

Nonlinear Dynamic Behavior Analysis of Spherical Tank Level System

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Abstract- Spherical tanks are widely used in industrial processes due to their structural strength and ability to withstand high pressure. However, their unique geometry introduces strong nonlinearities into the level–volume relationship, making the level control problem significantly more complex compared to linear tank structures. The objective of this study is to investigate and analyse the nonlinear dynamic behaviour of a spherical tank level system through experimental validation and mathematical characterization. An open-loop analysis was performed using a real-time setup consisting of a differential pressure transmitter, DAQ module, pump, and MATLAB/Simulink interface. The dynamic response was recorded for various inflow conditions to identify key parameters such as steady-state gain, time constant, and dynamic sensitivity across different level ranges. A conventional closed-loop PID controller was then implemented to observe its behaviour under nonlinear conditions. The experimental results show that the spherical tank has different dynamics depending on the region: faster level variation occurs in the bottom and top regions, and slower variation occurs in the mid-level region. This outcome points to the shortcoming of fixed-gain linear controllers in view of the necessity of advanced nonlinear or adaptive control methods for improved performance. In this respect, the comprehensive analysis presented herein constitutes a sound basis for future research work concerning the development of intelligent control strategies.

Keywords: Spherical tank, nonlinear system, PID control, dynamic modelling, liquid level system, real-time analysis

I. INTRODUCTION

Liquid-level control can find its application in any of a number of widely differing industrial processes, including chemical manufacture, wastewater treatment, pharmaceutical production, and storage of petrochemicals. For industries dealing with such

materials, liquid level regulation has to be precise and stable for safety, reliability, or even quality control. Although many of the industrial tanks operate in a linear or nearly linear manner, spherical tanks pose a special challenge because of their constantly changing cross-sectional area.

A key difference between spherical and cylindrical tanks is that the rate of level change is not uniform. In cylindrical tanks, a constant inflow increases the height at a relatively predictable rate. In contrast, in spherical tanks, the area available for storing liquid changes significantly with height. At the bottom and top regions, the area is small, causing faster changes in height, whereas in the middle region, the cross-sectional area is maximal, causing slower changes. This behaviour impacts:

- open-loop characteristics,
- controller design, and
- closed-loop performance.

Traditional linear controllers, such as PID, are only effective within a certain operating zone. If the operating point shifts, the dynamics become significantly different and may lead to instability, overshoot, or slow response.

Therefore, analysing and understanding nonlinear behaviour is crucial before attempting controller design. This paper focuses entirely on such analysis to:

1. Provide a complete dynamic characterization of the spherical tank.
2. Identify nonlinear effects from real-time experimental behaviour.
3. Establish baseline behaviour for future advanced controller research.

II. LITERATURE REVIEW AND RESEARCH MOTIVATION

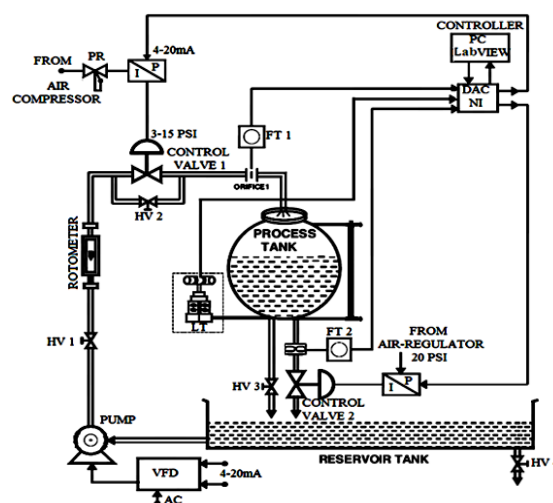
The nonlinear behaviour of liquid-level processes has been extensively examined in industrial control literature due to its relevance in chemical, petrochemical, and water treatment applications. Several studies have specifically focused on spherical and conical tanks because their varying cross-sectional area introduces strong nonlinear dynamics that challenge conventional linear control strategies.

Early works such as Padhee & Tyagi (2012) demonstrated that spherical tanks cannot be effectively controlled using fixed-gain PID controllers because the process gain and time constant vary significantly with height. Subsequent research by Sivakumar et al. (2016) experimentally established that nonlinear tanks require region-wise identification to accurately characterize dynamic behaviour. Similar conclusions were reported by Nisha & Chidambaram (2017), who showed that the mid-height region of spherical tanks exhibits the slowest response due to maximum cross-sectional area.

Advanced control strategies have been proposed to overcome these challenges. Model-based adaptive control (Zhao et al., 2019) and gain-scheduled control (Ranganathan et al., 2018) have shown improved tracking performance across varying operating points. In recent years, reinforcement learning and data-driven techniques have also been applied to nonlinear tank systems. Chen et al. (2021) demonstrated that Deep Q-Networks can autonomously adapt to nonlinearities without explicit modelling, while Proximal Policy Optimization (PPO) was used by Patel & Kumar (2022) for optimal inflow regulation in nonlinear storage systems.

Despite these advancements, most studies focus on simulation-based validation rather than real-time hardware experimentation. Only a limited number of works have analysed the open-loop and closed-loop behaviour of spherical tanks using real industrial instrumentation. Therefore, the present work contributes by experimentally characterizing the nonlinear dynamics of a spherical tank through real-time measurements, providing a practical foundation for future advanced control research.

III. SYSTEM DESCRIPTION



The experimental setup consists of the following components:

S. No.	Parts/Field instruments	Specifications/Description
1	Spherical Tank	Material-Stainless steel; Diameter-43cm;(LRV = 436mmH ₂ O, URV= 866mmH ₂ O), Volume-42 Liters
2	Pump and Variable Frequency Drive	VFD-ABB-ACS 350;3Φ;4-20mA to 0-50Hz; Pump – Grundfos – JP5 Centrifugal pump, 3-Phase
3	DPT for Flow measurement (FT)	6200 T series, Range: 0 to 6500 mmH ₂ O, Output: 4 to 20 mA + HART
4	DPT for Level measurement (LT)	6200 T series, Range: 0 to 6500 mmH ₂ O, Output: 4 to 20 mA + HART
5	I to P converter	Input – 4 to 20 mA; Output – 3 to 15 psi
6	Control valve	Linear, Air to open, Body – 1", trim ½"
7	Rotameter	150 – 1500 LPH

3.1 Spherical Tank

- Material: Stainless steel
- Diameter: 43 cm
- Total volume: 42 L

The spherical geometry causes the nonlinear level–area relationship, making the tank an ideal candidate for nonlinear system analysis.

3.2. Pump and VFD (Variable Frequency Drive)

Inlet flow is controlled by a centrifugal pump driven by a VFD. The VFD converts a 4–20 mA control signal into a frequency range of 0–50 Hz.

3.3 Differential Pressure Transmitter (DPT)

A 4–20 mA output transmitter measures the hydrostatic pressure corresponding to the liquid level.

Range: 0–6500 mmH₂O

Output: 4–20 mA proportional to level

3.4 Data Acquisition (DAQ) System

The ATmega2560 / NI USB 6211 DAQ board collects data from the DPT and interfaces directly with MATLAB/Simulink for real-time processing.

It performs:

- ADC conversion
- Signal conditioning
- Communication with Simulink models

3.5 Actuator (Control Valve / PWM Output)

For closed-loop operation, a PWM-driven actuator adjusts the inflow based on controller output.

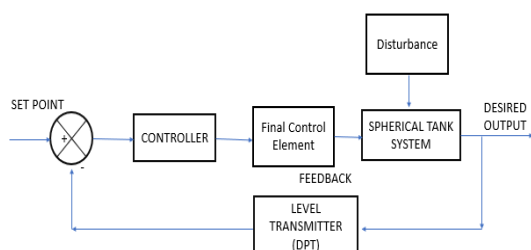
3.6 MATLAB/Simulink Environment

MATLAB is used for:

- Real-time plotting
- Data logging
- Controller implementation
- Curve fitting
- Parameter estimation

The system forms a complete real-time platform suitable for analyzing nonlinear liquid-level dynamics.

Block Diagram:



The block diagram below is the closed-loop control system that maintains liquid level in a spherical tank. A set point or desired liquid level serves as an input to the system. This set point is compared with the actual liquid level to generate an error signal. The controller processes this error signal to determine the corrective action required. The controller's output is then sent to the spherical tank system, which adjusts the inflow or outflow of liquid accordingly. As the liquid level changes, the level transmitter, also known as a Differential Pressure Transmitter (DPT), continuously measures the actual liquid level. This measurement is converted into a feedback signal and sent back to the input of the system. The feedback allows the system to correct any deviations from the desired output. By constantly comparing the set point with the measured level, the system ensures stability. The closed-loop structure helps minimize overshoot and reduces steady-state error. It also improves the system's response to disturbances. The controller can be of different types, such as PID, depending on the required precision and dynamics. The spherical tank geometry is considered in the level measurement to ensure accurate volume calculations. Overall, this feedback control loop enhances the reliability and efficiency of liquid level regulation. Such systems are widely applied in industrial processes, where precise liquid handling plays an important role.

IV. MATHEMATICAL MODELLING AND NONLINEAR DYNAMICS

Spherical tanks do not follow a linear area–height relationship. The cross-sectional area at any height (h) in a sphere of radius (R) is:

$$A(h) = \pi(2Rh - h^2)$$

This leads to the nonlinear dynamic equation:

$$dtdh = \pi(2Rh - h^2)Q_{in} - Q_{out}$$

Key Nonlinear Characteristics

1. Bottom Region (0–20% height)

- Cross-sectional area is small
- Level rises very quickly
- Highly sensitive to small disturbances

2. Middle Region (20%–80% height)

- Area is maximum
- Level rises slowly
- Natural damping observed
- Difficult region for linear controllers

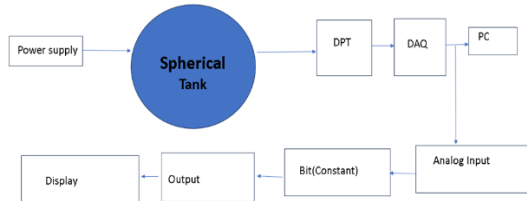
3. Top Region (80%–100% height)

- Area decreases again
- Level accelerates rapidly
- Increased risk of overshoot

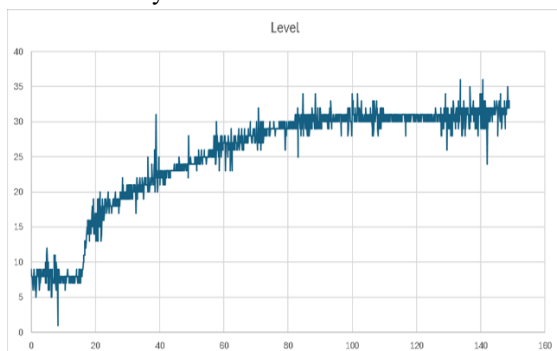
This behaviour directly impacts both open-loop and closed-loop performance. Understanding these nonlinearities is essential before designing any control strategy.

V. EXPERIMENTAL ANALYSIS

5.1 Open-Loop Response



A 24-volt power supply energizes all the components in the setup and thus serves as a primary electrical feed for the system. The DPT utilizes this power to produce a 4–20 mA output signal corresponding to the pressure differential across the tank. This current signal, in turn, is connected to the data acquisition unit (AT Mega 2560) interfacing between the transmitter and the computer for further processing. The microcontroller is connected to a computer running MATLAB Simulink, thus allowing online monitoring and control of the whole system. In Simulink, input values are derived from DAQ, where the analog voltage is converted into digital values ranging between 0 to 1023 bits. Since one volt is equivalent to roughly 204.6 bits, this digital representation is used to calculate the tank's level. Using the digital output block, the model also generates corresponding output signals. The Simulink environment includes visual display elements, allowing the system's response to be shown through graphs or other real-time visual indicators. This setup helps users continuously observe and interpret the behavior of the spherical tank control system.



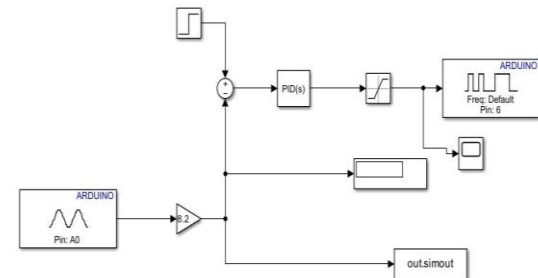
5.2 Closed-Loop Response

A conventional PID controller was used to evaluate feedback performance.

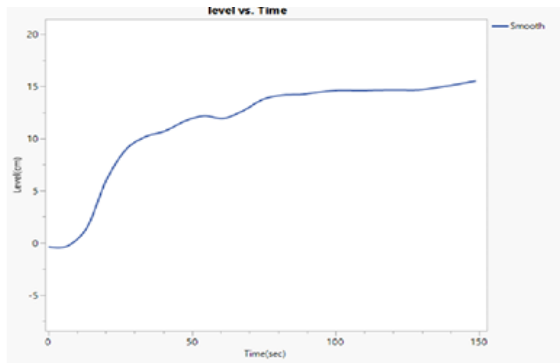
PID Tuning Parameters

- ($K_p = 1.5$)
- ($K_i = 0.03$)
- ($K_d = 0.1$)

Closed-Loop Behavior Observations



- The presented Simulink model demonstrates a real-time implementation of a PID controller to in a spherical tank using an Arduino interface. The control system utilizes an analog input block connected to Pin A0 of the Arduino to acquire real time data from a differential pressure transmitter, which senses the current level in the tank.
- Since the Arduino provides the analog input as a 10-bit value between 0 and 1023, a gain block of 0.041 scales this to actual tank levels in centimeters between 0 and 42 cm. A constant block defines the desired setpoint-e.g., 15 cm-and the difference between this setpoint and measured value is determined to provide the error signal.
- The PID block processes this error, where the proportional, integral, and derivative gains are assigned to calculate the control output. The output is then fed into a PWM output block configured on Pin 6 of the Arduino to control the final control element, typically a motor or valve, which adjusts the flow to maintain the desired water level.
- A scope block is used for real-time graphical monitoring of the system response, while the simout block exports data to Excel for further analysis. This integrated approach allows for effective tuning and testing of PID parameters in a real-world level control application using MATLAB Simulink.



5.3 Comparative Nonlinear Region Behaviors

1. Rise Time Variation

1. Slowest in mid region
2. Fastest near bottom and top

2. Overshoot Tendency

- Higher near top region
- Minimal in mid region

3. Disturbance Handling

- Nonlinearity amplifies effects differently at different heights

4. Control Effort Variation

- Actuator needs stronger effort near mid-region
- Very small adjustments produce large effects near top/bottom

These patterns reinforce that the spherical tank cannot be treated as a linear system.

VI. RESULTS AND DISCUSSION

Key Findings

1. Nonlinearity is dominant and unavoidable

The tank cannot be controlled with a single global linear model due to continuously varying cross-section.

2. A single PID tuning is insufficient

PID tuned for one operating point underperforms in other regions.

3. Open-loop behavior provides critical insight

Understanding the natural dynamic response is essential before controller design.

4. Closed-loop instabilities are geometry-driven

Nonlinear geometry creates unpredictable transient behavior under fixed-gain control.

5. Study forms baseline for advanced control

This analysis provides the required groundwork for methods like:

- Gain-scheduled control

- Adaptive control
- Reinforcement learning (future work)
- Multi-model control

This paper fulfills the goal of analyzing nonlinear spherical tank behavior without implementing advanced optimization.

VII. CONCLUSION

This work presented a comprehensive study of the nonlinear dynamic behavior of a spherical tank level system. Through experimental open-loop and closed-loop analysis, the inherent nonlinearity arising from spherical geometry was identified and characterized. The results clearly indicate that the system exhibits region-dependent dynamics, making fixed-gain linear controllers insufficient for maintaining uniform performance. The findings emphasize the need for advanced, adaptive, or intelligent control techniques for efficient level regulation. This detailed behavioral analysis forms a strong foundation for further research into optimization and machine learning-based control strategies.

VIII. FUTURE WORK

Based on the findings and limitations, the following directions are proposed for future research:

1. Implementing adaptive and gain-scheduled PID controllers
To handle region-dependent dynamics more effectively.
2. Developing multi-model identification
Creating separate models for bottom, mid, and top regions and switching between them dynamically.
3. Applying reinforcement learning techniques
Using algorithms such as Deep Q-Learning, PPO, and Actor-Critic methods for autonomous control.
4. Incorporating actuator and sensor nonlinearities
Detailed modeling of valve characteristics, pump head variations, and transmitter hysteresis.
5. Testing disturbance rejection capability
Introducing controlled outflow disturbances to examine robustness.
6. Extending the setup to multi-tank systems
Understanding coupled nonlinear dynamics, which is common in industrial plants.

7. Hardware-in-the-loop optimization
Real-time optimization-based control can be tested using MPC or RL-based MPC.

These enhancements will help transition the work from baseline experimental characterization to advanced nonlinear control development.

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