System Modelling and PID Controller Design for Quadruple Tank Water Level Regulation

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Abstract: When multiple tanks interact through shared flows, changes in one tank can alter the behaviour of the others, making level regulation more demanding than in single-tank systems. This work investigates such interactions using the Quadruple Tank System, a four-tank configuration commonly used to study multivariable control challenges. A mathematical model of the system is developed based on basic flow dynamics and mass-balance principles. Using this model, a PID controller is designed to regulate the levels of the two lower tanks, with controller gains determined through manual tuning and evaluated through simulation. The results show that the controller guides the tank levels toward their setpoints, although coupling effects cause overshoot and slower settling. These findings serve as a starting point for future improvement using optimization-based tuning methods.

Keywords: Quadruple Tank System, PID Control, Process Dynamics, Multivariable Modelling, Nonlinear Systems.

I. INTRODUCTION

Controlling liquid levels is crucial in industrial processes such as chemical production, wastewater management, and power generation. Real-world systems often consist of multiple interacting units where changes in one variable affect others. The Quadruple Tank System (QTS) serves as a representative model for such multivariable processes.

The QTS consists of four interconnected tanks fed by two pumps, with the water levels of the lower tanks being the main outputs of interest. The system can operate in minimum-phase or non-minimum-phase modes depending on flow split ratios.

Although advanced controllers such as Model Predictive Control (MPC) and evolutionary algorithms have been explored in literature, conventional PID controllers remain the preferred choice in industries due to simplicity, reliability, and ease of implementation. This paper focuses on modelling, linearization, and PID control of the system, providing a clear understanding of its dynamics and controller performance.

II. LITERATURE SURVEY

The Quadruple Tank System (QTS) is widely recognized in research and academia as a benchmark for studying interacting multivariable systems. Its unique structure, with two lower controlled tanks influenced by two upper tanks, allows researchers to explore both minimum-phase and non-minimum-phase behaviours depending on flow-splitting ratios. This characteristic makes it an ideal platform for evaluating classical and modern control strategies.

Several studies have demonstrated the challenges of controlling the QTS using conventional PID controllers due to strong interactions between loops. Open-loop experiments show that disturbances in one tank can propagate to others, making simple decoupled control insufficient for optimal performance. Researchers have also highlighted that tuning PID gains manually is time-consuming and sensitive to changes in system parameters.

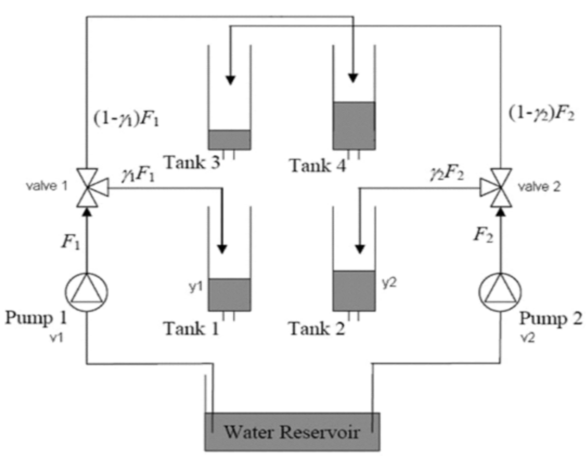
Advanced control methods have been explored to address these limitations. Adaptive control and fuzzy logic controllers have been used to handle nonlinearities, while Model Predictive Control (MPC) has proven effective in managing constraints and interactions. Optimization techniques such as Particle Swarm Optimization (PSO) and Genetic Algorithms

(GA) have been applied to automate PID tuning, improving transient response and reducing overshoot.

Despite these advancements, a clear consensus exists that accurate system modelling and classical PID implementation remain essential. Linearized models of the QTS provide insight into dynamic behaviour and interactions, which is crucial before attempting optimization or adaptive strategies. Hence, studying PID control on a well-understood linearized model provides a strong foundation for future advanced controller design.

III. SYSTEM DESCRIPTION

The Quadruple Tank System (QTS) consists of four interconnected vertical tanks supplied by two pumps, with water recirculated from a common reservoir. Tanks 1 and 2 are the primary controlled tanks, while Tanks 3 and 4 act as auxiliary tanks, influencing the lower tanks through gravity-driven flow.



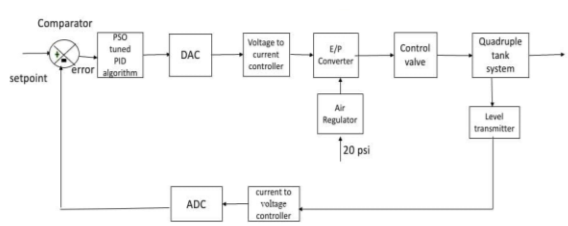
The experimental setup consists of the following components:

S. No	Field Instruments	Specifications
1.	Quadruple Tank	Diameter:30 cm Level:60 cm, Volume:3 Liters (per tank)
2.	Pump	270 RPM Voltage(0-230V) AC/DC,50Hz Discharge:225 to 400LPH
3.	Level Transmitter	DPT, 24V DC
4.	Pressure Regulator	Input pressure:18kg/cm^2 Output pressure:(0.2-1) kg/cm^2
5.	Control Valve	RK valves input signal (4 t0 20) mA DC*1
6.	Rotameter	Range:(10-100) Liter/Hour
7.	I/P Converter	ABB, Input:(4-20) mA DC ,20 Psi Output: 3 to 15 psi

3.1 SYSTEM COMPONENTS:

- ❖ Tanks (T1, T2, T3, T4): Transparent acrylic tanks for easy visualization of water levels.
- ❖ Pumps (P1, P2): Supply water to the tanks; pump flow is proportional to control voltage.
- ❖ Flow splitters: Adjust the fraction of pump flow delivered to upper and lower tanks.
- ❖ Orifices: Allow outflow from each tank based on water height.
- ❖ Level sensors: Measure real-time water levels for feedback control.
- ❖ Reservoir: Collects tank outflow and supplies pumps for recirculation

IV. BLOCK DIAGRAM



V. SYSTEM MATHEMATICAL MODELLING

The QTS is modelled using mass balance equations and flow dynamics.

5.1 MASS BALANCE:

For each tank, the rate of change of water level is:

$$A_i \frac{dh_i}{dt} = q_{in,i} - q_{out,i}$$

where:

- A_i= cross-sectional area of tank i
- h_i= water level in tank i
- $q_{in,i}$= inflow rate to tank i
- $q_{out,i}$= outflow rate from tank i

5.2 OUTFLOW DYNAMICS:

Outflow through each tank follows Torricelli's law:

$$q_{out,i} = a_i\sqrt{2gh_i}$$

where:

- a_i= outlet orifice area
- g= acceleration due to gravity

5.3 PUMP INFLOW:

Pump flow is proportional to input voltage:

$$q_{pump,j} = k_j u_j$$

Flow-splitting ratios $\gamma_1$and $\gamma_2$determine how the pump flow is distributed between upper and lower tanks:

$$q_{in,T3} = \gamma_1 q_{pump,1}, q_{in,T1} = (1-\gamma_1)q_{pump,1}$$

$$q_{in,T4} = \gamma_2 q_{pump,2}, q_{in,T2} = (1-\gamma_2)q_{pump,2}$$

These ratios directly influence the system's minimum-phase or non-minimum-phase behaviour.

5.4 LINEARIZATION:

The nonlinear equations are linearized around a steady-state operating point to design controllers. The linearized transfer function for each tank is approximated as:

$$G(s) = \frac{K}{\tau s + 1}$$

where:

- K= process gain
- τ= time constant

Off-diagonal transfer functions represent coupling between tanks, indicating interactions that make multivariable control necessary.

5.4 MASS BALANCE EQUATION:

The dynamic behaviour of the Quadruple Tank System is governed by the mass balance equations given below. Each equation represents the rate of change of water level in a tank, considering inflow and outflow.

$$\begin{aligned}
A_1 \frac{dh_1}{dt} &= (1-\gamma_1)k_1u_1 + a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1}, \\
A_2 \frac{dh_2}{dt} &= (1-\gamma_2)k_2u_2 + a_4\sqrt{2gh_4} - a_2\sqrt{2gh_2}, \\
A_3 \frac{dh_3}{dt} &= \gamma_1 k_1 u_1 - a_3\sqrt{2gh_3}, \\
A_4 \frac{dh_4}{dt} &= \gamma_2 k_2 u_2 - a_4\sqrt{2gh_4}.
\end{aligned}$$

VI. MATLAB MODELLING AND CONTROLLER IMPLEMENTATION

MATLAB was used as the primary tool for analyzing the behaviour of the Quadruple Tank System and for designing the PID controller. After developing the mathematical model, the system parameters were used to obtain the linearized transfer function. MATLAB's control toolbox provided an efficient environment to simulate the system dynamics, evaluate controller performance, and verify stability.

The modelling process began by defining the geometric and flow characteristics of the tanks and incorporating them into a state–space model. This model was then simplified into a first-order transfer function representation to support PID design. The software environment allowed iterative tuning and fine adjustments while observing the effect of each parameter change on the transient response.

Using MATLAB ensured consistent and accurate analysis without the uncertainties associated with physical hardware experiments during the initial design phase. It enabled rapid verification of the controller under different conditions and provided visual insights through step response plots and system characteristics.

VII. TRANSFER FUNCTION

By taking the Laplace transform of the linearized equations and expressing in standard form, the system can be represented as a Multi-Input Multi-Output (MIMO) transfer function matrix relating the tank levels (h1,h2) to the pump voltages (v1,v2).

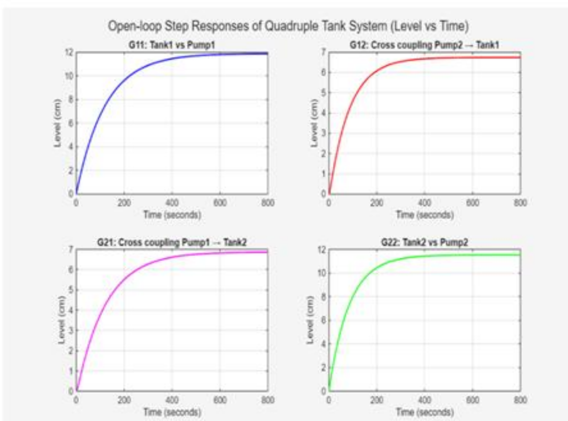
$$G(s) = \begin{bmatrix} \frac{11.89}{1+121.4\,s} & \frac{6.738}{(1+84.73\,s)(1+3.109\,s)} \\ \frac{6.875}{(1+121.4\,s)(1+3.967\,s)} & \frac{11.53}{1+84.73\,s} \end{bmatrix}$$

VIII. EXPERIMENTAL ANALYSIS

8.1 OPEN-LOOP RESPONSE:

The open-loop response analysis of the Quadruple Tank System was conducted to understand the natural behaviour of the interacting tanks without any controller action. This step is essential because the Quadruple Tank System exhibits strong coupling between tanks, meaning that a change in one pump input affects both tank levels.

When the pump voltages were applied individually, the resulting water-level changes in the lower tanks revealed the inherent dynamic characteristics of the process. The system includes two inlet pumps, each supplying flow to both upper and lower tanks depending on the valve position. As a result, the input-to-output behaviour does not follow a simple first-order pattern.



8.2 CLOSED LOOP RESPONSE:

The closed-loop response was analysed after implementing the PID controller for both tank levels. The aim was to evaluate how effectively the controller maintains the tank levels at their desired setpoints while compensating for interactions and disturbances.

8.2.1 PID CONTROLLER DESIGN:

A classical Proportional–Integral–Derivative (PID) controller is used to regulate the lower-tank levels of the Quadruple Tank System. The continuous-time control law is:

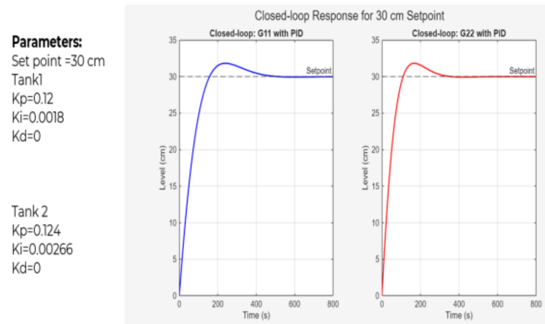
$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau + K_d \frac{de(t)}{dt}$$

Where;

- $e(t) = r(t) - y(t)$ is the tracking error (setpoint minus measured level),
- K_p is the proportional gain,
- K_i is the integral gain, and
- K_d is the derivative gain.

Parameters:

- $K_p = 5.0$
- $K_i = 1.0$
- $K_d = 0.1$



IX. RESULT

The overall analysis of the Quadruple Tank System showed that the process exhibits slow dynamics, nonlinear flow characteristics, and strong interaction between the tanks. These factors make precise level regulation challenging when operated without control. After implementing the PID controller, the system demonstrated a significant improvement in behaviour. The controlled response was faster, more stable, and showed smooth tracking of the desired water levels. Overshoot was minimized, and the water levels settled quickly with no noticeable long-term error. The controller also reacted effectively to disturbances, restoring the levels to the required setpoints with minimal deviation.

In addition, the PID controller was able to reduce the impact of coupling between the tanks, resulting in more coordinated and predictable behaviour across the entire system. Overall, the controller greatly enhanced the reliability and performance of the multivariable tank process.

X. CONCLUSION

The study successfully modelled and controlled the Quadruple Tank System using a PID-based strategy. The dynamic characteristics of the system—such as nonlinear outflow, interacting loops, and slow response—highlighted the need for an effective controller to maintain stable water-level regulation.

The implemented PID control approach improved the transient and steady-state performance, ensured smooth and stable operation, and provided strong disturbance-handling capability. The controller also compensated well for the interaction between the tanks, resulting in consistent and accurate level maintenance.

The findings confirm that PID control is an effective and reliable method for managing the behaviour of an interacting multivariable tank system. This work provides a strong foundation for exploring advanced control strategies and potential real-time implementation in future development.

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