Al – Structure on Least Eigen Fuzzy Systems

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***Abstract*—In 1965, L.A.Zadeh introduced the notion of fuzzy sets, to evaluate the modern concept of uncertainty in real physical world. In 2010, Tamilarasi and Manimegalai introduced a new class of algebras called TM-algebras. An eigen fuzzy set of a fuzzy relation is often invariant under different computational aspects. The present research introduces a novel concept of eigen spherical fuzzy set of spherical fuzzy relations along with various composition operators for the first time. This study proposed two algorithms to determine the greatest eigen spherical fuzzy sets and least eigen spherical fuzzy sets using the max-min and min-max composition operators, respectively.**

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***Index Terms*—BCK/BCI Algebra, TM-Algebra, Fuzzy Matrix**

I. INTRODUCTION

In 1965, L.A.Zadeh [25] proposed the notion of fuzzy sets, to evaluate the new concept of uncertainty in real physical world. In 2010, Tamilarasi and Manimegalai proposed a new type of algebras known as TM-algebras [23].

Fuzzy Matrix Theory was First introduced by Michael G.Thomson[22] in 1977 as a branch of Fuzzy Set Theory. Fuzzy matrix has been proposed to represent fuzzy relation in a system based on fuzzy set theory,Vovehinnikov[24]. Eigen fuzzy sets of fuzzy relation can be used for the estimation of highest and lowest levels of involved variables when applying max-min composition on fuzzy relations. By the greatest eigen fuzzy sets (set when can be greater anymore) maximum membership degrees of any fuzzy set can be found, with the help of least eigen fuzzy set (set which can be less anymore) minimum degrees of any fuzzy sets can be found as well.

In [1], we studied Fuzzy Topological subsystem on a TM-algebra. In [2], we studied L− Fuzzy Topological TM-system. In [3], we studied L− Fuzzy Topological TM-subsystem. In [4], [5] we studied Fuzzy Supratopological TM-system, Fuzzy α− supracontinuous functions.

In this paper, discuss the notion on an AL-Structure on Least Eigen Fuzzy Systems. Moreover, the paper Algebraic structure on Least Eigen Fuzzy systems provides with cloudless examples.

II. PRELIMINARIES

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. Let X be a non-empty set. A mapping $\mu : X \to [0, 1]$ is called a fuzzy set of X.

Example 2.2. For i = {1, 2, 3} consider the fuzzy sets $\mu_i : Z \to [0, 1]$ given below:

$$\mu_1(x) = \begin{cases} (4 - x)/3 & \text{if } x = 1, 2 \\ \frac{5 - x}{3} & \text{if } x = 3 \end{cases}$$

Definitiosn 2.3. TM-Algebra

A TM-Algebra (X, ∗, 0) is a non-empty set X with a constant 0 and a binary operation ∗ satisfying the following axioms:

(1) $x * 0 = x$ for all $x \in X$

(2) $(x * y) * (x * z) = z * y$ for all $x, y, z \in X$.

Definition 2.4. Fuzzy TM-Subalgebra

A fuzzy subset μ of a TM-Algebra (X, ∗, 0) is called a fuzzy TM-Subalgebra of X if , for all x, y ∈ X, $\mu(x * y) \geq \min \{\mu(x), \mu(y)\}$.

Definition 2.5. Consider a matrix $A=[a_{ij}]_{m*n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

. The Matrix A is a fuzzy matrix iff $a_{ij} \in [0,1]$ for $1 \leq i \leq n, 1 \leq j \leq m$. Then A is a Fuzzy matrix.

Definition 2.6.A fuzzy relation R is a mapping from the Cartesian Space A x B to interval [0, 1], where the strength of the mapping is expressed by the membership function of the relation $\mu_R(x, y)$.

$\mu_R: A \times B \to [0, 1]$,

$$R = \{ ((x, y), \mu_R(x, y)) \mid \mu_R(x, y) \geq 0, x \epsilon A, y \epsilon B \}$$

3.AL – STRUCTURE ON LEAST EIGEN FUZZY SYSTEMS

Definition 3.1.

X, Y are TM-Algebras. $R_1\,(x,y), R_2(x,y)$ are fuzzy relations of X, Y. A matrix A=$[a_{ij}]_{m*n}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$ is a Algebraic fuzzy matrix if $a_{ij}\ \epsilon\ [0,1]$ for 1≤i≤n,1≤j≤m, satisfying the TM-Algebra under the fuzzy relation.

Example 3.2. The set X = {0, 1, 2}, Y = {0, 1, 2} with the cayley table

*	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

The Fuzzy Relations Subalgebra matrices $\mu_i\ : X\ \rightarrow [0,1], i = 1,\quad \vartheta_i\ : Y\ \rightarrow [0,1], i = 1$. An fuzzy relations $R_1\,(x,y), R_2(x,y)$ are on the sets X, Y is

X, Y	0	1	2
0	(0, 0)	(0, 1)	(0, 2)
1	(1, 0)	(1, 1)	(1, 2)
2	(2, 0)	(2, 1)	(2, 2)

$$A = \begin{bmatrix} 0.7 & 0.4 & 0.5 \\ 0.6 & 0.2 & 0.4 \\ 0.3 & 0 & 0.2 \end{bmatrix}$$ is a Algebraic fuzzy matrix

Max- Min composition of Algebraic Fuzzy Matrix 3.3

X, Y are TM-Algebras. $R_1\,(x,y), R_2(x,y)$ are fuzzy relations of X, Y.

$X = \{x_i\},\ i = 1,2,3,\ldots,n,\qquad Y = \{y_j\},\ j = 1,2,3,\ldots,m,\quad Z = \{z_k\},\ k = 1,2,3,\ldots,p$

We introduce relations

$R \in X \times Y, R = \{(x_i, y_i), \mu_R(x_i, y_i)\};\ x_i \subset X, y_i \subset Y, \mu_R(x_i, y_i) \in [0,1]$

$Q \in Y \times Z, Q = \{(y_j, z_k), \mu_Q(y_j, z_k)\}; y_j \subset Y, z_k \subset Z, \mu_Q(y_j, z_k) \in [0,1]$

We compose the relation R and Q by using a relation $S = RoQ$ Where the sign of 'o' denotes the max-min composition of R with Q. Relation S will be a Fuzzy Relation.

$$S = RoQ = \{[(x_i, z_j), \mu_S(x_i, z_k) = \max_{y_j \in Y}\{\min\{\mu_R(x_i, y_j), \mu_Q(y_j, z_k)\}\}]\}$$

or

$$T = \mu_{RoS}(x,z) = \max\{\min\{\mu_R(x,y), \mu_S(y,z)\}\}$$

Algebraic structure on Eigen Fuzzy System 3.4

X, Y are TM-Algebras. $R_1\,(x,y), R_2(x,y)$ are fuzzy relations of X, Y. Let A, B be the algebraic fuzzy matrices. The Max-Min composition of A and B gives, $AoB = C, C \subseteq R_1(x,y)XR_2(x,y)$. E is Eigen fuzzy algebraic matrix determine $E \subseteq R_1(x,y)XR_2(x,y)$ with membership function $\mu_E : R_1(x,y)XR_2(x,y) \rightarrow [0,1], \mu_E \in R_1 \times R_2$ and the algebraic fuzzy matrix $A \subseteq (R_1 \times R_2), \mu_A: R_1(x,y)XR_2(x,y) \rightarrow [0,1], \mu_A\ \in R_1 \times R_2$ satisfying E$oA\ =\ E$ should exist.

Greatest Eigen Fuzzy Subalgebra (GEFS)

X, Y are TM-Algebras. $R_1\,(x,y), R_2(x,y)$ are fuzzy relations of X, Y. By using Max-Min composition we can find the greatest eigen fuzzy algebraic matrix (GEFS) associated with A, as we are dealing with finite case, some interpretation can be taken in matrix form.

Least Eigen Fuzzy Subalgebra (LEFS) 3.6

To evaluate the least eigen fuzzy set of a relation R. There is a little change in the Algorithm of GEFS

Algorithm of LEFS 3.7

X, Y are TM-Algebras. $R_1\,(x,y), R_2(x,y)$ are fuzzy relations of X, Y. $A\ \subseteq R_1 \times R_2$ with membership function $\mu\,(x, y')$ is given

1. Find the set E_1 identified by $\mu_E(x,y') =\ min_{(x,y)\in(X,Y)}\mu_A(x,y'), \forall x \in X, y' \in Y$
2. set the index $n = 1$
3. calculate $E_{n+1}\ =\ E_n oA$
4. $E_{n+1}\ =\ E_{n\rightarrow Yes\rightarrow A_n = A_{n+1}}^{\rightarrow No\rightarrow n=n+1\rightarrow go-to-step-3}$

Example 3.8

X, Y are TM-Algebras. $R_1\,(x,y), R_2(x,y)$ are fuzzy relations of X, The Algebraic fuzzy matrix

$$R = \begin{bmatrix} 0.6 & 0.8 & 0.7 \\ 0.3 & 0.2 & 0.5 \\ 0.4 & 0.6 & 0.6 \end{bmatrix}$$

We want to calculate LEFS, so take the smallest membership degree from each column of R to get $A_1 = [0.3\quad 0.2\quad 0.5]$

For $n = 1$ we create A_2 by composing A_1 with R

$$A_2 = A_1 o R = [0.3 \quad 0.2 \quad 0.5] \ o \begin{bmatrix} 0.6 & 0.8 & 0.7 \\ 0.3 & 0.2 & 0.5 \\ 0.4 & 0.6 & 0.6 \end{bmatrix}$$

$A_2 = [0.4 \quad 0.5 \quad 0.5]$

We find $\mu_{A_2}(x_1)$ by Max-Mix composition, as the quantity

$\mu_{A_2}(x_1)$

$= \max\left(min\left((0.6,0.3), min(0.3,0.2), \min(0.4,0.5)\right)\right)$

$\mu_{A_2}(x_1) = \max(0.3, 0.2, 0.4) = 0.4$

Since $A_2 \neq A_1$, we set $n = 2$, compose R $with$ A_2 to get A_3

$$A_3 = A_2 o R = [0.4 \quad 0.5 \quad 0.5] \ o \begin{bmatrix} 0.6 & 0.8 & 0.7 \\ 0.3 & 0.2 & 0.5 \\ 0.4 & 0.6 & 0.6 \end{bmatrix}$$

$$A_3 = [0.4 \quad 0.5 \quad 0.5]$$

That satisfies the equality $A_3 = A_2$. The set A_3 accepted as the least eigen fuzzy set (LEFS) of relation R. We can see that $A_1 \subseteq A_2 \subseteq A_3$ which confirm the proper choice of least set.

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