An Inventory Model for Deteriorating Items with Price and Advertisement Dependent Demand in Fuzzy, Intuitionistic Fuzzy and Neutrosophic Fuzzy Environments

Supriya S. Wagh[1], Hemant P. Umap[2]
[1,2]*Department of Statistics, Karmaveer Bhaurao Patil University, Satara (M.S.) India*
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***Abstract*— In this paper an inventory model for deteriorating items with shortages is developed in Fuzzy, Intuitionistic fuzzy and Neutrosophic fuzzy environments. Deterioration rate, holding cost, shortage cost and advertisement cost are considered to be uncertain in nature and represented by triangular fuzzy numbers. In practice, demand is rarely constant and is influenced by various factors. The demand for items is assumed to depend on two important factors: the selling price of the product and the frequency of advertisement, both of which play a significant role in real-world market conditions. The model incorporates the effect of deterioration, which naturally occurs over time, thereby influencing the total inventory cost and optimal policy decisions. Removal area method is used for defuzzification. Model is illustrated by a numerical example and then results are compared on the basis of their environments. Sensitivity analysis is also carried out to study the responsiveness of the model to parameter changes, thereby providing valuable managerial insights for decision-making.**

***Index Terms*— Inventory, Fuzzy, Intuitionistic, Neutrosophic, Triangular, Deterioration**.

I. INTRODUCTION

In classical inventory model, all inventory parameters are considered to be deterministic. But in real life situations the values of these parameters are variates within a certain interval. Among these uncertain parameters, some have historical data to assume probability distributions. Such models are developed in stochastic environment. To tackle the problem of uncertain parameters without historical data, Zadeh [19] introduced 'Fuzzy set theory' in 1965. Initially, Sommer [13] treated inventory parameters as fuzzy parameters and inventory model is developed in fuzzy environment. Many researchers developed inventory models in fuzzy environment with different situations and concluded that the results are more realistic than of deterministic models. These models are known as fuzzy inventory models. In fuzzy inventory models, the fuzzy set is characterised by only membership degree.

The concept of fuzzy sets was generalized by Atanassov [1] and introduced the notion of Intuitionistic fuzzy sets which is characterized by not only membership degree but also non-membership degree of fuzzy parameter. It provides a richer framework to handle uncertainty. Researchers applied this theory in different areas of decision-making problems. Chakraborty et al. [2] proposed intuitionistic fuzzy optimization techniques for the solution of the EOQ model in which carrying cost, setup cost of demand quantity is considered as fuzzy members. Kaur and Mahuya [6] developed an inventory model to determine the optimal cost and an optimum order quantity of inventory by taking uncertain parameters as triangular intuitionistic fuzzy numbers. Singh and Kumar [11] formulate an inventory model that includes waste disposal cost under the intuitionistic fuzzy environment and triangular intuitionistic fuzzy number is used to represent all the costs and defuzzified by α-cut method.

Smarandache [12] presented idea of neutrosophic set and neutrosophic logic. Neutrosophy extends fuzzy logic and intuitionistic fuzzy logic by introducing indeterminacy as a third dimension. To overcome the limitations of fuzzy sets and intuitionistic fuzzy sets, neutrosophic sets are used for more flexible and dynamic representation of imprecise data. Neutrosophic sets independently define truth (T), falsity (F) and indeterminacy (I) without restrictive conditions provides a more generalized, more advanced and more realistic approach for handling complex, uncertain and conflicting information. Wang et al. [18] presented the concept of single valued neutrosophic set. Mullai and Broumi [8] firstly introduced neutrosophic approach in inventory models. They developed inventory model without shortages where demand and ordering cost are assumed to triangular neutrosophic numbers. Chakraborty et al. [3] introduced the concept of different types of triangular neutrosophic numbers and de-neutrosophication techniques. Mullai and Surya [9] introduced neutrosophic inventory backorder model where shortage cost and carrying cost are represented by triangular neutrosophic numbers and sign distance method is used for defuzzification. Sen and Chakrabarti [10] developed an inventory model for decaying items under fuzzy and neutrosophic environment with price dependent demand and without shortage. Surya et al. [15] implemented neutrosophic inventory model for deteriorating items with demand that is influenced by price. The objective of model is finding the neutrosophic optimal total cost and neutrosophic optimal time interval for inventory system.

The demand is most important factor in inventory management for success and there are so many ideas to enhance the demand of items. Among them advertisements are very beneficial to increase the demand of products and attract the customers. Umap [16] developed inventory model with shortage for deteriorating items with selling price and advertisement dependent demand.

In this paper, an inventory model is developed in fuzzy, intuitionistic fuzzy and neutrosophic fuzzy environments. Deterioration rate, holding cost, shortage cost and advertisement cost are represented in above three environments and defuzzified by removal area method. The model is illustrated by numerical example and the results are compared. The sensitivity analysis of the optimum solution with respect to the changes in the different parameter values is also discussed.

II. PRELIMINARIES

Definition 2.1. (Fuzzy set) [7], Let $\widetilde{A}$ be a fuzzy subset of the set of real numbers $\mathbb{R}$. Then $\widetilde{A}$ is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \emptyset$ such that

$$\mu_{\widetilde{A}}(x) = \begin{cases} 1\ , & \text{for } x \in [a, b] \\ l(x), & \text{for } x \in (-\infty, a) \\ r(x), & \text{for } x \in (b, \infty) \end{cases}$$

Definition 2.2. (Triangular fuzzy number) [7], A fuzzy number $\widetilde{A}$ is defined to be a triangular fuzzy number (TFN) if its membership function $\mu_{\widetilde{A}}: \mathbb{R} \to [0,1]$ is equal to

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & \text{if } x \in [a, b] \\ \dfrac{c-x}{c-b}\ , & \text{if } x \in [b, c] \\ 0\ , & \text{otherwise} \end{cases}$$

where $a \leq b \leq c$. This fuzzy number $\widetilde{A}$ is denoted by (a, b, c).

Definition 2.3. (Intuitionistic fuzzy sets) [1], Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set in X is an object given by

$$\widetilde{A}^I = \{\left(x, \mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x)\right) : x \in X\}$$

where $\mu_{\widetilde{A}^I}: X \to [0,1]$ and $\nu_{\widetilde{A}^I}: X \to [0,1]$ satisfy the condition $0 \leq \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leq 1$, for every $x \in X$.

Different research works were done over intuitionistic fuzzy numbers (IFNs). IFN is the generalization of fuzzy number and so it can be represented in the following manner.

Definition 2.4. (Intuitionistic fuzzy numbers) [5], An intuitionistic fuzzy subset

$\widetilde{A}^I = \{\left(x, \mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x)\right) : x \in \mathbb{R}\}$ of the real line $\mathbb{R}$ is called an IFN if the following holds:

(i) There exist $b \in \mathbb{R}$ such that $\mu_{\widetilde{A}^I}(b) = 1$ and $\nu_{\widetilde{A}^I}(b) = 0$

(ii) $\mu_{\widetilde{A}^I}$ is a continuous mapping from $\mathbb{R} \to [0,1]$ and for every $x \in \mathbb{R}$, the relation $0 \leq \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leq 1$ holds

(iii) The membership and non-membership functions of A are of the following form:

$$\mu_{\widetilde{A}^I}(x) = \begin{cases} 0, & -\infty < x \leq a \\ f(x)\ , & a \leq x \leq b \\ 1\ , & x = b \\ g(x)\ , & b \leq x \leq c \\ 0\ , & c \leq x < \infty \end{cases}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} 1, & -\infty < x \le e \\ h(x), & e \le x \le b,\ 0 \le f(x) + h(x) \le 1 \\ 0, & x = b \\ k(x), & b \le x \le g,\ 0 \le g(x) + k(x) \le 1 \\ 1, & g \le x < \infty \end{cases}$$

where $f(.)$, $g(.)$, $h(.)$, $k(.)$ are functions from $\mathbb{R} \to [0,1]$, $f(.)$ and $k(.)$ are strictly increasing functions and $g(.)$ and $h(.)$ are strictly decreasing functions.

It is worth noting that each IFN $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) \mid x \in \mathbb{R}\}$ is a conjunction of two fuzzy numbers: A^+ with a membership function $\mu_{A^+}(x) = \mu_{\tilde{A}^I}(x)$ and A^- with a membership function $\mu_{A^-}(x) = 1 - \nu_{\tilde{A}^I}(x)$.

Note. Here $e \le a$ and $c \le g$. For $x \le b$, if $e > a$, then there exists real number m such that $a < m < e$,

$$\mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1 \Rightarrow f(m) \le 0,$$

a contradiction. Hence $e \le a$. Similarly, $c \le g$.

Definition 2.5. (Triangular intuitionistic fuzzy number) [17]. An IFN may be defined as a triangular intuitionistic fuzzy number (TIFN) if and only if its membership and non-membership functions take the following form:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases}$$

$$\nu_{\tilde{A}^I}(x) = \begin{cases} 1, & x < e \\ \dfrac{b-x}{b-e}, & e \le x \le b \\ \dfrac{x-b}{g-b}, & b \le x \le g \\ 1, & x > g \end{cases}$$

where $e \le a$ and $c \le g$. Symbolically, TIFN $\tilde{A}^I$ is represented as $\{(a, b, c); (e, b, g)\}$.

Definition 2.6 (Neutrosophic Set) [12] A set $\tilde{S}^N$ in the universal discourse X, which is denoted by x, is said to be a neutrosophic set if $\tilde{S}^N = \{\langle x; [T_{\tilde{S}^N}(x), I_{\tilde{S}^N}(x), F_{\tilde{S}^N}(x)]\rangle : x \in X\}$

where $T_{\tilde{S}^N}(x): X \to [0,1]$ is called the truth membership function which represents the degree of confidence, $I_{\tilde{S}^N}(x): X \to [0,1]$ is called the indeterminacy membership function which represents the degree of uncertainty and $F_{\tilde{S}^N}(x): X \to [0,1]$ is called the falsity membership function which represents the degree of scepticism on the decision given the decision maker.

$T_{\tilde{S}^N}(x), I_{\tilde{S}^N}(x), F_{\tilde{S}^N}(x)$ exhibit the following relation:

$$0 \le T_{\tilde{S}^N}(x) + I_{\tilde{S}^N}(x) + F_{\tilde{S}^N}(x) \le 3$$

Definition 2.7 (Single Valued Neutrosophic Set) [18], Neutrosophic set $\tilde{S}^N$ is called a Single Valued Neutrosophic Set ($\tilde{S}^N$) if x is a single valued independent variable. Thus,

$\tilde{S}^N = \{\langle x; [T_{\tilde{S}^N}(x), I_{\tilde{S}^N}(x), F_{\tilde{S}^N}(x)]\rangle : x \in X\}$, where $T_{\tilde{S}^N}(x), I_{\tilde{S}^N}(x)$ and $F_{\tilde{S}^N}(x)$ represent the truth, indeterminacy and falsity membership functions respectively and also exhibit the same relationship as stated earlier.

If there exist three points a_0, b_0 & c_0, for which $T_{\tilde{S}^N}(a_0) = 1$, $I_{\tilde{S}^N}(b_0) = 1$ & $F_{\tilde{S}^N}(c_0) = 1$ then the $\tilde{S}^N$ is called neut-normal.

A $\tilde{S}^N$ is said to be neut-convex, which implies that it is a subset of a real line, by satisfying the following conditions:

1. $T_{\tilde{S}^N}(\rho a_1 + (1-\rho)a_2) \ge \min\left(T_{\tilde{S}^N}(a_1), T_{\tilde{S}^N}(a_2)\right)$
2. $I_{\tilde{S}^N}(\rho a_1 + (1-\rho)a_2) \le \max\left(I_{\tilde{S}^N}(a_1), I_{\tilde{S}^N}(a_2)\right)$
3. $F_{\tilde{S}^N}(\rho a_1 + (1-\rho)a_2) \le \max\left(F_{\tilde{S}^N}(a_1), F_{\tilde{S}^N}(a_2)\right)$

where a_1 & $a_2 \in \mathbb{R}$ and $\rho \in [0,1]$.

Definition 2.8 (Single Valued Neutrosophic Number) [14], A Single Valued Neutrosophic Number ($\tilde{A}^N$) is defined as $\tilde{A}^N = ([(a^1, b^1, c^1, d^1); \alpha], [(a^2, b^2, c^2, d^2); \beta], [(a^3, b^3, c^3, d^3); \gamma])$ where $\alpha, \beta, \gamma \in [0,1]$. The truth membership function $(T_{\tilde{A}^N}): \mathbb{R} \to [0, \alpha]$, the indeterminacy membership function $(I_{\tilde{A}^N}): \mathbb{R} \to [0, \beta]$ and the falsity membership function $(F_{\tilde{A}^N}): \mathbb{R} \to [\gamma, 1]$ are given as:

$$T_{\tilde{A}^N}(x) = \begin{cases} T_{\tilde{A}^N l}(x), & a^1 \le x \le b^1 \\ \alpha, & b^1 \le x \le c^1 \\ T_{\tilde{A}^N u}(x), & c^1 \le x \le d^1 \\ 0, & \text{otherwise} \end{cases}$$

$$I_{\tilde{A}^N}(x) = \begin{cases} I_{\tilde{A}^N l}(x), & a^2 \le x \le b^2 \\ \beta, & b^2 \le x \le c^2 \\ I_{\tilde{A}^N u}(x), & c^2 \le x \le d^2 \\ 1, & \text{otherwise} \end{cases}$$

$$F_{\tilde{A}^N}(x) = \begin{cases} F_{\tilde{A}^N l}(x), & a^3 \le x \le b^3 \\ \gamma, & b^3 \le x \le c^3 \\ F_{\tilde{A}^N u}(x), & c^3 \le x \le d^3 \\ 1, & \text{otherwise} \end{cases}$$

Definition 2.9 (Triangular Single Valued Linear Neutrosophic Number –Type 1) [3], The quantity of the truth, indeterminacy and falsity are not dependent. A Triangular Single Valued Neutrosophic Number of Type 1 is defined as $\widetilde{A}^N = (a, b, c; d, b, f; g, b, h)$ whose truth membership, indeterminacy and falsity membership are defined as follows:

$$T_{\widetilde{A}^N}(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x < b \\ 1, & x = b \\ \frac{c-x}{c-b}, & b < x \le c \\ 0, & \text{otherwise} \end{cases}$$

$$I_{\widetilde{A}^N}(x) = \begin{cases} \frac{b-x}{b-d}, & d \le x < b \\ 0, & x = b \\ \frac{x-b}{f-b}, & b < x \le f \\ 1, & \text{otherwise} \end{cases}$$

and

$$F_{\widetilde{A}^N}(x) = \begin{cases} \frac{b-x}{b-g}, & g \le x < b \\ 0, & x = b \\ \frac{x-b}{h-b}, & b < x \le h \\ 1, & \text{otherwise} \end{cases}$$

Were, $0 \le T_{\widetilde{A}^N}(x) + I_{\widetilde{A}^N}(x) + F_{\widetilde{A}^N}(x) \le 3$; $x \in \widetilde{A}^N$

III. ASSUMPTIONS

1) The scheduling period is constant and no lead-time.
2) Demand rate R is dependent linearly on the unit selling price and non-linearly on frequency of advertisement i.e., $R = (a-bp)N^{\alpha}$ where a, b and α are non-negative constants.
3) Shortages are allowed and backlogged.
4) Deteriorating rate is age specific failure rate.
5) The advertisement cost is fraction of the total selling price per cycle.

IV. NOTATIONS

T : Scheduling time of one cycle.
R : Demand rate per unit time; $R = (a-bp)N^{\alpha}$
θ : Deterioration rate.
I(t) : Inventory level at time t.
C_H : Total Holding cost per cycle.
C_1 : Holding cost per unit.
C_S : Total Shortage cost per cycle.
C_2 : Shortage cost per unit.
S_d : Total deteriorating units.
C_D : Total deteriorating cost per cycle.
C_d : Deteriorating cost per unit.
C_A : Advertisement cost per cycle.
P : Selling price per unit.
N : Number of advertisements.
μ : Advertisement cost $(0 < \mu < 1)$
S : Initial stock level.
S_1 : Maximum shortage level.
TC : Total inventory cost per cycle.

(wavy bar (~) represents the fuzzification of the parameters)

(~I represents the intuitionistic fuzzification of the parameters)

(~N represents the neutrosophic fuzzification of the parameters)

Figure

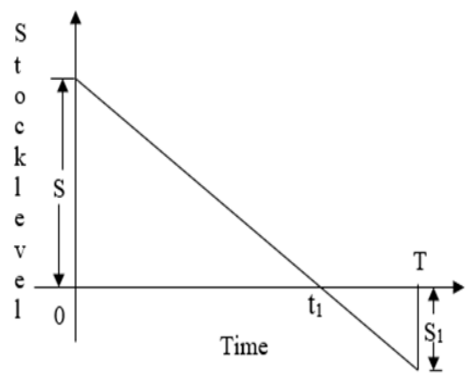

Fig. 1

V. MATHEMATICAL ANALYSIS

5.1 Crisp Model

Initially stock is filled at level S, thereafter inventory level decreases due to both demand and deterioration. At time $t = t_1$, the inventory is fully depleted. Then starts to occur shortages and accumulate to the level S_1 at time T.

The differential equation of the state of inventory in the time interval $(0, t_1)$ is described by,

$$\frac{dI(t)}{dt} + \theta I(t) = -(a - bp)N^{\alpha}\ ;\ 0 \le t \le t_1 \quad \text{---} (1)$$

Above differential equation is simplified using the boundary condition at $t = 0$, $I(t) = S$

$$I(t) = -\frac{(a-bp)N^{\alpha}}{\theta} + \left(\frac{S\theta+(a-bp)N^{\alpha}}{\theta}\right)e^{-\theta t}\ ;\ 0 \le t \le t_1 \quad \text{---} (2)$$

Used boundary condition at $t = t_1$, $I(t_1) = 0$, to get

$$t_1 = \frac{1}{\theta}\log\left(1 + \frac{S\theta}{(a-bp)N^{\alpha}}\right) \text{--- (3)}$$

The differential equation describing the state of shortages in the time interval (t_1, T) is,

$$\frac{dI(t)}{dt} = -(a-bp)N^{\alpha}\ ;\ t_1 \le t \le T \text{--- (4)}$$

This differential equation is simplified using the boundary condition at $t = t_1$, $I(t_1) = 0$,

$$I(t) = -(a-bp)N^{\alpha}t + (a-bp)N^{\alpha}t_1\ ;\ t_1 \le t \le T \text{--- (5)}$$

Then at $t = T$, $I(T) = -S_1$,

$$S_1 = (a-bp)N^{\alpha}T - (a-bp)N^{\alpha}\cdot\frac{1}{\theta}\log\left(1 + \frac{S\theta}{(a-bp)N^{\alpha}}\right) \text{--- (6)}$$

During the time interval $(0, T)$, total deteriorating units are

$$S_d = \int_0^{t_1} \theta I(t)\, dt;\ \ 0 \le t \le t_1$$

Using equation (2),

$$S_d = -(a-bp)N^{\alpha}t_1 - \left(\frac{S\theta + (a-bp)N^{\alpha}}{\theta}\right)(e^{-\theta t_1} - 1)$$

The deteriorating cost is $C_D = C_d S_d$

Holding cost over the time period $(0, T)$ is $C_H = C_1\int_0^{t_1} I(t)dt$

Shortage cost is $C_S = C_2\left[\int_{t_1}^{T} - I(t)dt\right]$

And Advertisement cost per cycle is $C_A = \mu(S - S_d)PN$

By adding above all costs, the total inventory cost is

$$TC = C_H + C_D + C_S + C_A$$

$$TC = (C_1 + C_d\theta)\left[-\frac{(a-bp)N^{\alpha}t_1}{\theta} - \left(\frac{S\theta + (a-bp)N^{\alpha}}{\theta^2}\right)(e^{-\theta t_1} - 1)\right] + C_2\left[\frac{(a-bp)N^{\alpha}}{2}(T - t_1)^2\right] + \mu\left[S - \frac{S^2\theta}{(a-bp)N^{\alpha}}\right]PN \quad \text{---(7)}$$

Ignoring higher-order terms, the simplified total inventory cost becomes:

$$TC = (C_1 + C_d\,\theta)\frac{S^2}{(a-bp)N^{\alpha}} + C_2\left[\frac{(a-bp)N^{\alpha}}{2}\left(T - \frac{S}{(a-bp)N^{\alpha}}\right)^2\right] + \mu\left[S - \frac{S^2\theta}{(a-bp)N^{\alpha}}\right]PN \quad \text{---(8)}$$

And the optimum order level is,

$$S = \frac{(a-bp)N^{\alpha}(C_2T - \mu PN)}{2(C_1 + C_d\theta - \mu PN\theta) + C_2} \text{---(9)}$$

5.2 Fuzzy Model

The preceding crisp model assumes complete certainty in parameter values, real-life systems are influenced by uncertainty. Hence, incorporating variability is essential for a more realistic representation. Recognizing the vagueness associated with deterioration rates in practical scenarios, this study extends the EOQ model by incorporating a fuzzy number to represent the rate of deterioration. Holding cost, shortage cost and advertisement cost are also represented by fuzzy numbers.

In fuzzy model deterioration rate, holding cost, shortage cost and advertisement cost are represented by triangular fuzzy numbers.

Then fuzzy total inventory cost and fuzzy optimum ordered quantity are,

$$\widetilde{TC} = (\tilde{C}_1 \oplus C_d \otimes \tilde{\theta})\frac{\tilde{S}^2}{(a-bp)N^{\alpha}} \oplus \tilde{C}_2\left[\frac{(a-bp)N^{\alpha}}{2}\left(T - \frac{\tilde{S}}{(a-bp)N^{\alpha}}\right)^2\right] \oplus \tilde{\mu} \otimes \left[\tilde{S} - \frac{\tilde{S}^2 \otimes \tilde{\theta}}{(a-bp)N^{\alpha}}\right]PN$$

$$\tilde{S} = \frac{(a-bp)N^{\alpha} \otimes (\tilde{C}_2 \otimes T - \tilde{\mu} \otimes PN)}{2(\tilde{C}_1 + C_d \otimes \tilde{\theta} - \tilde{\mu} \otimes \tilde{\theta} \otimes PN) \oplus \tilde{C}_2}$$

5.2.1 Defuzzification by Removal Area Method of Triangular Fuzzy numbers

Using removal area method [4] total cost and optimum ordered quantity are then defuzzified.

Defining a triangular fuzzy number as: $\tilde{A}$ = {(a, b, c)} which represents graphically as follows:

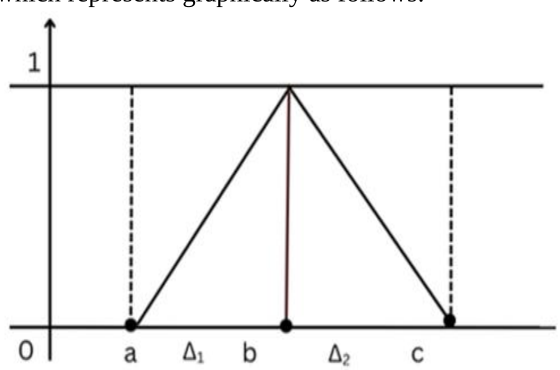

Fig. 2

Fig.3, 4 and 5 shows the graphical representation of defuzzification of triangular fuzzy number.

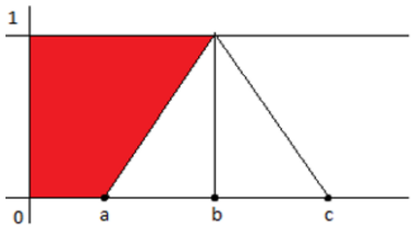

Fig. 3

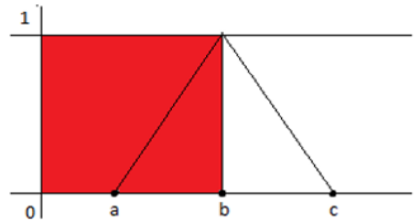

Fig. 4

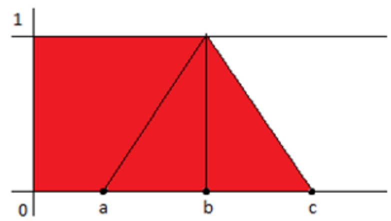

Fig. 5

By considering different types of areas of the corresponding triangular fuzzy number as shown below:

$R_1\ (\tilde{A}, 0)$ = Area of shaded region for Fig. 3 = $\frac{(a+b)}{2}$

$R_2\ (\tilde{A}, 0)$ = Area of shaded region for Fig. 4 = b

$R_3\ (\tilde{A}, 0)$ = Area of shaded region for Fig. 5 = $\frac{(b+c)}{2}$

Then Defuzzified value of fuzzy number $\tilde{A}$ = (a, b,c) can be generally defined by

$R_{\tilde{A}}\ (\check{D}, 0) = \frac{R_1\ (\tilde{A},0)+R_2\ (\tilde{A},0)+R_3\ (\tilde{A},0)}{3}$

$R_{\tilde{A}}\ (\check{D}, 0) = \frac{(a+4b+c)}{6}$

$R_{\tilde{A}}\ (\check{D}, 0) = b + \frac{1}{6}(\Delta_2 - \Delta_1)$ where $a = b - \Delta_1$ and $c = b + \Delta_2$ and $0 < \Delta_1\ \Delta_2$

In real life, it is very difficult to consider the deterioration rate θ over a total time period T. The deterioration rate can be located in the interval $\{(\theta - <\Delta_1, \theta, \theta + \Delta_2)\}$ where $0 < \Delta_1 < \theta$ and $0 < \Delta_1\ \Delta_2$. The values of $< \Delta_1, \Delta_2$ will be decided by decision maker whereas θ is a known number. Then the Defuzzified value of $\tilde{\theta}$ is,

$R_{\tilde{\theta}}\ (\check{D}, 0) = \theta + \frac{1}{6}(\Delta_2 - \Delta_1)$

Where $\Delta_1 = \theta - a$, $\Delta_2 = c - \theta$.

Similarly, Defuzzified values of $\widetilde{C_1}$, $\widetilde{C_2}$ and $\tilde{\mu}$ are,

$R_{\widetilde{C_1}}\ (\check{D}, 0) = C_1 + \frac{1}{6}(\Delta_2 - \Delta_1)$

Where $\Delta_1 = C_1 - a$, $\Delta_2 = c - C_1$

$R_{\widetilde{C_2}}\ (\check{D}, 0) = C_2 + \frac{1}{6}(\Delta_2 - \Delta_1)$

Where $\Delta_1 = C_2 - a$, $\Delta_2 = c - C_2$

$R_{\tilde{\mu}}\ (\check{D}, 0) = \mu + \frac{1}{6}(\Delta_2 - \Delta_1)$

Where $\Delta_1 = \mu - a$, $\Delta_2 = c - \mu$

5.3 Intuitionistic Fuzzy Model:

Generally vague and imprecise parameters are treated as fuzzy parameters. However, Atanassov [1] treated such parameters as intuitionistic fuzzy parameters in an appropriate way to get more accurate results.

In this model deterioration rate, holding cost, shortage cost and advertisement cost are considered as intuitionistic fuzzy parameters and represented by triangular intuitionistic fuzzy numbers. Then the Intuitionistic fuzzy total inventory cost is,

$$\widetilde{TC}^{I} = \left(\widetilde{C_1}^{I} \oplus C_d \otimes \tilde{\theta}^{I}\right)\frac{\tilde{S}^{I^2}}{(a-bp)N^{\alpha}} \oplus \widetilde{C_2}^{I}\left[\frac{(a-bp)N^{\alpha}}{2}\left(T - \frac{\tilde{S}^{I}}{(a-bp)N^{\alpha}}\right)^2\right] \oplus \tilde{\mu}^{I} \otimes \left(\tilde{S}^{I} - \frac{\left(\tilde{S}^{I}\right)^2 \otimes \tilde{\theta}^{I}}{(a-bp)N^{\alpha}}\right)PN$$

Were,

$$\tilde{S}^{I} = \frac{(a-bp)N^{\alpha} \otimes \left(\widetilde{C_2}^{I} \otimes T - \tilde{\mu}^{I} \otimes PN\right)}{2\left(\widetilde{C_1}^{I} + C_d \otimes \tilde{\theta}^{I} - \tilde{\mu}^{I} \otimes \tilde{\theta}^{I} \otimes PN\right) \oplus \widetilde{C_2}^{I}}$$

5.3.1: Defuzzification by Removal Area Method of Triangular Intuitionistic Fuzzy numbers

Using removal area method [4] total cost and optimum ordered quantity are then defuzzified.

Defining a triangular intuitionistic fuzzy number as: $\tilde{A}^{I}$ ={(a, b, c); (e, b, g)} which represents graphically as follows:

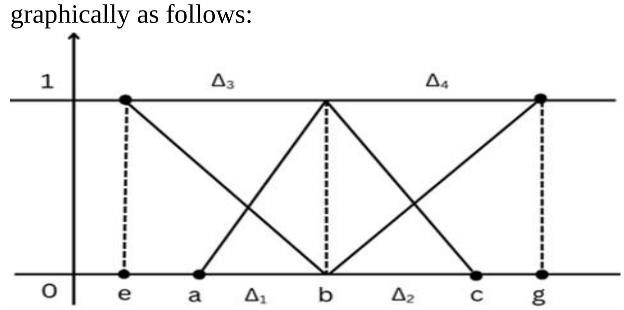

Fig. 6

Considering fuzzy number $\widetilde{A}$ = (a,b,c) for membership degree and fuzzy number $\widetilde{B}$ = (e,b,g) for non-membership degree. Fig. 7, 8, 9, 10, 11 and 12 shows the graphical representation of defuzzification of triangular intuitionistic fuzzy number.

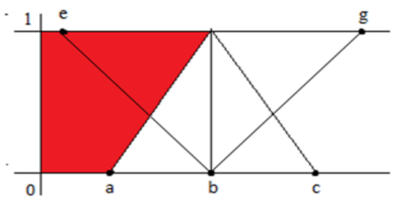

Fig. 7

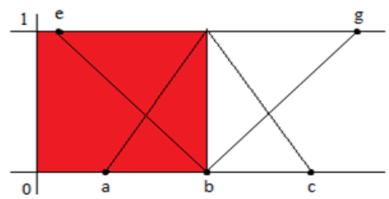

Fig. 8

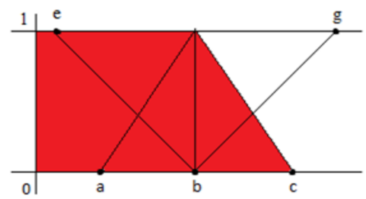

Fig. 9

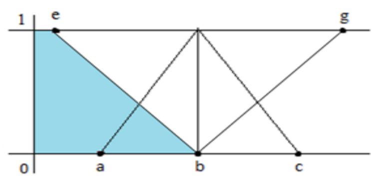

Fig. 10

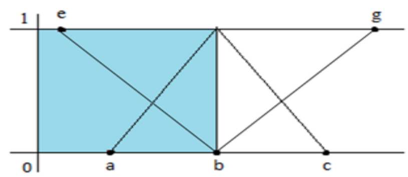

Fig. 11

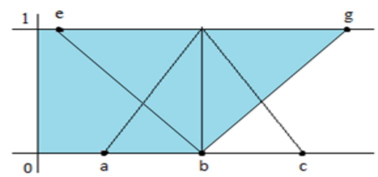

Fig. 12

By considering different types of areas of the corresponding triangular intuitionistic fuzzy number as shown below:

$R_1\ (\widetilde{A}, 0)$ = Area of shaded region for Fig. 7 = $\frac{(a+b)}{2}$

$R_2\ (\widetilde{A}, 0)$ = Area of shaded region for Fig. 8 = b

$R_3\ (\widetilde{A}, 0)$ = Area of shaded region for Fig. 9 = $\frac{(b+c)}{2}$

$R_1\ (\widetilde{B}, 0)$ = Area of shaded region for Fig. 10 = $\frac{(e+b)}{2}$

$R_2\ (\widetilde{B}, 0)$ = Area of shaded region for Fig. 11 = b

$R_3\ (\widetilde{B}, 0)$ = Area of shaded region for Fig. 12 = $\frac{(b+g)}{2}$

Defuzzified value of fuzzy number $\widetilde{A}$ = (a, b, c) for membership degree and fuzzy number $\widetilde{B}$ = (e, b, g) for non-membership degree are defined by

For membership degree, $R(\widetilde{A}, 0) = \frac{R_1(\widetilde{A},0)+R_2(\widetilde{A},0)+R_3(\widetilde{A},0)}{3}$

And for non-membership degree, $R(\widetilde{B}, 0) = \frac{R_1(\widetilde{B},0)+R_2(\widetilde{B},0)+R_3(\widetilde{B},0)}{3}$

Then Defuzzified value of intuitionistic fuzzy number $\widetilde{A}^I = \{(a, b, c);\ (e, b, g)\}$ can be generally defined by,

$R_{\widetilde{A}^I}(\check{D}, 0) = \frac{R(\widetilde{A},0)+R(\widetilde{B},0)}{2}$

$R_{\widetilde{A}^I}(\check{D}, 0) = \frac{a+4b+c+d+4b+f}{12}$

Let Δ_1 = b - a, Δ_2= c - b, Δ_3 = b - e, Δ_4 = g - b

$R_{\widetilde{A}^I}(\check{D}, 0) = A + \frac{1}{12}(\Delta_2 + \Delta_4 - \Delta_3 - \Delta_1)$

Generally, in real life, it is very difficult to consider the deterioration rate θ over a total time period T. Then it is easy to locate the deterioration rate in an interval $\{(\theta - \Delta_1, \theta, \theta + \Delta_2); (\theta - \Delta_3, \theta, \theta + \Delta_4)\}$ where $0 < \Delta_1$, $\Delta_3 < \theta$ and $0 < \Delta_1 \Delta_2$, $0 < \Delta_3\Delta_4$ then $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ will be decided by decision maker and θ is a known number.

Then the defuzzified value of θ is,

$R_{\widetilde{\theta}^I}(\check{D}, 0) = \theta + \frac{1}{12}(\Delta_2 + \Delta_4 - \Delta_1 - \Delta_3)$

Where $\Delta_1 = \theta$ - a, Δ_2 = c - θ, $\Delta_3 = \theta$ - e, Δ_4 = g - θ

Similarly, defuzzified values of $\widetilde{C_1}^I$, $\widetilde{C_2}^I$ and $\widetilde{\mu}^I$ are

$R_{\widetilde{C_1}^I}(\check{D}, 0) = C_1 + \frac{1}{12}(\Delta_2 + \Delta_4 - \Delta_1 - \Delta_3)$

Where $\Delta_1 = C_1$ - a, Δ_2 = c - C_1, $\Delta_3 = C_1$ - e, Δ_4 = g - C_1

$R_{\widetilde{C_2}^I}(\check{D}, 0) = C_2 + \frac{1}{12}(\Delta_2 + \Delta_4 - \Delta_1 - \Delta_3)$

Where $\Delta_1 = C_2$ - a, Δ_2 = c - C_2, $\Delta_3 = C_2$ - e, Δ_4 = g - C_2

$R_{\widetilde{\mu}^I}(\check{D}, 0) = \mu + \frac{1}{12}(\Delta_2 + \Delta_4 - \Delta_1 - \Delta_3)$

Where $\Delta_1 = \mu$ - a, Δ_2 = c - μ, $\Delta_3 = \mu$ - e, Δ_4 = g - μ

5.4 Neutrosophic Model

In this model deterioration rate, holding cost, shortage cost and advertisement cost are considered as neutrosophic parameters and represented by triangular neutrosophic numbers. Then the Neutrosophic total inventory cost is,

$$\widetilde{TC}^N = \left(\widetilde{C_1}^N \oplus C_d \otimes \tilde{\theta}^N\right)\frac{\tilde{S}^{N^2}}{(a-bp)N^\alpha} \oplus \widetilde{C_2}^N\left[\frac{(a-bp)N^\alpha}{2}\left(T - \frac{\tilde{S}^N}{(a-bp)N^\alpha}\right)^2\right] \oplus \tilde{\mu}^N \otimes \left(\tilde{S}^N - \frac{\left(\tilde{S}^N\right)^2 \otimes \tilde{\theta}^N}{(a-bp)N^\alpha}\right)PN$$

Were,

$$\tilde{S}^N = \frac{(a-bp)N^\alpha \otimes \left(\widetilde{C_2}^N \otimes T - \tilde{\mu}^N \otimes PN\right)}{2\left(\widetilde{C_1}^N + C_d \otimes \tilde{\theta}^N - \tilde{\mu}^N \otimes \tilde{\theta}^N \otimes PN\right) \oplus \widetilde{C_2}^N}$$

5.4.1 De-Neutrosophication of Triangular Neutrosophic numbers by Removal area method

Using removal area method [3] total cost and optimum ordered quantity are then defuzzified.

Defining a triangular intuitionistic fuzzy number as: $\widetilde{A}^N = \{(a,b,c);\ (d,b,f);\ (g,b,h)\}$ which represents graphically as follows:

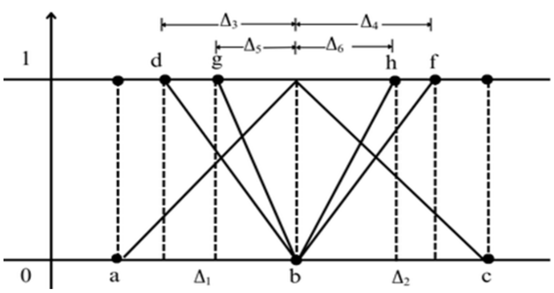

Fig. 13

Then the De-neutrosophication value of Triangular Neutrosophic number $\widetilde{A}^N$ is

$R_{\widetilde{A}^N}(\check{D}, 0) = \frac{a+2b+c+d+2b+f+g+2b+h}{12}$

Let Δ_1 = b - a, Δ_2 = c - b, Δ_3 = b - d, Δ_4 = f - b, Δ_5 = b - g, Δ_6 = h - b

$R_{\widetilde{A}^N}(\check{D}, 0) = b + \frac{1}{12}(\Delta_2 + \Delta_4 + \Delta_6 - \Delta_1 - \Delta_3 - \Delta_5)$

Generally, in real life, it is very difficult to consider the deterioration rate θ over a total time period T. Then it is easy to locate the deterioration rate in an interval {(θ - Δ_1, θ, θ + Δ_2); (θ - Δ_3, θ, θ + Δ_4); (θ - Δ_5, θ, θ + Δ_6)} where 0 < Δ_1, Δ_3, Δ_5 < θ, 0 < $\Delta_1\Delta_2$, 0 < Δ_3 Δ_4 and 0 < $\Delta_5\Delta_6$ then Δ_1, Δ_2, Δ_3, Δ_4, Δ_5, Δ_6 will be decided by decision maker and θ is a known number.

Then the De-Neutrosophication value of θ is,

$R_{\tilde{\theta}^N}(\check{D}, 0) = \theta + \frac{1}{12}(\Delta_2 + \Delta_4 + \Delta_6 - \Delta_1 - \Delta_3 - \Delta_5)$

Where Δ_1 = θ - a, Δ_2 = c - θ, Δ_3 = θ - d, Δ_4 = f - θ, Δ_5 = θ - g, Δ_6 = h - θ

Similarly, De-Neutrosophication value of $\widetilde{C_1}^N$, $\widetilde{C_2}^N$ and $\tilde{\mu}^N$ are,

$R_{\widetilde{C_1}^N}(\check{D}, 0) = C_1 + \frac{1}{12}(\Delta_2 + \Delta_4 + \Delta_6 - \Delta_1 - \Delta_3 - \Delta_5)$

Where $\Delta_1 = C_1$ - a, Δ_2 = c - C_1, $\Delta_3 = C_1$ - d, Δ_4 = f - C_1, $\Delta_5 = C_1$ - g, Δ_6 = h - C_1

$R_{\widetilde{C_2}^N}(\check{D}, 0) = C_2 + \frac{1}{12}(\Delta_2 + \Delta_4 + \Delta_6 - \Delta_1 - \Delta_3 - \Delta_5)$

Where $\Delta_1 = C_2$ - a, Δ_2 = c - C_2, $\Delta_3 = C_2$ - d, Δ_4 = f - C_2, $\Delta_5 = C_2$ - g, Δ_6 = h - C_2

$R_{\tilde{\mu}^N}(\check{D}, 0) = \mu + \frac{1}{12}(\Delta_2 + \Delta_4 + \Delta_6 - \Delta_1 - \Delta_3 - \Delta_5)$

Where Δ_1 = μ - a, Δ_2 = c - μ, Δ_3 = μ - d, Δ_4 = f - μ, Δ_5 = μ - g, Δ_6 = h - μ

VI. NUMERICAL EXAMPLES

6.1 Crisp model

Input values:

a = 100, b = 0.5, P = 4, N = 2, α = 0.3, C_1 = 0.5, C_2 = 5, C_d = 4, T = 1, θ = 0.5, μ = 0.4

Output values:

S = 31.93733, t_1 = 0.2486, S_1 = 90.6586, TC = 272.887

6.2 Defuzzification by Removal Area Method

Table 1

Centroid values of θ for various cases of distance from centre									
Case	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Centroid values of Fuzzy ‘θ’	Centroid values of Intuitionistic Fuzzy ‘θ’	Centroid values of Neutrosophic Fuzzy ‘θ’
1	0.1	0.05	0.1	0.1	0.05	0.05	0.491667	0.495833	0.495833
2	0.12	0.08	0.2	0.15	0.15	0.2	0.493333	0.4925	0.496667
3	0.2	0.25	0.3	0.15	0.2	0.25	0.508333	0.491667	0.495833
4	0.25	0.3	0.3	0.35	0.2	0.2	0.508333	0.508333	0.508333
5	0.05	0.1	0.125	0.25	0.15	0.1	0.508333	0.514583	0.510417

Table 2

Centroid values of C_1 for various cases of distance from centre									
Case	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Centroid values of Fuzzy ‘C_1’	Centroid values of Intuitionistic Fuzzy ‘C_1’	Centroid values of Neutrosophic Fuzzy ‘C_1’
1	0.1	0.05	0.1	0.1	0.05	0.05	0.4916667	0.495833	0.495833
2	0.15	0.1	0.2	0.15	0.15	0.2	0.4916667	0.491667	0.495833
3	0.2	0.25	0.2	0.3	0.2	0.25	0.5083333	0.5125	0.516667
4	0.25	0.3	0.3	0.4	0.2	0.15	0.5083333	0.5125	0.508333
5	0.05	0.1	0.125	0.25	0.15	0.1	0.5083333	0.514583	0.510417

Table 3

Centroid values of C_2 for various cases of distance from centre									
Case	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Centroid values of Fuzzy ‘C_2’	Centroid values of Intuitionistic Fuzzy ‘C_2’	Centroid values of Neutrosophic Fuzzy ‘C_2’
1	0.1	0.05	0.2	0.1	0.05	0.05	4.991667	4.9875	4.9875
2	0.15	0.1	0.25	0.15	0.15	0.2	4.991667	4.9875	4.991667
3	0.2	0.25	0.3	0.2	0.2	0.25	5.008333	4.995833	5
4	0.25	0.3	0.4	0.45	0.2	0.15	5.008333	5.008333	5.004167
5	0.05	0.1	0.125	0.25	0.15	0.1	5.008333	5.014583	5.010417

Table 4

Centroid values of μ for various cases of distance from centre									
Case	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Centroid values of Fuzzy ‘μ’	Centroid values of Intuitionistic Fuzzy ‘μ’	Centroid values of Neutrosophic Fuzzy ‘μ’
1	0.1	0.05	0.1	0.1	0.05	0.03	0.391667	0.395833	0.394167
2	0.12	0.1	0.2	0.15	0.1	0.15	0.396667	0.394167	0.398333
3	0.2	0.25	0.2	0.15	0.2	0.25	0.408333	0.4	0.404167
4	0.25	0.2	0.3	0.4	0.2	0.15	0.391667	0.404167	0.4
5	0.05	0.1	0.125	0.25	0.15	0.1	0.408333	0.414583	0.410417

6.2.1 Comparison of Fuzzy, Intuitionistic Fuzzy, Neutrosophic Fuzzy Model

Table 5: Total Cost (for a = 100, b = 0.5, P = 4, T = 1 and C_d = 4)

N	α	Fuzzy				Intuitionistic				Neutrosophic			
		S	TC	t_1	S_1	S	TC	t_1	S_1	S	TC	t_1	S_1
1	0.3	40.1122	175.8995	0.3729	61.4529	39.8305	176.5926	0.37027	61.7139	39.8771	176.3805	0.3707	61.6752
		39.9334	176.605	0.3713	61.6124	39.9939	176.1812	0.37187	61.5567	39.7705	177.0149	0.3697	61.7691
		39.2297	179.2082	0.3644	62.2913	39.7087	177.3738	0.3695	61.7891	39.4872	178.2008	0.3673	61.9998
		39.6856	177.1161	0.3682	61.9129	39.3056	178.7526	0.36502	62.2283	39.4297	178.0915	0.3661	62.1252
		39.2297	179.2082	0.3644	62.2913	38.9074	180.423	0.36126	62.5969	39.1218	179.6147	0.3633	62.3937
	0.4	40.1122	175.8995	0.3729	61.4529	39.8305	176.5926	0.37027	61.7139	39.8771	176.3805	0.3707	61.6752

		39.9334	176.605	0.3713	61.6124	39.9939	176.1812	0.37187	61.5567	39.7705	177.0149	0.3697	61.7691
		39.2297	179.2082	0.3644	62.2913	39.7087	177.3738	0.3695	61.7891	39.4872	178.2008	0.3673	61.9998
		39.6856	177.1161	0.3682	61.9129	39.3056	178.7526	0.36502	62.2283	39.4297	178.0915	0.3661	62.1252
		39.2297	179.2082	0.3644	62.2913	38.9074	180.423	0.36126	62.5969	39.1218	179.6147	0.3633	62.3937
2	0.3	32.8409	270.613	0.2555	89.8305	32.2806	271.4875	0.25123	90.3404	32.4539	271.1133	0.2525	90.1874
		32.3071	271.7552	0.2515	90.3079	32.5206	271.0522	0.25309	90.1163	32.0673	272.1869	0.2496	90.5318
		31.0367	275.1053	0.2418	91.4835	31.8283	272.7999	0.24804	90.7252	31.3754	273.9155	0.2446	91.1409
		32.7569	271.4235	0.2543	89.9671	31.4347	274.2348	0.24467	91.1318	31.8512	273.1493	0.2477	90.7643
		31.0367	275.1053	0.2418	91.4835	30.3627	276.7335	0.23664	92.1007	30.8119	275.6514	0.2401	91.6893
	0.4	35.198	290.0359	0.2555	96.278	34.5975	290.9731	0.25123	96.8244	34.7833	290.5721	0.2525	96.6605
		34.6259	291.26	0.2515	96.7896	34.8547	290.5066	0.25309	96.5842	34.3689	291.7227	0.2496	97.0296
		33.2643	294.8505	0.2418	98.0496	34.1127	292.3797	0.24804	97.2369	33.6273	293.5753	0.2446	97.6824
		35.108	290.9045	0.2543	96.4243	33.6909	293.9176	0.24467	97.6726	34.1373	292.7541	0.2477	97.2787
		33.2643	294.8505	0.2418	98.0496	32.5419	296.5956	0.23664	98.711	33.0234	295.4359	0.2401	98.2702

VII. CONCLUSION

Table 1, Table 2, Table 3 and Table 4 shows the centroid values of θ, C_1, C_2 and μ respectively in fuzzy, intuitionistic fuzzy and neutrosophic fuzzy environments for various cases of distance from centre. Using centroid values of θ, C_1, C_2 and μ, sensitivity analysis for optimum values of total cost, order level and shortage level of fuzzy, intuitionistic fuzzy and neutrosophic fuzzy models are presented in Table 5. The optimum values of Neutrosophic model are less fluctuate as compare to optimum values of fuzzy, intuitionistic fuzzy models. Hence Neutrosophic approach seems to be more realistic. Decision maker can select any appropriate case for better results.

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