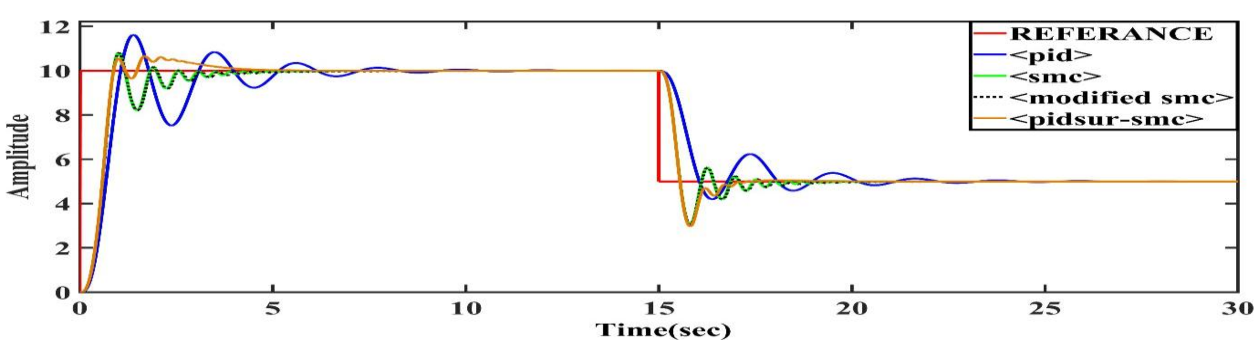


FIG 18.PID, SMC, M-SMC, PID-SURFACE SMC CONTROLLERS WITH LOW-PASS FILTER

$G(s) = \frac{8}{S^2+4S+8}$

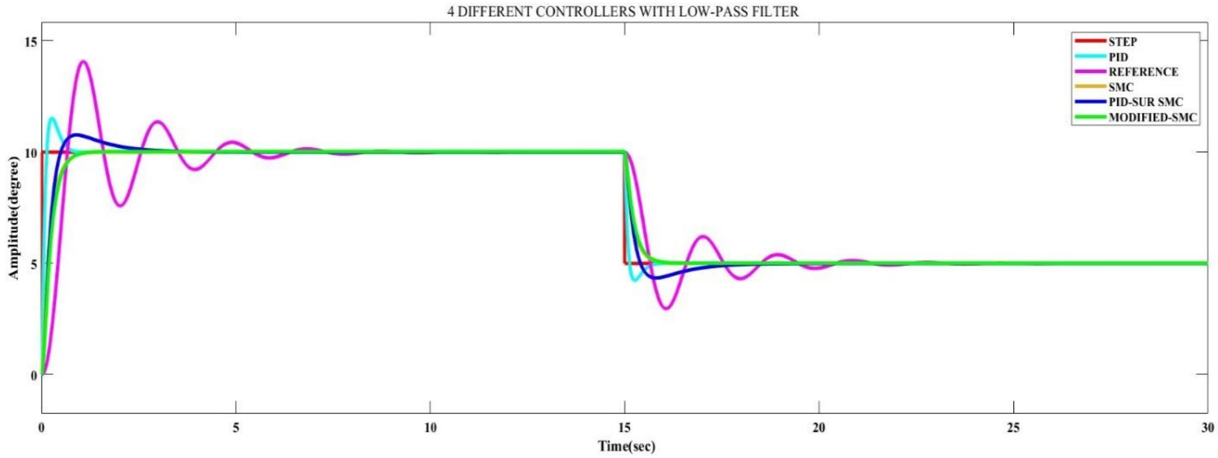


FIG 19. PID, SMC, M-SMC CONTROLLERS WITH LOW-PASS FILTER

$$G(s) = \frac{8}{S^2+4S+8}$$

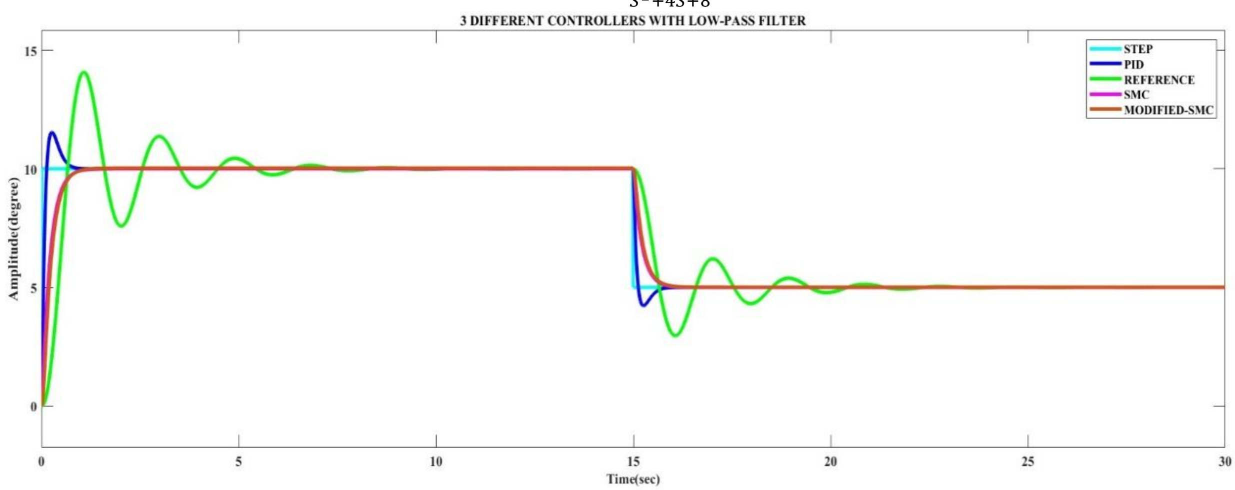


FIG 20.PID, SMC CONTROLLERS WITH LOW-PASS FILTER

$$G(s) = \frac{18.6}{S^3+9.9S^2+23.4S+26.6}$$

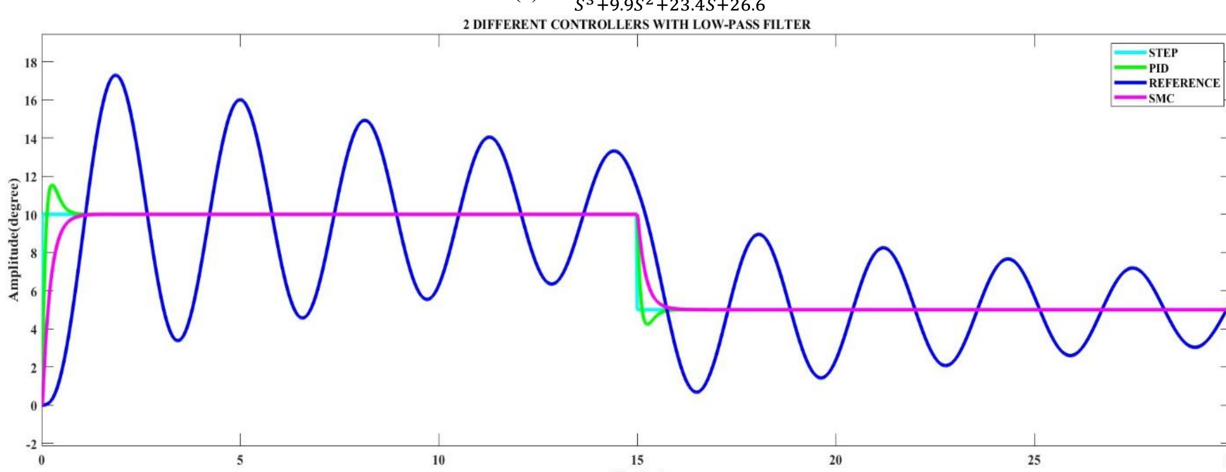
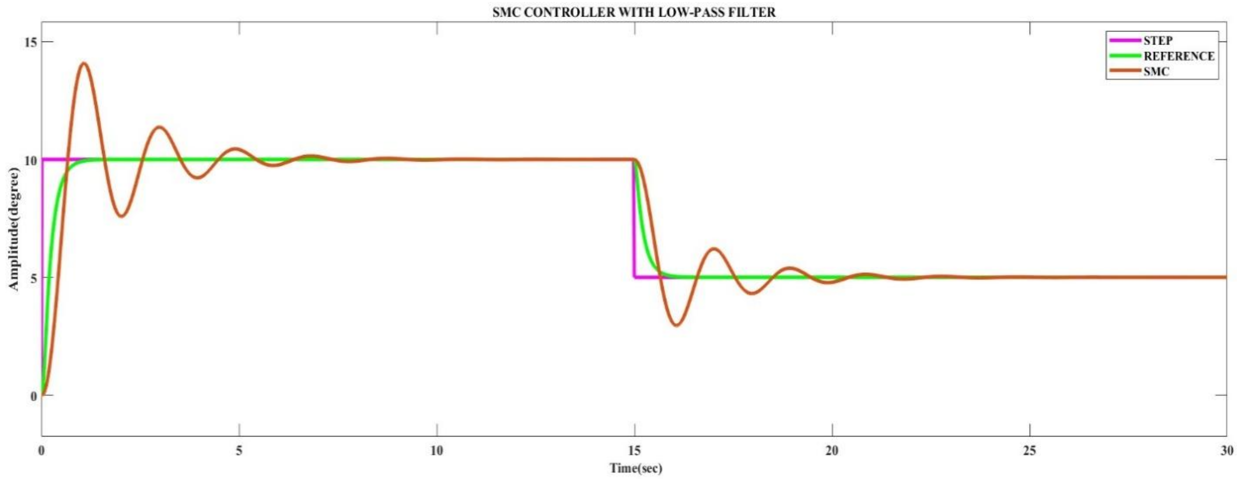


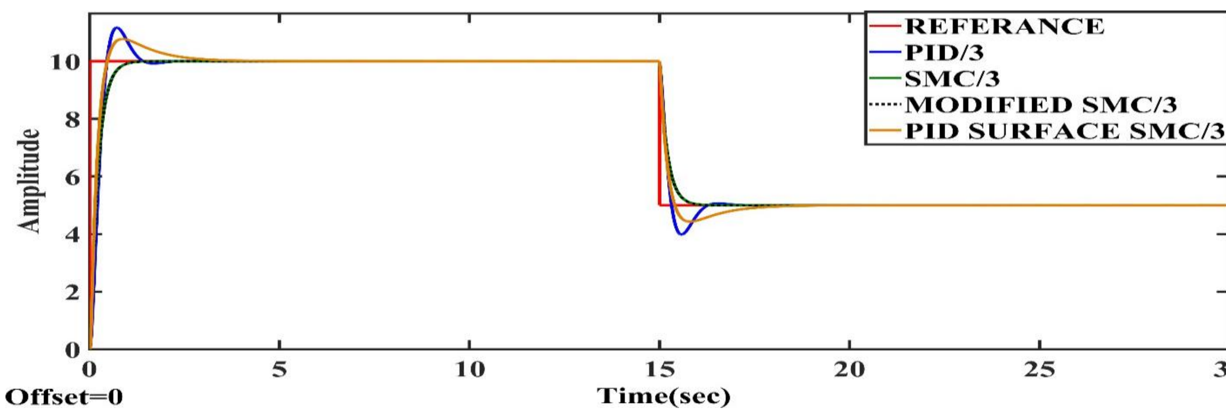
FIG 21. SMC CONTROLLER WITH LOW-PASS FILTER

$$G(s) = \frac{8}{S^2+4S+8}$$



$$P(s) = \frac{0.6}{0.00676S^2+0.1622S+0.36}$$

FIG. 22



XVII. CONCLUSION

The design of a disturbance observer for third-order interval plants presented in this study has proven effective in enhancing system robustness and dynamic response under parametric uncertainties. Drawing upon advanced disturbance observer architectures [3], [9], and velocity-based implementations [10], the proposed framework ensures accurate estimation of external disturbances, thereby maintaining system stability and performance. Inspired by adaptive and sliding-mode control techniques [1], [2], the observer integrates seamlessly with nonlinear compensation strategies for improved resilience. Classical stabilization approaches for interval plants [12] were extended with modern adaptive fuzzy logic and neural network enhancements [7], [11], ensuring scalability to high-order dynamics. Practical feasibility and control precision were validated, indicating strong potential for real-time applications in high-performance servo systems and robotics [4]. Future directions may explore hybrid learning-based observer designs for further adaptation to complex and time-varying environments.

This research effectively validates the performance of a PID Surface Sliding Mode Control (PID-SMC) strategy, enhanced with a Low-Pass Filter (LPF), for the precise regulation of second and third-order interval plants. The integrated approach demonstrates strong robustness and control accuracy, addressing both system uncertainties and external disturbances with improved dynamic response. The proposed method achieved robust performance against system uncertainties while significantly reducing chattering, a common drawback in traditional SMC [1], [10]. Drawing inspiration from higher-order sliding mode applications [2] and hybrid PID-SMC frameworks [9], the control design ensured fast convergence and minimized steady-state error. The integration of the LPF further enhanced output smoothness, confirming its effectiveness in handling disturbances. Compared to conventional PID and standalone SMC approaches

[5], [7], the presented method offers an optimized balance between robustness and smooth control effort. These findings pave the way for real-time deployment in dynamic industrial systems, with future work aimed at adaptive tuning and hardware implementation for increased versatility.

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