Study of Free Convective MHD Flow of Jeffrey Fluid Past Over an Oscillating Moving Vertical Plate in the Presence of Heat Generation

Ramesh Patoliya
Department of Mathematics, KPGU – Vadodara

***Abstract*—In this article, the unsteady free convective MHD flow of Jeffrey fluid past over an oscillating moving vertical plate in the presence of heat generation has been considered. It is assumed that the bounding plate have ramped temperature and ramped surface concentration profile through uniform porous medium in a rotating system. Analytic solution of governing non-dimensional equation for primary and secondary fluid velocities, temperature and concentration is obtained using Laplace transform technique for ramped surface concentration and constant surface concentration. The Skin friction, Nusselt Number and Sherwood Number are express in exact form. The features of the fluid flow, heat and mass transfer characteristics are analyzed by plotting graphs and the physical aspects are discussed in detail. It is observed that, velocity profile decreases with increase in Magnetic field and Jeffrey fluid parameter. It is also seen that, Heat transfer process improves with Heat generation parameter.**

***Index Terms*—Chemical reaction, free convective, Heat generation, Jeffrey fluid, MHD.**

I. INTRODUCTION

In recent days, due to vast application of non-Newtonian fluids in engineering and several industries, researchers have keen interest to investigate the thermo-physical properties of different parameter to enhance the heat transfer properties of these fluids. In practical applications non-Newtonian fluids have great advantages than that of the Newtonian fluids. Among all the non-Newtonian models, Jeffrey model is one of the remarkable models. This model has proved quite successful [1]. Jeffrey fluid is a famous non-Newtonian fluid that falls into the category of viscoelastic or rate type fluids. This fluid model explains the properties of the ratio between relaxation and retardation times. In the Jeffrey fluid model, the two parameters λ_1 and λ_2 represent the behavior of retardation and relaxation times, respectively [2]. It debases on a non-Newtonian fluid at a very high wall shear stress, also have noticeable applications in metallurgical materials processing, chemical Engineering, etc. Hayat et al. [3] carried out one of the first studies on the Jeffrey model. Combinations of the Jeffrey fluid model and other non-Newtonian fluids have also been investigated. Sreenadh et al. [4] considered a two fluid model consisting of the power-law fluid in contact with a Jeffrey fluid flowing through an inclined channel. Over the past few years, researchers have also explored the peristaltic transport of Jeffrey fluid considering different geometrical configurations [5–7].

Moreover, Magnetohydrodynamic (MHD) effects on peristalsis are significant in magneto therapy, hyperthermia, arterial flow, compressor, etc. MHD is used in the study of electrically conducting fluids; examples of such fluids include electrolytes, salt water, liquid metals and plasmas. A number of researchers have applied magnetic field on peristaltic mechanisms, one way or the other. Aman et al. [8] investigate magnetohydrodynamic stagnation-point flow to- wards a stretching/shrinking sheet with slip effects [8]. On the possibility of over stable motion of a rotating viscoelastic fluid layer heated from below under the effect of magnetic field with one relaxation time studied by Othman [9]. Mansur and Ishak [10] studied the MHD boundary layer flow of a nano fluid numerically. Ahmed et al. [11] applied the linearization method to the study of radiation effects on MHD boundary layer convective heat transfer in a porous media with low pressure gradient. A number of researchers discussed the MHD effects on Jeffrey fluid. Hayat et al. [12] found the chain solutions of MHD Jeffrey fluid in a channel. Das et al. [13] discussed Jeffrey fluid with MHD and slip condition. Jena et al. [14] found heat generation effects on MHD

Jeffery fluid through a porous medium. Imtiaz et al. [15] found the effects of heterogeneous and homogenous reactions on MHD Jeffrey fluid. Ahmad and Ishak [16] studied the effects of viscous dissipation effects on a Jeffrey fluid with MHD. An extensive literature on the said topics is now available but we can only mention a few recent interesting investigations here [17-22].

II. MATHEMATICAL FORMULATION

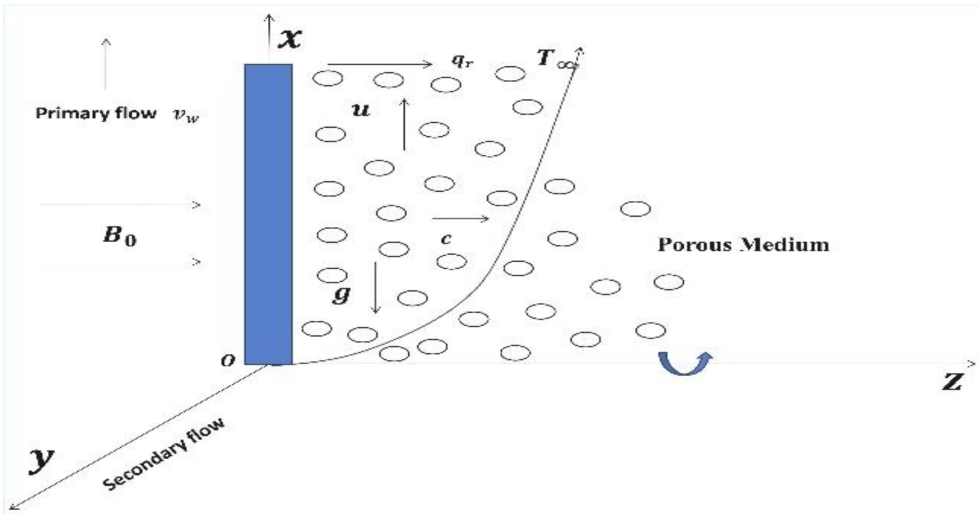

Figure 1: Physical Sketch of the Problem

Consider an incompressible, unsteady, free convection flow of MHD Jeffrey fluid lying above the exponentially accelerated vertical plate of infinite length with flexible temperature and concentration at the boundary and also to consider, first-order chemical reaction parameter and rate of heat absorption coefficient. The plate is positioned in the co-ordinate (x, z) flat surface of a Cartesian co-ordinate structure of x, y and z as delineated in Fig. 1. The fluid is electrically stimulated in the presence of a constant magnetic field B_0 which is applied normal to the plate.

The continuity, momentum and energy equations of unsteady Jeffrey fluid are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{v}{1+\lambda_1}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2} - \frac{B_0 J_y}{\rho} - \frac{v\varphi}{k(1+\lambda_1)}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) u + g\beta(T - T_\infty) + g\beta(C - C_\infty) \quad (2)$$

$$\frac{\partial u}{\partial t} + 2\Omega v = \frac{v}{1+\lambda_1}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2} - \frac{B_0 J_x}{\rho} - \frac{v\varphi}{k(1+\lambda_1)}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) v \quad (3)$$

$$\frac{\partial T}{\partial t} = \frac{k_1}{\rho C_p}\frac{\partial^2 T}{\partial z^2} - \frac{B_0 J_y}{\rho} - \frac{\partial q_r}{\partial z} + \frac{Q_0}{\rho C_p}(T - T_\infty) \quad (4)$$

$$\frac{\partial C}{\partial t} = D_M \frac{\partial^2 C}{\partial z^2} + D_T \frac{\partial^2 T}{\partial y^2} \quad (5)$$

With sufficient boundary conditions,

$$u = v = 0, T = T_{\infty,}\ C = C_{\infty,}\ \text{at } t = 0, Z > 0 \quad (6)$$

$$u = \cos wt, v = \sin wt, \quad \text{at } t > 0, z = 0 \quad (7)$$

$$T = \begin{cases} T_\infty + (T_w - T_\infty)\left({}^{t}/_{t_0}\right), & 0 < t < t_0 \\ T_w, & t \geq t_0 \end{cases} \quad \text{at } z = 0 \quad (8)$$

$$C = \begin{cases} C_\infty + (C_w - C_\infty)\left({}^{t}/_{t_0}\right), & 0 < t < t_0 \\ C_w, & t \geq t_0 \end{cases} \quad \text{at } z = 0 \quad (9)$$

$$u \to 0, v \to 0, T \to T_\infty, C \to C_\infty \quad \text{as } Z \to \infty, t > 0 \quad (10)$$

$$J = \sigma(E + V \times B) - \frac{\omega_e \tau_e \beta_i}{B_0^2}((J \times B) \times B) \quad (11)$$

$$(1 + \beta_i \beta_e) J_x + \beta_e J_y = \sigma B_0 v \quad (12)$$

$$(1 + \beta_i \beta_e) J_y - \beta_e J_x = -\sigma B_0 v \quad (13)$$

Resolving the equation (12) and (13), we accomplished,

$$J_x = \sigma B_0(\alpha_2 u + \alpha_1 v) \quad (14)$$

$$J_y = -\sigma B_0(\alpha_2 v - \alpha_1 u) \quad (15)$$

Where $\alpha_1 = \frac{1+\beta_i\beta_e}{(1+\beta_i\beta_e)^2+\beta_e^2}$ and $\alpha_2 = \frac{\beta_e}{(1+\beta_i\beta_e)^2+\beta_e^2}$

Substituting the equations (14)-(15) in equations (2)-(3) accordingly, we acquired,

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{v}{1+\lambda_1}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial z^2} + \frac{\sigma {B_0}^2(\alpha_2 v - \alpha_1 u)}{\rho} - \frac{v\varphi}{k(1+\lambda_1)}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) u + g\beta(T - T_\infty) \quad (16)$$

$$\frac{\partial u}{\partial t} + 2\Omega v = \frac{v}{1+\lambda_1}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\sigma {B_0}^2(\alpha_2 u + \alpha_1 v)}{\rho} - \frac{v\varphi}{k(1+\lambda_1)}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right) v \quad (17)$$

Combining the equations (16) and (17), let $F = u + iv$ we achieved,

$$\frac{\partial F}{\partial t} + 2i\Omega v = \frac{v}{1+\lambda_1}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\frac{\partial^2 F}{\partial z^2} + \left(\frac{\sigma {B_0}^2(\alpha_1 + i\alpha_2)}{\rho} + \frac{v\varphi}{k(1+\lambda_1)}\left(1 + \lambda_2 \frac{\partial}{\partial t}\right)\right) F + g\beta(T - T_\infty) \quad (18)$$

$$q_r = -\frac{4\sigma^*}{3k_2}\frac{\partial T^4}{\partial z} \quad (19)$$

$$\frac{\partial T}{\partial t} = \frac{k_1}{\rho C_p}\left(1 + \frac{16\sigma^* T_\infty^3}{3k_1 k_2}\right)\frac{\partial^2 T}{\partial z^2} \quad (20)$$

Introducing non-Dimensional quantities,

$$Z^* = \frac{zU_0}{v}, F = \frac{F}{F_0}, t^* = \frac{t{U_0}^2}{v}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, C = \frac{C - C_\infty}{C_w - C_\infty}, Pr = \frac{\mu C_p}{k_1}, Nr = \frac{16\sigma^* T_\infty^3}{3k_1 k_2}$$

$$Gr = \frac{g\beta v(T_w - T_\infty)}{U_0^3}, \lambda = \frac{\lambda_2 U_0^2}{v}, M^2 = \frac{\sigma {B_0}^2 v}{\rho U_0^2}, K = \frac{kU_0^2}{v\varphi}, R = \frac{\Omega v}{U_0^2}$$

Making use of the non-dimensional variables, the governing equations reduces to,

$$\left(1 + \frac{\lambda}{k(1+\lambda_1)}\right)\frac{\partial F}{\partial t} = \frac{1}{1+\lambda_1}\frac{\partial^2 F}{\partial z^2} + \frac{\lambda}{(1+\lambda_1)}\frac{\partial}{\partial t}\left(\frac{\partial^2 F}{\partial z^2}\right) - \left(M^2(\alpha_1 + i\alpha_2) + 2iR + \frac{1}{k(1+\lambda_1)}\right) F + Gr\theta + GmC \quad (21)$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{1+Nr}{Pr}\right)\frac{\partial^2 \theta}{\partial z^2} + q'' \quad (22)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} + Sr\frac{\partial^2 \theta}{\partial z^2} \quad (23)$$

With initial and boundary condition

$$F = \theta = C = 0, z > 0, t = 0 \text{ and } F \to 0, \theta \to 0, C \to 0 \text{ at } z \to \infty, t > 0$$

$$F = e^{iwt}, \ \theta = \begin{cases} t, & 0 < t < 1 \\ 1 & t \geq 1 \end{cases}, C = 1 \text{ at } z = 0, \ t > 0 \quad (24)$$

It is fascinating the Laplace transforms towards the equations (21) to (23) with initial boundary conditions (24), produced the subsequent transformed equations be,

$$a_0 \frac{\partial F}{\partial t} = a_1 \frac{\partial^2 F}{\partial z^2} + a_2 \frac{\partial^3 F}{\partial t\, \partial z^2} - a_3 F + Gr\theta + GmC \quad (25)$$

$$\frac{\partial \theta}{\partial t} = a_4 \frac{\partial^2 \theta}{\partial z^2} + a_5 \theta \quad (26)$$

$$\frac{\partial C}{\partial t} = a_6 \frac{\partial^2 C}{\partial z^2} + Sr\frac{\partial^2 \theta}{\partial z^2} \quad (27)$$

III. RESULT AND DISCUSSION

The effect of Jeffrey fluid parameter λ on primary velocity distribution can be seen in Fig. 2. It is observed that with the increase in Jeffrey fluid parameter λ primary velocity is increase. Fig. 3 show that effect of Jeffrey fluid parameter λ on secondary velocity. It is observed that with the increase in Jeffrey fluid parameter λ up to 0.3 secondary velocity is decrease and between $0.3 < \lambda < 2$ secondary velocities are increase. Fig. 4 shows that the effect of λ_1 by making the other flow parameters constant. It is clear from the figures that primary velocity increasing whereas the secondary velocity is increase function of λ_1.The influence of porosity parameter is shown in Fig. 5-7.It concludes that porosity parameter k decrease velocity for both the ramped temperature and isothermal plate. We observe that the drag force reduce is the increase in permeability of porosity medium. Fig. 8-10 shows altered values of magnetic parameter Sr . As expected, by increasing the value of Sr , primary velocity increase in both ramped and constant concentration cases. Whereas Sr parameter up to 0.3 secondary velocity is decrease and between $0.3 < Sr < 2$ secondary velocities are increase. Fig. 11-12 shows that different value of thermal Grashof number Gr . Here, as increase in Grashof number secondary velocity is decrease and primary velocity is increase. Fig. 13-14 shows altered values of Mass Grashof number Gm . It is also worked as thermal Grashof number. Fig. 15-17 shows that the when we increase the value of Nusselt number Nr , the fluid velocity and temperature increases. Fig. 18-19 shows the due to effects of rotational parameter, velocity decreases. Fig. 20-21 shows that hall parameter is opposite working as rotational parameter. It is observed that with the increase in hall parameter β_1 up to 0.3 secondary velocity is decrease and between $0.3 < \beta_1 < 2$ secondary velocities are increase. Fig. 22-23 shows the behavior of flow by ion slip parameter β_e . It is also working as hall parameter, both the cases primary velocity increase.

IV. CONCLUSION

MHD free convective flow of Jeffrey fluid in a porous medium is analyzed analytically by using the ramped wall condition for both velocity and temperature fields respectively. It is observed that the literature is scarce in simultaneous use of these conditions. Some graphical results are also plotted and discussed. Some observations are as follow:

- Effect of all parameters is similar in all cases, Ramped surface concentration and Constant surface concentration
- It is observed that magnitude of primary and secondary velocity, heat and mass transfer in case of ramped surface concentration is less than that of Constant surface concentration.
- The effects of increasing value of magnetic parameter M and parameter λ on velocity profile, result in decreasing the momentum boundary layer thickness whereas momentum boundary layer increases with increasing value of parameters Gm and Gr
- Mass transfer process delayed with increasing the values of thermo diffusion whereas, improved with increasing the values of chemical reaction.

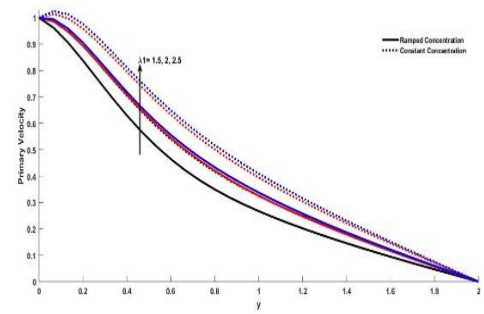

Figure 2: Primary Velocity for different values of λ

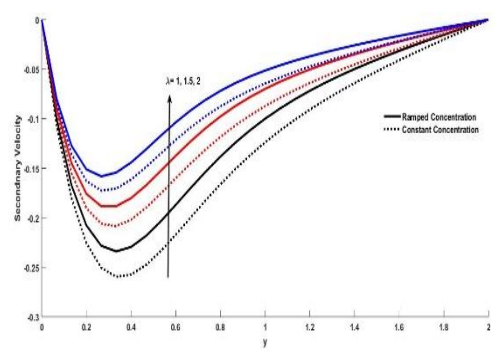

Figure 3: Secondary Velocity for different values of λ

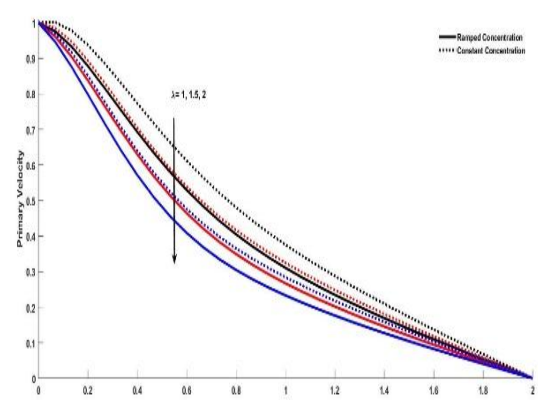

Figure 4: Primary Velocity for different values of λ_1

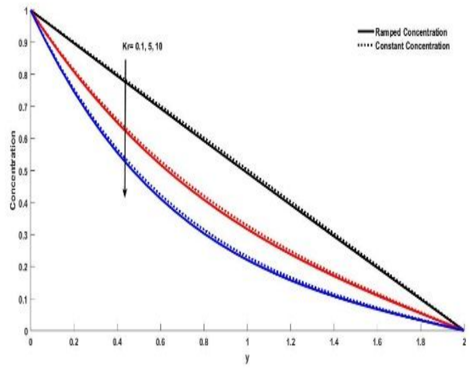

Figure 5: Contradiction profile for different values of Kr

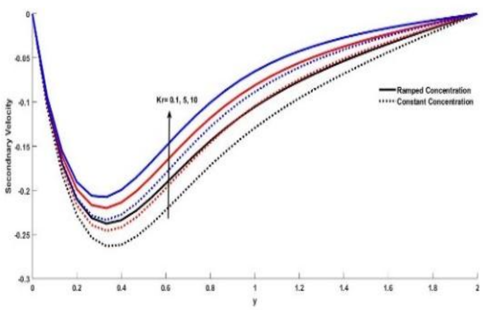

Figure 6: Secondary Velocity for different values of Kr

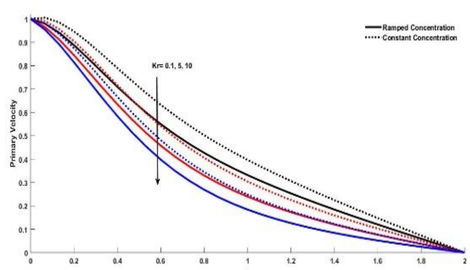

Figure 7: Primary Velocity for different values of Kr

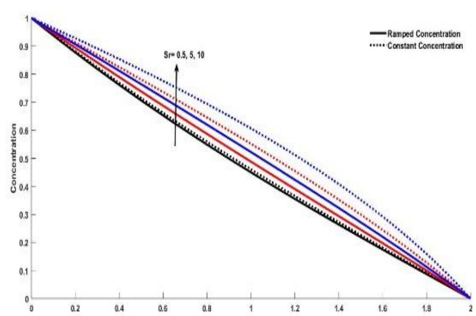

Figure 8: Contradiction profile for different values of Sr

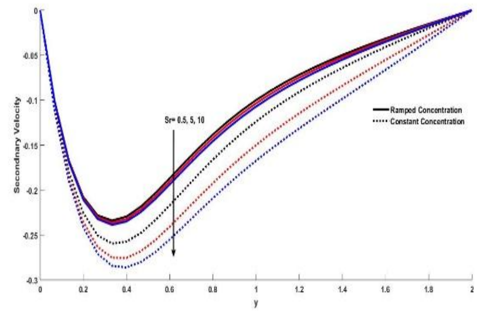

Figure 9: Secondary Velocity for different values of Sr

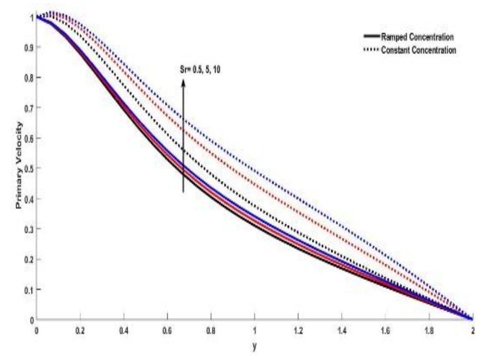

Figure 10 Primary Velocity for different values of Sr

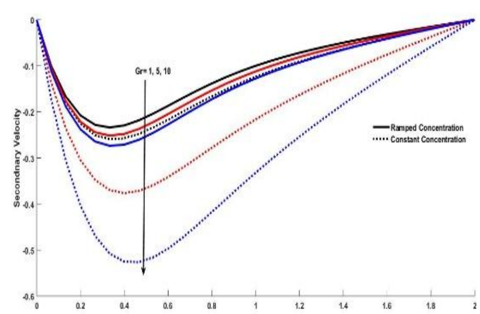

Figure 11: Secondary Velocity for different values of Gr

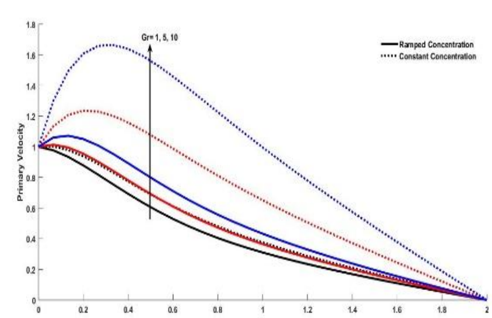

Figure 12: Primary Velocity for different values of Gr

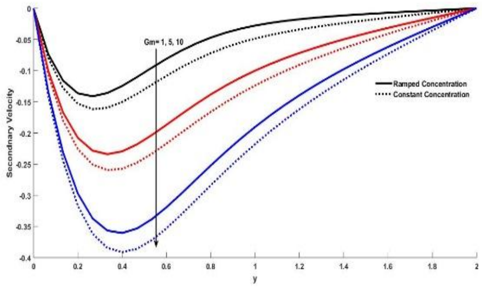

Figure 13: Secondary Velocity for different values of Gm

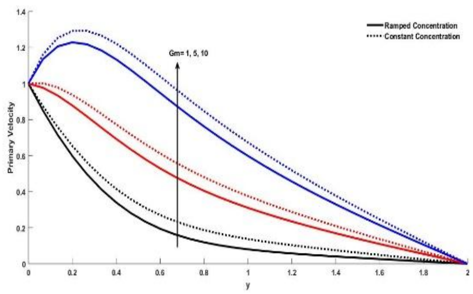

Figure 14: Primary Velocity for different values of Gm

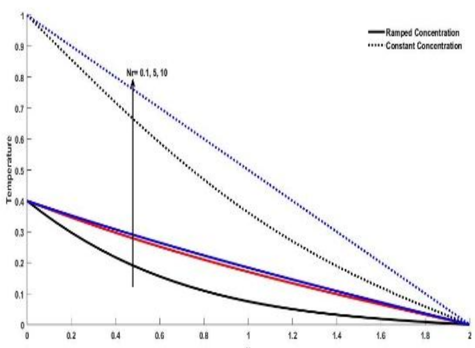

Figure 15: Temperature for different values of Nr

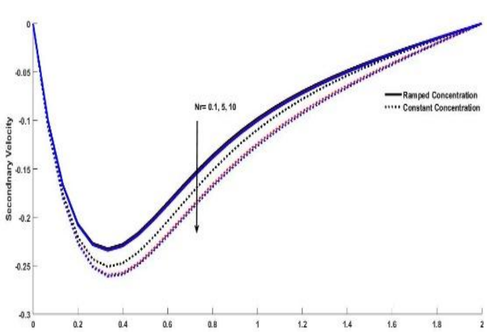

Figure 16: Secondary Velocity for different values of Nr

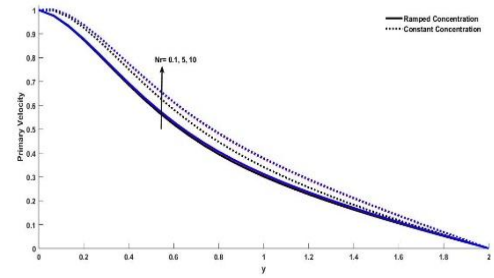

Figure 17: Primary Velocity for different values of Nr

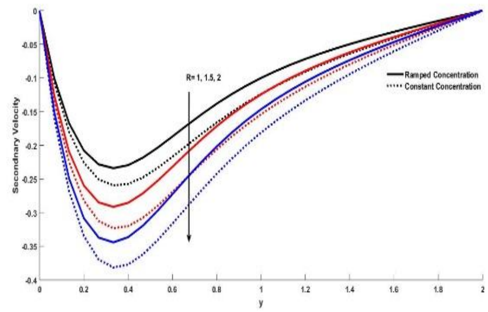

Figure 18: Secondary Velocity for different values of R

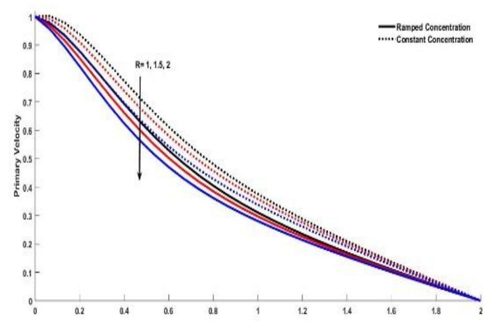

Figure 19 Primary Velocity for different values of R

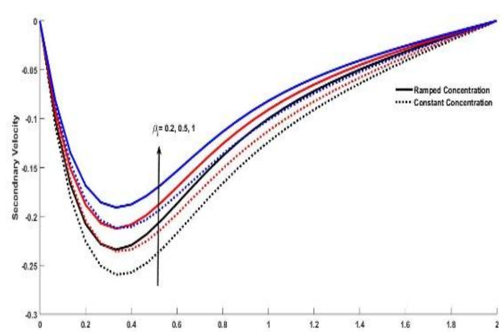

Figure 20: Secondary Velocity for different values of $\beta 1$

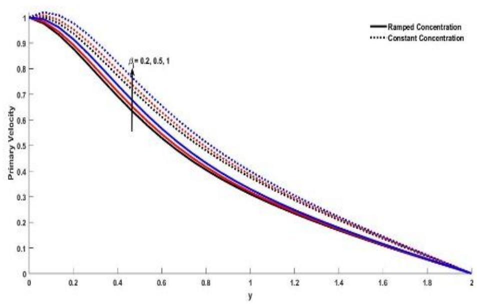

Figure 21 Primary Velocity for different values of $\beta 1$

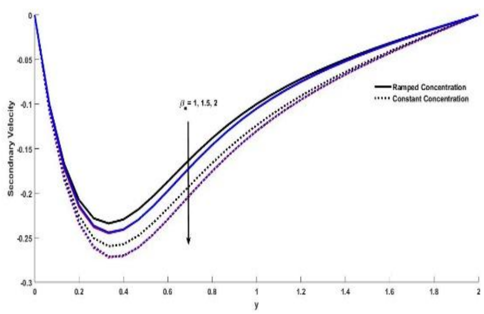

Figure 22: Secondary Velocity for different values of βe

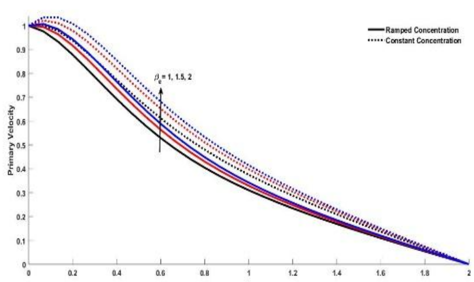

Figure 23: Primary Velocity for different values of βe

REFERENCES

1. Kottakkaran Sooppy Nisar, R. Mohapatra, S.R. Mishra, M. Ganeswar Reddy,Semi-analytical solution of MHD free convective Jeffrey fluid flow in the presence of heat source and chemical reaction,Ain Shams Engineering Journal, https://doi.org/10.1016/j.asej.2020.08.015.
2. Hu Ge-JiLe, Mubbashar Nazeer, Farooq Hussain, Two-phase flow of MHD Jeffrey fluid with the suspension of tiny metallic particles incorporated with viscous dissipation and Porous Medium, https://doi.org/10.1177%2F16878140211005960
3. T. Hayat, N. Ali, S. Asghar, An analysis of peristaltic transport for flow of a jeffrey fluid, Acta Mech. 193 (1–2) (2007) 101–112.
4. S. Sreenadh, K. Komala, A.N.S. Srinivas, Peristaltic pumping of a power–law fluid in contact with a jeffrey fluid in an inclined channel with permeable walls, Ain Shams Eng. J. 8 (4) (2017) 605–611.
5. M.M. Bhatti, M. Ali Abbas, Simultaneous effects of slip and mhd on peristaltic blood flow of jeffrey fluid model through a porous medium, Alexandria Eng. J. 55 (2) (2016) 1017–1023.
6. C.K. Selvi, C. Haseena, A.N.S. Srinivas, S. Sreenadh, The effect of heat transfer on peristaltic flow of jeffrey fluid in an inclined porous stratum, IOP Conference Series: Materials Science and Engineering, vol. 263, IOP Publishing, 2017, p. 062027.
7. A. Kavitha, R. Hemadri Reddy, R. Saravana, S. Sreenadh, Peristaltic transport of a jeffrey fluid in contact with a Newtonian fluid in an inclined channel, Ain Shams Eng. J. 8 (4) (2017) 683–687.
8. F.Aman, A.Ishak, I.Pop, Magnetohydrodynamic stagnation-point flow to- wards a stretching/shrinking sheet with slip effects, Int.Commun.HeatMass Transf.47 (2013)68–72.
9. M.I.A. Othman, On the possibility of over stable motion of a rotating viscoelastic fluid layer heated from below under the effect of magnetic field with one relaxationtime, Mech. Mech. Eng.7(2004)41–52.
10. S. Mansur, A. Ishak, The megnetohydrodynamic boundary layer flow of a nanofluid past a stretching sheet with slip boundary conditions, J. Appl. Math, Vol 9, 1073-1079, (2016)
11. M.A.M. Ahmed, M.E. Mohammad, A.A. Khidir, On linearization method to MHD boundary layer convective heat transfer with low pressure gradient, Prop. Power. Res, Vol 4, 105-123, (2015).
12. T. Hayat, R. Sajjad, S. Asghar, Series solution for MHD channel flow of Jeffery fluid,

Commun. Nonlinear Sci. Numer. Simul, Vol 15, 2400, (2010).
13. K. Das, N. Acharya, P.K. Kundu, Thermal radiation in unsteady MHD free convection flow of Jeffery fluid, Alex. Eng. J, Vol 54, 815, (2015).
14. S. Jena1, S.R. Mishra, G.C. Dash, Influence of non-integer order parameter and Hartmann number on the heat and mass transfer flow on Jeffery fluid over an oscillating vertical plate with Caputo-Fabrizio time fractional derivatives, Int. J. Appl. Comp. Math., (2016).
15. M. Imtiaz, T. Hayat, A. Alsaedi, PLoS ONE, MHD convection flow of Jeffery fluid due to a curved strechting surface with Homogenous Reactions, (2016).
16. K. Ahmad, A. Ishak, MHD flow and heat transfer of a Jeffery fluid over a stretching sheet with viscous dissipation, Malays. J. Math. Sci, Vol 10, 311-323, (2016).
17. M. Sheikholeslami, M.Gorji-Bandpy,D. D.Ganji, P.Rana, Soheil Soleimani, Magnetohydrodynamic free convection of Al2O3-water nano fluid considering Thermophoresis and Brownian motion effects,Comput.Fluids94(2014) 147–160.
18. M. Hameed, S.Nadeem, Unsteady MHD flow of a non-Newtonian fluid on a porous plate, J.Math. Anal.Appl.325(1)(2007)724–733
19. Y.Abd Elmaboud, Influence of induced magnetic field on peristaltic flow in an annulus, Commun. Nonlinear Sci. Numer.Simul.17(2012)685–698.
20. M. Kothandapani, S.Srinivas, Peristaltic transport of a Jeffery fluid under the effect of magnetic field in an asymmetric channel, Int.J.Non-LinearMech.43 (2008)915.
21. Ebaid, Effects of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel, Phys. Lett.A372 (2008)4493–4499.
22. R. Taneja, N.C.Jain, MHD flow with slip effects and temperature-dependent heat source in a viscous incompressible fluid confined between along vertical wavy wall and a parallel flat wall, Def.Sci.J.54 (2004)21–29.