

# Dominating Sets and Domination Polynomials of Triangular Ladders

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**Abstract**—Let  $G = (V, E)$  be a simple graph. A set  $D \subseteq V$  is a dominating set of  $G$  if every vertex in  $V - D$  is adjacent to atleast one vertex in  $D$ . Let  $TL_n$  be a triangular ladder graph with order  $2n$ , and let  $\mathcal{D}(TL_n, i)$  denote the family of all dominating sets of  $TL_n$  with cardinality  $i$ . Let  $d(TL_n, i) = |\mathcal{D}(TL_n, i)|$ . In this paper, we obtain a recursive formula for  $d(TL_n, i)$  and study the dominating sets of  $TL_n$ . Using this formula, we construct the polynomial,

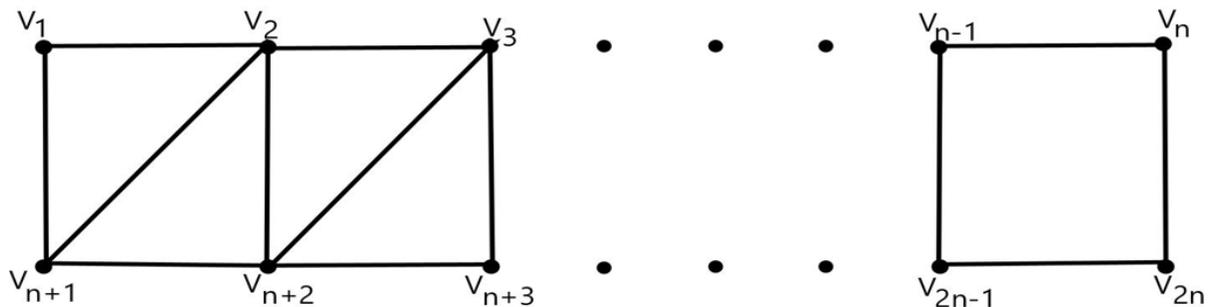
$$D(TL_n, x) = \sum_{i=\lfloor \frac{n}{3} \rfloor + 1}^n d(TL_n, i) x^i$$

which we call domination polynomial of  $TL_n$  and some properties of this polynomial are studied.

**Index Terms**—Dominating sets, Domination Number, Domination Polynomials

## I. INTRODUCTION

Let  $G = (V, E)$  be a simple graph of order  $|V| = n$ . A set  $D \subseteq V$  is a dominating set of  $G$ , if every vertex in  $V - D$  is adjacent to at least one vertex in  $D$ . The domination number  $\gamma(G)$  is the minimum cardinality



$$V(TL_n) = \{v_1, v_2, \dots, v_n, v_{n+1}, v_{n+2}, \dots, v_{2n}\} \ \& \ E(TL_n) = \{(v_1, v_2), (v_2, v_3), \dots, (v_{n-1}, v_n), (v_{n+1}, v_{n+2}), (v_{n+2}, v_{n+3}), \dots, (v_{2n-1}, v_{2n}), (v_1, v_{n+1}), (v_2, v_{n+2}), \dots, (v_n, v_{2n}), (v_2, v_{n+1}), (v_3, v_{n+2}), \dots, (v_n, v_{2n-1})\}$$

of a dominating set in  $G$ . Let  $G_n$  be graph with order  $n$  and let  $G_n^i$  be the family of dominating sets of a graph  $G_n$  with cardinality  $i$  and let  $d(G_n, i) = |G_n^i|$ . We call the polynomial  $(G_n, x) = \sum_{i=\gamma(G)}^n d(G_n, i) x^i$ , the domination polynomial of graph  $G$ . (Alikhan, Peng, 2010). Let  $TL_n^i$  be the family of dominating sets of a triangular ladder  $TL_n$  with cardinality  $i$  and let  $d(TL_n, i) = |TL_n^i|$ . We call the polynomial  $D(TL_n, x) = \sum_{i=\lfloor \frac{n}{3} \rfloor + 1}^n d(TL_n, i) x^i$ , the domination polynomial of triangular ladder.

## II. DOMINATING SETS OF TRIANGULAR LADDERS

Consider two paths  $v_1, v_2, \dots, v_n$  and  $v_{n+1}, v_{n+2}, \dots, v_{2n}$ . Join each pair of vertices  $v_i v_{i+1}$ ;  $i = 1, 2, \dots, n-1, n+1, \dots, 2n-1$ ;  $v_i v_{n+i-1}$ ;  $i = 2, \dots, n$ , and  $v_i v_{n+i}$ ;  $i = 1, 2, \dots, n$ . The resulting graph is a triangular ladder. Let  $TL_n$  be a triangular ladder with  $2n$  vertices. Label the vertices of  $TL_n$  as given in the following figure

As usual we use  $\lfloor x \rfloor$  for the largest integer less than or equal to  $x$  and  $\lceil x \rceil$  for the smallest integer greater than or equal to  $x$ . Also we denote the set  $\{1, 2, \dots, n\}$  by  $[n]$ , throughout this paper.

For the construction of the dominating sets of the triangular ladder,  $TL_n$ , we need to investigate the dominating sets of  $TL_n - \{2n\}$ . In this section we investigate dominating sets of  $TL_n$ . Let  $D(TL_n, i)$  be the family of dominating sets of  $TL_n$  with cardinality  $i$ . We shall find recursive formula for  $|D(TL_n, i)|$ .

Lemma 2.1

For every  $n \in \mathbb{N}$ ,

$$\gamma(TL_n) = \lfloor \frac{n}{3} \rfloor + 1$$

By Lemma 2.1 and the definition of domination number, one has the following lemma.

Lemma 2.2

Let  $TL_n, n \geq 2$  be the triangular ladder of order  $2n$ . Then

$$D(TL_n, i) = \emptyset \text{ iff } i < \lfloor \frac{n}{3} \rfloor + 1 \text{ or } i > 2n$$

$$D(TL_n - \{2n\}, i) = \emptyset \text{ iff } i < \lfloor \frac{n}{2} \rfloor \text{ or } i > 2n - 1$$

Proof:

Since  $TL_n$  is a triangular ladder of order  $2n$  & by lemma 2.1, the domination number of  $TL_n$  is  $\gamma(TL_n) = \lfloor \frac{n}{3} \rfloor + 1$

$$\therefore d(TL_n, i) = 0 \text{ iff } \lfloor \frac{n}{3} \rfloor + 1 \text{ or } i > 2n$$

$$\text{ie } D(TL_n, i) = \emptyset \text{ iff } i < \lfloor \frac{n}{3} \rfloor + 1 \text{ or } i > 2n$$

Since  $TL_n - \{2n\}$  is a triangular ladder of order  $2n - 1$

$$\therefore d(TL_n - \{2n\}, i) = 0 \text{ iff } i > 2n - 1$$

by lemma 2.1, the domination number of  $TL_n$  is  $\gamma(TL_n - \{2n\}) = \lfloor \frac{n}{2} \rfloor$

$$\therefore d(TL_n - \{2n\}, i) = 0 \text{ iff } i < \lfloor \frac{n}{2} \rfloor \text{ or } i > 2n - 1$$

$$\text{ie } D(TL_n - \{2n\}, i) = \emptyset \text{ iff } i < \lfloor \frac{n}{2} \rfloor \text{ or } i > 2n - 1$$

Table 1:  $d(TL_n, i)$  The number of dominating sets of  $TL_n$  with cardinality  $i$

$i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$TL_1 - \{2\}$	1														
$TL_1$	2	1													
$TL_2 - \{4\}$	3	3	1												
$TL_2$	2	6	4	1											
$TL_3 - \{6\}$	1	8	10	5	1										
$TL_3$	0	9	18	15	6	1									
$TL_4 - \{8\}$	0	8	27	33	21	7	1								
$TL_4$	0	6	34	60	54	28	8	1							
$TL_5 - \{10\}$	0	3	37	93	114	82	36	9	1						
$TL_5$	0	1	34	126	206	196	118	45	10	1					
$TL_6 - \{12\}$	0	0	27	150	327	401	314	163	55	11	1				
$TL_6$	0	0	18	159	462	722	714	477	218	66	12	1			
$TL_7 - \{14\}$	0	0	10	150	588	1163	1429	1190	695	284	78	13	1		
$TL_7$	0	0	4	126	678	1697	2564	2611	1884	979	362	91	14		
$TL_8 - \{16\}$	0	0	1	93	711	2261	4179	5139	4486	2862	1341	453	105	15	1

Theorem: 2.3

Let  $TL_n$  be a triangular ladder graph of order  $2n$ , then

- (i)  $d(TL_n, 2n) = 1$
- (ii)  $d(TL_n - \{2n\}, 2n - 1) = 1$
- (iii)  $d(TL_n, 2n - 1) = 2n$
- (iv)  $d(TL_n - \{2n\}, 2n - 2) = 2n - 1$
- (v)  $d(TL_n, 2n - 2) = n(2n - 1)$
- (vi)  $d(TL_n - \{2n\}, 2n - 3) = (n - 1)(2n - 1)$

$$(vii) d(TL_n, 2n - 3) = \frac{n(2n-1)(2n-2)}{3} - 2$$

$$(viii) d(TL_n - \{2n\}, 2n - 4) = \frac{(n-1)(2n-1)(2n-3)}{3} - 2$$

Proof:

(i) Let  $TL_n$  be a triangular ladder graph of order  $2n$ , then  $d(TL_n, 2n) = 1$  [Since  $|G_n^n| = 1$ ]

(ii) Since for any graph G with n vertices,  $d(G, n) =$   
 1. Let  $TL_n - \{2n\}$  be a triangular ladder graph of order  
 $2n - 1$ , then  $d(TL_n - \{2n\}, 2n - 1) = 1$ .

$$= (n - 1)(2n - 1) + 2(n - 1) + 1$$

(iii) We prove this by induction on 'n'

If  $n = 1$ ,

$$R.H.S = 2n = 2$$

$$L.H.S = d(TL_n, i) = 2 \text{ (from table)}$$

∴ The result is true for  $n = 1$ .

Suppose the result is true for all numbers less than 'n'  
 and we prove it for .

we have,

$$\begin{aligned} d(TL_n, i) &= d(TL_n - \{2n\}, i - 1) + d(TL_{n-1}, i - 1) \\ &\quad + d(TL_{n-1} - \{2n - 2\}, i - 1) \\ &\quad + d(TL_{n-2}, i - 1) \\ &\quad + d(TL_{n-2} - \{2n - 4\}, i - 1) \\ &= 2n - 1 + 1 = 2n. \end{aligned}$$

(iv) If  $n = 2$ ,

$$L.H.S = d(TL_2 - \{4\}, 2) = 3 \text{ (from table)}$$

$$R.H.S = 2n - 1 = 3$$

$$L.H.S = R.H.S$$

∴ the result is true for  $n = 2$

We have,

$$\begin{aligned} d(TL_n - \{L_n\}, 2n - 2) &= d(TL_{n-1}, 2n - 3) \\ &\quad + d(TL_{n-1} - \{2n - 2\}, 2n - 3) \\ &\quad + d(TL_{n-2}, 2n - 3) \\ &\quad + d(TL_{n-2} - \{2n - 4\}, 2n - 3) + d(TL_{n-3}, 2n - 3) \\ &= 2(n - 1) + 1 \\ &= 2n - 1 \end{aligned}$$

(v) If  $n = 2$

$$L.H.S = d(TL_n, 2n - 2)$$

$$= d(TL_2, 2) = 6 \text{ (from table)}$$

$$R.H.S = n(2n - 1) = 6$$

$$L.H.S = R.H.S$$

∴ the result is true for  $n = 2$

Suppose the result is true for all numbers less than 'n'  
 and we prove for 'n'

We have,

$$\begin{aligned} d(TL_n, 2n - 2) &= d(TL_n - \{2n\}, 2n - 3) + d(TL_{n-1}, 2n - 3) \\ &\quad + d(TL_{n-1} - \{2n - 2\}, 2n - 3) \\ &\quad + d(TL_{n-2}, 2n - 3) \\ &\quad + d(TL_{n-2} - \{2n - 4\}, 2n - 3) \end{aligned}$$

$$\begin{aligned} &= (n - 1)(2n - 1) + 2n - 2 + 1 \\ &= (n - 1)(2n - 1) + 2n - 1 \\ &= (2n - 1)(n - 1) \end{aligned}$$

(vi) If  $n = 3$

$$L.H.S = d(TL_2 - \{4\}, 2) = 3 \text{ (from table)}$$

$$R.H.S = 1 \times 3 = 3$$

$$L.H.S = R.H.S$$

∴ the result is true for  $n = 3$

Suppose that the result is true for all numbers less than  
 'n' and we prove it for 'n'

We have,

$$\begin{aligned} d(TL_n - \{2n\}, 2n - 3) &= d(TL_{n-1}, 2n - 4) + \\ &\quad d(TL_{n-1} - \{2n - 2\}, 2n - 4) + d(TL_{n-2}, 2n - 4) + \\ &\quad d(TL_{n-2} - \{2n - 4\}, 2n - 4) + d(TL_{n-3}, 2n - 4) \end{aligned}$$

$$= (n - 1)(2n - 3) + (2n - 3) + 1$$

$$\begin{aligned} &= (n - 1)(2n - 3) + 2n - 3 + 1 \\ &= (n - 1)(2n - 3) + 2n - 2 \\ &= (n - 1)(2n - 3) + 2(n - 1) \\ &= (n - 1)(2n - 3 + 2) = (n - 1)(2n - 1) \end{aligned}$$

(vii) If  $n = 2$

$$L.H.S = d(TL_2, 1) = 2 \text{ (from table)}$$

$$R.H.S = \frac{2(3)(2)}{3} - 2 = 2$$

$$L.H.S = R.H.S$$

∴ the result is true for  $n = 2$

Suppose that the result is true for all numbers less than  
 'n' and we prove it for 'n'

We have ,

$$\begin{aligned} d(TL_n, 2n - 3) &= d(TL_n - \{2n\}, 2n - 4) \\ &\quad + d(TL_{n-1}, 2n - 4) \\ &\quad + d(TL_{n-1} - \{2n - 2\}, 2n - 4) \\ &\quad + d(TL_{n-2}, 2n - 4) \\ &\quad + d(TL_{n-2} - \{2n - 4\}, 2n - 4) \\ &= \frac{(n - 1)(2n - 1)(2n - 3)}{3} - 2 \\ &\quad + (n - 1)(2(n - 1) - 1) \\ &\quad + 2(n - 1) - 1 + 1 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(n-1)(2n-1)(2n-3)}{3} \\
 &+ (n-1)(2n-2-1) + 2n-2 \\
 &- 1 + 1 - 2 \\
 &= \frac{(n-1)(2n-1)(2n-3)}{3} \\
 &+ (n-1)(2n-3) + (2n-2) \\
 &- 2 \\
 &= \frac{(n-1)(2n-1)(2n-3)}{3} \\
 &+ (n-1)(2n-3) + 2(n-1) \\
 &- 2 \\
 &= \frac{(n-1)(4n^2 - 6n - 2n + 3)}{3} \\
 &+ (n-1)(2n-3) + (2n-2) \\
 &- 2 \\
 &= \frac{4n^3 - 6n^2 - 2n^2 + 3n - 4n^2 + 6n + 2n - 3}{3} \\
 &+ 2n^2 - 3n - 2n + 3 + 2n - 2 - 2 \\
 &= \frac{4n^3 - 12n^2 + 11n - 3 - 6 + 6n^2 - 9n + 9}{3} - 2 \\
 &= \frac{4n^3 - 6n^2 + 2n}{3} - 2 \\
 &= \frac{n(4n^2 - 6n + 2)}{3} - 2 \\
 &= \frac{2n(2n^2 - 3n + 1)}{3} - 2 \\
 &= \frac{2n(2n^2 - 2n - n + 1)}{3} - 2 \\
 &= \frac{2n(2n(n-1) - (n-1))}{3} - 2 \\
 &= \frac{2n}{3} (n-1)(2n-1) - 2 \\
 &= \frac{n}{3} (2n-2)(2n-1) - 2 \\
 &= \frac{n(2n-1)(2n-2)}{3} - 2
 \end{aligned}$$

(viii) If  $n = 3$ ,

L.H.S =  $d(TL_3 - \{6\}, 2) = 8$  (from table)

R.H.S =  $\frac{2 \times 5 \times 3}{3} - 2 = 8$

L.H.S = R.H.S

∴ the result is true for  $n = 3$

Suppose that the result is true for all numbers less than 'n' and we prove it for 'n'

We have ,

$$\begin{aligned}
 &d(TL_n - \{2n\}, 2n-4) \\
 &= d(TL_{n-1}, 2n-5) \\
 &\quad + d(TL_{n-1} - \{2n-2\}, 2n-5) \\
 &\quad + d(TL_{n-2}, 2n-5) \\
 &\quad + d(TL_{n-2} - \{2n-4\}, 2n-5) \\
 &\quad + d(TL_{n-3}, 2n-5) \\
 &= \frac{(n-1)(2(n-1)-1)(2(n-1)-2)}{3} - 2 \\
 &+ 2(n-2) + (n-1-1)(2(n-1)-1) + 1 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-4) \\
 &\quad + 6(n-2) + 3(n-2)(2n-3) \\
 &\quad + 3] - 2 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-4) \\
 &\quad + 3(n-2)(2n-3) \\
 &\quad + 3(2(n-2)+1)] - 2 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-4) \\
 &\quad + 3(n-2)(2n-3) + 3(2n-3)] \\
 &\quad - 2 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-4) \\
 &\quad + 3(2n-3)(n-2+1)] - 2 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-4) \\
 &\quad + 3(2n-3)(n-1)] - 2 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-4+3)] \\
 &\quad - 2 \\
 &= \frac{1}{3} [(n-1)(2n-3)(2n-1)] - 2 \\
 &= \frac{1}{3} [(n-1)(2n-1)(2n-3)] - 2
 \end{aligned}$$

### III. DOMINATION POLYNOMIAL OF A TRIANGULAR LADDER

In this section we introduce and investigate the domination polynomial of triangular ladder

Definition 3.1

Let  $TL_i^n$  be the family of dominating sets of a triangular ladder  $TL_n$  with cardinality  $i$  and let  $d(TL_n, i) = |TL_i^n|$

and  $\gamma(TL_n) = \left\lfloor \frac{n}{3} \right\rfloor + 1$ . Then the domination

polynomial  $D(TL_n, x)$  of  $TL_n$  is defined as  $D(TL_n, x) = \sum_{i=\lfloor \frac{n}{3} \rfloor + 1}^n d(TL_n, i) x^i$

**Theorem3.2**

Let  $TL_n$  be a triangular ladder graph of order  $2n$ , then

(i)  $D(TL_n, x) = x\{D(TL_n - \{2n\}, x) + D(TL_{n-1}, x) + D(TL_{n-1} - \{2n - 2\}, x) + D(TL_{n-2}, x) +$

$D(TL_{n-2} - \{2n - 4\}, x)\}$  for every  $n \in Z^+, n \geq 2$ .

(ii)  $D(TL_n - \{2n\}, x) = x\{D(TL_{n-1}, x) + D(TL_{n-1} - \{2n - 2\}, x) + D(TL_{n-2}, x) +$

$D(TL_{n-2} - \{2n - 4\}, x) + D(TL_{n-3}, x)\}$  for every  $n \in Z^+, n \geq 2$ .

**Proof:**

(i) By definition

$$\begin{aligned} D(TL_n, x) &= \sum_{i=1}^n d(TL_n, i) x^i \\ &= \sum_{i=1}^n d(TL_n, i) x^{i-1} \\ &= x \sum_{i=1}^n \{d(TL_n - \{2n\}, x) + d(TL_{n-1}, x) + d(TL_{n-1} - \{2n - 2\}, x) + d(TL_{n-2}, x) + d(TL_{n-2} - \{2n - 4\}, x)\} x^{i-1} \\ &= x \{ \sum_{i=1}^n d(TL_n - \{2n\}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-1}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-1} - \{2n - 2\}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-2}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-2} - \{2n - 4\}, x) x^{i-1} \} \\ D(TL_n, x) &= x\{D(TL_n - \{2n\}, x) + D(TL_{n-1}, x) + D(TL_{n-1} - \{2n - 2\}, x) + D(TL_{n-2}, x) + D(TL_{n-2} - \{2n - 4\}, x)\} \end{aligned}$$

(ii) By definition

$$\begin{aligned} D(TL_n - \{2n\}, x) &= \sum_{i=1}^n d(TL_n - \{2n\}, i) x^i \\ &= \sum_{i=1}^n d(TL_n - \{2n\}, i) x^{i-1} \\ &= x \{ \sum_{i=1}^n \{d(TL_{n-1}, x) + d(TL_{n-1} - \{2n - 2\}, x) + d(TL_{n-2}, x) + d(TL_{n-2} - \{2n - 4\}, x) + d(TL_{n-3}, x)\} x^{i-1} \\ &= x \{ \sum_{i=1}^n d(TL_{n-1}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-1} - \{2n - 2\}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-2} - \{2n - 4\}, x) x^{i-1} + \sum_{i=1}^n d(TL_{n-3}, x) x^{i-1} \} \\ D(TL_n - \{2n\}, x) &= x\{D(TL_{n-1}, x) + D(TL_{n-1} - \{2n - 2\}, x) + D(TL_{n-2}, x) + D(TL_{n-2} - \{2n - 4\}, x) + D(TL_{n-3}, x)\} \end{aligned}$$

**Example3.3**

Let  $TL_6$  be a triangular ladder graph of order 12, then (by theorem3.2(i)) we have  $D(TL_6, x) = x\{D(TL_6 - \{12\}, x) + D(TL_5, x) + D(TL_5 - \{10\}, x) +$

$$\begin{aligned} D(TL_4, x) + D(TL_4 - \{8\}, x) &= 18x^3 + 159x^4 + 462x^5 + 722x^6 + 714x^7 + 477x^8 + 218x^9 + 66x^{10} + 12x^{11} + x^{12} \end{aligned}$$

**Verification:**

$$\begin{aligned} D(TL_6 - \{12\}, x) &= 27x^3 + 150x^4 + 327x^5 + 401x^6 + 314x^7 + 163x^8 + 55x^9 + 11x^{10} + x^{11} \dots \dots \dots \text{I} \end{aligned}$$

$$\begin{aligned} D(TL_5, x) &= x^2 + 34x^3 + 126x^4 + 206x^5 + 196x^6 + 118x^7 + 45x^8 + 10x^9 + x^{10} \dots \dots \dots \text{II} \end{aligned}$$

$$\begin{aligned} D(TL_5 - \{10\}, x) &= 3x^2 + 37x^3 + 93x^4 + 114x^5 + 82x^6 + 36x^7 + 9x^8 + x^9 \dots \dots \dots \text{III} \end{aligned}$$

$$\begin{aligned} D(TL_4, x) &= 6x^2 + 34x^3 + 60x^4 + 54x^5 + 28x^6 + 8x^7 + x^8 \dots \dots \dots \text{IV} \end{aligned}$$

$$\begin{aligned} D(TL_4 - \{8\}, x) &= 8x^2 + 27x^3 + 33x^4 + 21x^5 + 7x^6 + x^7 \dots \dots \dots \text{V} \end{aligned}$$

Adding equations I,II,III,IV and V, we get

$$\begin{aligned} D(TL_6 - \{12\}, x) + D(TL_5, x) + D(TL_5 - \{10\}, x) + D(TL_4, x) + D(TL_4 - \{8\}, x) &= 18x^2 + 159x^3 + 462x^4 + 722x^5 + 714x^6 + 477x^7 + 218x^8 + 66x^9 + 12x^{10} + x^{11} \\ x\{D(TL_6 - \{12\}, x) + D(TL_5, x) + D(TL_5 - \{10\}, x) + D(TL_4, x) + D(TL_4 - \{8\}, x)\} &= 18x^3 + 159x^4 + 462x^5 + 722x^6 + 714x^7 + 477x^8 + 218x^9 + 66x^{10} + 12x^{11} + x^{12} \end{aligned}$$

Hence the result.

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