An Economic Order Quantity Model with Quartic Demand, Weibull Deterioration and Time-Dependent Partial Backlogging

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***Abstract*—This paper develops an Economic Order Quantity (EOQ) model for deteriorating items characterized by a quartic demand pattern, Weibull distributed deterioration rate, and time-dependent partial backlogging. The demand rate is assumed to follow a quartic function of time, representing products whose demand may rise and fall in multiple phases during their life cycle. The deterioration of items is modelled using a two-parameter Weibull distribution, which effectively captures a wide range of deterioration behaviours. Shortages are permitted and partially backlogged, with the backlogging rate considered as a decreasing function of the waiting time. The total cost function of the inventory system is formulated by integrating ordering, holding, deterioration, and shortage costs. Analytical expressions are derived to determine the optimal cycle time and order quantity that minimize the total average cost. A numerical example is provided to illustrate the applicability of the model, followed by a sensitivity analysis to examine the influence of key parameters on the optimal solution. The results reveal that both demand and deterioration parameters significantly affect the optimal inventory policy, offering valuable insights for decision-makers handling time-sensitive and deteriorating items.**

***Index Terms*—Quartic Demand, Weibull Deterioration and Time-Dependent Partial Backlogging**

I. INTRODUCTION

Inventory management plays a vital role in modern business operations, helping organizations maintain an optimal balance between demand and supply while minimizing associated costs. The classical Economic Order Quantity (EOQ) model proposed by Harris (1913) assumes a constant demand rate and instantaneous replenishment, which rarely reflects real-world conditions. In practical situations, the demand rate for many products changes with time due to factors such as market fluctuations, product life cycles, technological advancement, and consumer preferences. To capture such variations, researchers have extended the EOQ framework by considering different forms of time-dependent demand patterns, including linear, quadratic, and exponential types. However, for certain products exhibiting complex market behaviour such as luxury goods, seasonal items, or fashion products a higher-degree demand pattern like the quartic function provides a more realistic representation.

In addition to time-varying demand, the phenomenon of deterioration is an inevitable factor in many inventory systems. Items such as food products, pharmaceuticals, chemicals, and electronic components gradually lose their utility over time. To model this deterioration, the Weibull distribution has been widely adopted due to its flexibility and ability to represent various deterioration rates under different conditions.

Shortages also frequently occur in practical situations, and allowing for backlogging can prevent loss of customer goodwill. In reality, the backlogging rate is rarely constant; it depends on the waiting time for the next replenishment. Customers are more willing to wait for a short duration but less likely to do so if the waiting time increases. Hence, considering a time-dependent partial backlogging rate provides a more realistic framework for inventory control.

Motivated by these observations, this paper develops an EOQ model incorporating quartic time-dependent demand, Weibull deterioration, and time-dependent partial backlogging. The total cost function includes ordering, holding, deterioration, and shortage costs, and the optimal inventory policy is derived by minimizing the total average cost. A numerical example and sensitivity analysis are conducted to demonstrate the applicability and managerial relevance of the model.

II. LITERATURE REVIEW

Several researchers have extended the classical Economic Order Quantity (EOQ) model to incorporate time-varying demand patterns, deterioration effects, and shortages. The study of inventory models with higher-order demand functions has gained considerable attention in recent years due to their ability to represent complex real-world market situations.

Suman and Vinod Kumar (2021) developed a deterministic inventory model for deteriorating items with a biquadratic demand rate and constant deterioration rate, providing analytical solutions for the optimal order quantity and total cost. Their work demonstrated that higher-degree demand functions offer more flexibility in modelling product life cycles. Extending this approach, Pradeepa, Sobia, and Valliathal (2025a) proposed a similar model that included salvage value for deteriorated items, emphasizing the economic benefits of recovering value from unsold stock. In another study, the same authors (2025b) examined a system with biquadratic demand, linear deterioration rate, and constant holding cost, illustrating how time-dependent deterioration affects total cost and replenishment decisions.

Pooja Soni and Rajender Kumar (2022) analyzed a deterministic inventory system with biquadratic demand, variable deterioration rate, and carrying cost, further enriching the understanding of non-linear demand structures. Similarly, Suman and Vinod Kumar (2022) incorporated the Weibull distribution for deterioration alongside a biquadratic demand rate, concluding that the Weibull model provides a more realistic depiction of deterioration over time.

Earlier related works with lower-degree demand functions have also contributed to the theoretical foundation of such models. Rahman and Uddin (2020) studied an EOQ model with quadratic demand and time-variable deterioration without shortages, while Tripathi and Tomar (2018) formulated a model involving quadratic time-sensitive demand and parabolic holding cost under salvage value considerations. These studies highlighted the sensitivity of total cost to changes in demand and holding cost parameters.

Further, Mandal (2020) analyzed an EOQ model with cubic demand and time-varying deterioration under salvage value and shortages, showing that cubic functions can describe gradual and smooth demand variations. Sahoo, Paul, and Sahoo (2021) extended this idea by proposing a model with cubic demand and three-parameter Weibull deterioration, highlighting the versatility of the Weibull distribution in deterioration analysis.

Recent advancements also include multi-warehouse and time-dependent cost structures. Anthony et al. (2024) developed a three-warehouse inventory model for non-instantaneous deteriorating items with quadratic demand, time-varying holding cost, and backlogging, revealing the managerial importance of distribution structures in inventory control. Amutha (2017) proposed a model incorporating quadratic demand, time-dependent holding cost, and salvage value, while Sharma, Gill, and Taneja (2024) explored quadratic demand with non-instantaneous deterioration under a mixed cash and advance payment scheme, integrating financial and inventory decisions.

From the review of these studies, it is evident that most existing works have focused on linear, quadratic, cubic, or biquadratic demand functions. However, very few models have addressed quartic (fourth-degree polynomial) demand patterns in conjunction with Weibull deterioration and time-dependent partial backlogging. The present study aims to fill this research gap by formulating a generalized EOQ model that captures more complex demand variations and realistic deterioration and shortage behaviours. This contributes to a deeper understanding of inventory dynamics in environments where demand and deterioration follow nonlinear and time-sensitive trends.

III. ASSUMPTIONS AND NOTATIONS

3.1 Assumptions

- The model deals with a single type of item over a fixed and finite planning horizon, represented by T.

- Replenishment is assumed to be instantaneous, meaning that ordered items are received immediately without any lead time.
- The demand rate follows a quartic function of time and is given by

$D(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$ where $a_0, a_1, a_2, a_3 \& a_4$ are constant coefficients representing the demand parameters.

- The deterioration of items over time (t > 0) follows a two-parameter Weibull distribution, expressed as $\theta(t) = \alpha\beta t^{\beta-1}$, where α ($0 < \alpha << 1$) is the scale parameter and $\beta > 0$ is the shape parameter.
- Shortages are allowed and during the shortage period, the backlogging rate depends on the waiting time until the next replenishment. The backorder rate is defined as $B(t) = \frac{1}{1+\delta(T-t)}$ where δ (0 ≤ δ ≤ 1) is the backlogging parameter, and (T−t) represents the waiting time for restocking.
- The holding cost is assumed to remain constant throughout the cycle.

3.2 Notations

- $I_1(t)$ – Inventory level at time t for the interval $0 \le t \le t_1$.
- $I_2(t)$ – Inventory level at time t for the interval $t_1 \le t \le T$.
- Q – Quantity ordered in each cycle.
- W– Initial inventory level at the beginning of the cycle.
- T – Length of the entire inventory cycle.
- t_1 – Time period during which the available inventory is sufficient to satisfy demand

without any shortages

- C_o – Ordering cost per cycle
- Cp – Purchasing cost per unit item
- Ch – Holding cost per unit per unit time
- Cd – Cost associated with deterioration
- CS – Shortage cost for backordered items
- CL – Cost corresponding to lost sales
- TC – Total cost per inventory cycle

IV. MATHEMATICAL FORMULATION OF THE MODEL

At the beginning of each inventory cycle, an order of quantity Q is made. Starting with an initial stock level W, the inventory gradually decreases over the interval [0, t_1] as a result of demand and deterioration, eventually reaching zero. After this point, during the interval [t_1, T], shortages arise since demand persists and a portion of this unmet demand is considered for partial backordering.

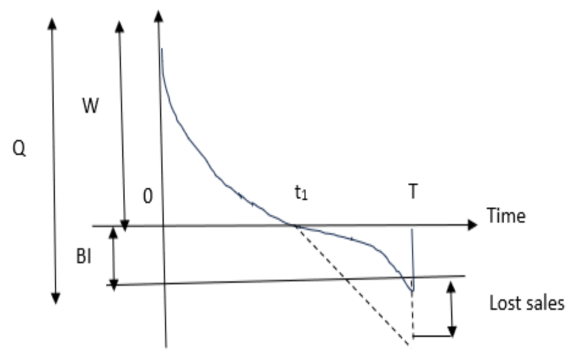

Figure 1. Representation of the inventory system

The rate of change of the inventory level can be represented as follows:

$$\frac{dI_1(t)}{dt} + \alpha\beta t^{\beta-1} I_1(t) = -(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4), \quad 0 \le t \le t_1 \qquad (1)$$

This differential equation is solved under the initial condition $I_1(0) = W$ and the terminal condition $I_1(t_1)=0$.

The solution to equation (1) is

$$I_1(t) = \frac{1}{1+\alpha t^\beta}\left\{\begin{array}{l} a_0(t_1 - t) + \frac{a_1}{2}(t_1^2 - t^2) + \frac{a_2}{3}(t_1^3 - t^3) + \frac{a_3}{4}(t_1^4 - t^4) + \frac{a_4}{5}(t_1^5 - t^5) + \frac{a_0\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) \\ + \frac{a_1\alpha}{\beta+2}\left(t_1^{\beta+2} - t^{\beta+2}\right) + \frac{a_2\alpha}{\beta+3}\left(t_1^{\beta+3} - t^{\beta+3}\right) + \frac{a_3\alpha}{\beta+4}\left(t_1^{\beta+4} - t^{\beta+4}\right) + \frac{a_4\alpha}{\beta+5}\left(t_1^{\beta+5} - t^{\beta+5}\right) \end{array}\right\}$$

and (2)

$$W = \left\{a_0 t_1 + \frac{a_1 t_1^2}{2} + \frac{a_2 t_1^3}{3} + \frac{a_3 t_1^4}{4} + \frac{a_4 t_1^5}{5} + \frac{a_0 \alpha t_1^{\beta+1}}{\beta+1} + \frac{a_1 \alpha t_1^{\beta+2}}{\beta+2} + \frac{a_2 \alpha t_1^{\beta+3}}{\beta+3} + \frac{a_3 \alpha t_1^{\beta+4}}{\beta+4} + \frac{a_4 \alpha t_1^{\beta+5}}{\beta+5}\right\} \qquad (3)$$

During the time interval from t_1 to T, the system encounters shortages, where a portion of the demand is backordered. Let $I_2(t)$ denote the inventory level at any time t within this range. The change in inventory during this period can be described by the following differential equation:

$$\frac{dI_2(t)}{dt} = -(a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4)\frac{1}{1+\delta(T-t)}, \qquad t_1 \le t \le T \tag{4}$$

At $t = t_1$, the inventory level becomes zero, which provides the boundary condition for this phase of the model: $I_2(t_1) = 0$

The solution to equation (4) is

$$I_2(t) = \Big\{a_0(t_1 - t) + \frac{a_1}{2}(t_1^2 - t^2) + \frac{a_2}{3}(t_1^3 - t^3) + \frac{a_3}{4}(t_1^4 - t^4) + \frac{a_4}{5}(t_1^5 - t^5) + a_0\delta T(t - t_1) + \frac{a_1\delta T}{2}(t^2 - t_1^2) + \frac{a_2\delta T}{3}(t^3 - t_1^3) + \frac{a_3\delta T}{4}(t^4 - t_1^4) + \frac{a_4\delta T}{5}(t^5 - t_1^5) + \frac{a_0\delta}{2}(t_1^2 - t^2) + \frac{a_1\delta}{3}(t_1^3 - t^3) + \frac{a_2\delta}{4}(t_1^4 - t^4) + \frac{a_3\delta}{5}(t_1^5 - t^5) + \frac{a_4\delta}{6}(t_1^6 - t^6)\Big\} \tag{5}$$

The maximum level of backordered inventory, represented by BI, is reached at the conclusion of the cycle, that is, at time t = T.

$$BI = -I_2(T) = -\Big\{a_0(t_1 - t) + \frac{a_1}{2}(t_1^2 - t^2) + \frac{a_2}{3}(t_1^3 - t^3) + \frac{a_3}{4}(t_1^4 - t^4) + \frac{a_4}{5}(t_1^5 - t^5) + a_0\delta T(t - t_1) + \frac{a_1\delta T}{2}(t^2 - t_1^2) + \frac{a_2\delta T}{3}(t^3 - t_1^3) + \frac{a_3\delta T}{4}(t^4 - t_1^4) + \frac{a_4\delta T}{5}(t^5 - t_1^5) + \frac{a_0\delta}{2}(t_1^2 - t^2) + \frac{a_1\delta}{3}(t_1^3 - t^3) + \frac{a_2\delta}{4}(t_1^4 - t^4) + \frac{a_3\delta}{5}(t_1^5 - t^5) + \frac{a_4\delta}{6}(t_1^6 - t^6)\Big\} \tag{6}$$

Consequently, the total quantity ordered over the entire time horizon [0,T] is:

Q = W + BI

$$Q = \Big\{a_0 t_1 + \frac{a_1 t_1^2}{2} + \frac{a_2 t_1^3}{3} + \frac{a_3 t_1^4}{4} + \frac{a_4 t_1^5}{5} + \frac{a_0\alpha t_1^{\beta+1}}{\beta+1} + \frac{a_1\alpha t_1^{\beta+2}}{\beta+2} + \frac{a_2\alpha t_1^{\beta+3}}{\beta+3} + \frac{a_3\alpha t_1^{\beta+4}}{\beta+4} + \frac{a_4\alpha t_1^{\beta+5}}{\beta+5}\Big\} - \Big\{a_0(t_1 - t) + \frac{a_1}{2}(t_1^2 - t^2) + \frac{a_2}{3}(t_1^3 - t^3) + \frac{a_3}{4}(t_1^4 - t^4) + \frac{a_4}{5}(t_1^5 - t^5) + a_0\delta T(t - t_1) + \frac{a_1\delta T}{2}(t^2 - t_1^2) + \frac{a_2\delta T}{3}(t^3 - t_1^3) + \frac{a_3\delta T}{4}(t^4 - t_1^4) + \frac{a_4\delta T}{5}(t^5 - t_1^5) + \frac{a_0\delta}{2}(t_1^2 - t^2) + \frac{a_1\delta}{3}(t_1^3 - t^3) + \frac{a_2\delta}{4}(t_1^4 - t^4) + \frac{a_3\delta}{5}(t_1^5 - t^5) + \frac{a_4\delta}{6}(t_1^6 - t^6)\Big\} \tag{7}$$

Ordering Cost (OC) = C_0

Purchase Cost (PC) = C_p Q

$$PC = C_p\Bigg\{\Big\{a_0 t_1 + \frac{a_1 t_1^2}{2} + \frac{a_2 t_1^3}{3} + \frac{a_3 t_1^4}{4} + \frac{a_4 t_1^5}{5} + \frac{a_0\alpha t_1^{\beta+1}}{\beta+1} + \frac{a_1\alpha t_1^{\beta+2}}{\beta+2} + \frac{a_2\alpha t_1^{\beta+3}}{\beta+3} + \frac{a_3\alpha t_1^{\beta+4}}{\beta+4} + \frac{a_4\alpha t_1^{\beta+5}}{\beta+5}\Big\} - \Big\{a_0(t_1 - t) + \frac{a_1}{2}(t_1^2 - t^2) + \frac{a_2}{3}(t_1^3 - t^3) + \frac{a_3}{4}(t_1^4 - t^4) + \frac{a_4}{5}(t_1^5 - t^5) + a_0\delta T(t - t_1) + \frac{a_1\delta T}{2}(t^2 - t_1^2) + \frac{a_2\delta T}{3}(t^3 - t_1^3) + \frac{a_3\delta T}{4}(t^4 - t_1^4) + \frac{a_4\delta T}{5}(t^5 - t_1^5) + \frac{a_0\delta}{2}(t_1^2 - t^2) + \frac{a_1\delta}{3}(t_1^3 - t^3) + \frac{a_2\delta}{4}(t_1^4 - t^4) + \frac{a_3\delta}{5}(t_1^5 - t^5) + \frac{a_4\delta}{6}(t_1^6 - t^6)\Big\}\Bigg\} \tag{8}$$

Holding Cost (HC) $= \int_0^{t_1} C_h I_1(t)dt$

$$HC = C_h\left\{\begin{array}{c} t_1^2\left(\frac{a_0}{2} + \frac{a_1 t_1}{3} + \frac{a_2 t_1^2}{4} + \frac{a_3 t_1^3}{5} + \frac{a_4 t_1^4}{6}\right) + \frac{\beta\alpha t_1^{\beta+2}}{\beta+1}\left(\frac{a_0}{\beta+2} + \frac{a_1 t_1}{\beta+3} + \frac{a_2 t_1^2}{\beta+4} + \frac{a_3 t_1^3}{\beta+5} + \frac{a_4 t_1^4}{\beta+6}\right) \\ -\frac{\alpha^2 t_1^{2\beta+2}}{\beta+1}\left(\frac{a_0}{2\beta+2} + \frac{a_1 t_1}{2\beta+3} + \frac{a_2 t_1^2}{2\beta+4} + \frac{a_3 t_1^3}{2\beta+5} + \frac{a_4 t_1^4}{2\beta+6}\right)\end{array}\right\} \tag{9}$$

Deterioration Cost (DC) = $C_d\left\{W - \int_0^{t_1} D(t)dt\right\}$

$$DC = C_d\left\{\frac{a_0\alpha t_1^{\beta+1}}{\beta+1}+\frac{a_1\alpha t_1^{\beta+2}}{\beta+2}+\frac{a_2\alpha t_1^{\beta+3}}{\beta+3}+\frac{a_3\alpha t_1^{\beta+4}}{\beta+4}+\frac{a_4\alpha t_1^{\beta+5}}{\beta+5}\right\} \quad (10)$$

Shortage Cost (SC) = $-C_s\int_{t_1}^{T} I_2(t)dt$

$$SC = -C_s\left\{T(1-\delta T)\left[a_0\left(t_1-\frac{T}{2}\right)+\frac{a_1}{2}\left(t_1^2-\frac{T^2}{3}\right)+\frac{a_2}{3}\left(t_1^3-\frac{T^3}{4}\right)+\frac{a_3}{4}\left(t_1^4-\frac{T^4}{5}\right)+\frac{a_4}{5}\left(t_1^5-\frac{T^5}{6}\right)\right]+\delta T\left[\frac{a_0}{2}\left(t_1^2-\frac{T^2}{3}\right)+\frac{a_1}{3}\left(t_1^3-\frac{T^3}{4}\right)+\frac{a_2}{4}\left(t_1^4-\frac{T^4}{5}\right)+\frac{a_3}{5}\left(t_1^5-\frac{T^5}{6}\right)+\frac{a_4}{6}\left(t_1^6-\frac{T^6}{7}\right)\right]-t_1^2(1-\delta T)\left[\frac{a_0}{2}+\frac{a_1t_1}{3}+\frac{a_2t_1^2}{4}+\frac{a_3t_1^3}{5}+\frac{a_4t_1^4}{6}\right]-\delta t_1^3\left[\frac{a_0}{3}+\frac{a_1t_1}{4}+\frac{a_2t_1^2}{5}+\frac{a_3t_1^3}{6}+\frac{a_4t_1^4}{7}\right]\right\} \quad (11)$$

Lost Sales Cost (LSC) = $C_L\int_{t_1}^{T}\left[1-\frac{1}{1+\delta(T-t)}\right](a_0+a_1t+a_2t^2+a_3t^3+a_4t^4)dt$

$$LSC = C_L\delta\left(\frac{a_0T^2}{2}+\frac{a_1T^3}{6}+\frac{a_2T^4}{12}+\frac{a_3T^5}{20}+\frac{a_4T^6}{30}-a_0Tt_1-\frac{t_1^2}{2}(a_1T-a_0)-\frac{t_1^3}{3}(a_2T-a_1)-\frac{t_1^4}{4}(a_3T-a_2)-\frac{t_1^5}{5}(a_4T-a_3)+\frac{et_1^6}{6}\right) \quad (12)$$

Total cost (TC) = Ordering cost + Holding cost + Purchase cost + Deterioration cost + Shortage cost + Lost sales cost

$$= C_o + C_h\left\{t_1^2\left(\frac{a_0}{2}+\frac{a_1t_1}{3}+\frac{a_2t_1^2}{4}+\frac{a_3t_1^3}{5}+\frac{a_4t_1^4}{6}\right)+\frac{\beta\alpha t_1^{\beta+2}}{\beta+1}\left(\frac{a_0}{\beta+2}+\frac{a_1t_1}{\beta+3}+\frac{a_2t_1^2}{\beta+4}+\frac{a_3t_1^3}{\beta+5}+\frac{a_4t_1^4}{\beta+6}\right)-\frac{\alpha^2t_1^{2\beta+2}}{\beta+1}\left(\frac{a_0}{2\beta+2}+\frac{a_1t_1}{2\beta+3}+\frac{a_2t_1^2}{2\beta+4}+\frac{a_3t_1^3}{2\beta+5}+\frac{a_4t_1^4}{2\beta+6}\right)\right\}$$

$$+ C_p\left\{\left\{a_0t_1+\frac{a_1t_1^2}{2}+\frac{a_2t_1^3}{3}+\frac{a_3t_1^4}{4}+\frac{a_4t_1^5}{5}+\frac{a_0\alpha t_1^{\beta+1}}{\beta+1}+\frac{a_1\alpha t_1^{\beta+2}}{\beta+2}+\frac{a_2\alpha t_1^{\beta+3}}{\beta+3}+\frac{a_3\alpha t_1^{\beta+4}}{\beta+4}+\frac{a_4\alpha t_1^{\beta+5}}{\beta+5}\right\}\right.$$
$$-\left\{a_0(t_1-t)+\frac{a_1}{2}(t_1^2-t^2)+\frac{a_2}{3}(t_1^3-t^3)+\frac{a_3}{4}(t_1^4-t^4)+\frac{a_4}{5}(t_1^5-t^5)+a_0\delta T(t-t_1)+\frac{a_1\delta T}{2}(t^2-t_1^2)+\frac{a_2\delta T}{3}(t^3-t_1^3)+\frac{a_3\delta T}{4}(t^4-t_1^4)+\frac{a_4\delta T}{5}(t^5-t_1^5)+\frac{a_0\delta}{2}(t_1^2-t^2)\right.$$
$$\left.\left.+\frac{a_1\delta}{3}(t_1^3-t^3)+\frac{a_2\delta}{4}(t_1^4-t^4)+\frac{a_3\delta}{5}(t_1^5-t^5)+\frac{a_4\delta}{6}(t_1^6-t^6)\right\}\right\}$$
$$+ C_d\left\{\frac{a_0\alpha t_1^{\beta+1}}{\beta+1}+\frac{a_1\alpha t_1^{\beta+2}}{\beta+2}+\frac{a_2\alpha t_1^{\beta+3}}{\beta+3}+\frac{a_3\alpha t_1^{\beta+4}}{\beta+4}+\frac{a_4\alpha t_1^{\beta+5}}{\beta+5}\right\}$$
$$-C_s\left\{T(1-\delta T)\left[a_0\left(t_1-\frac{T}{2}\right)+\frac{a_1}{2}\left(t_1^2-\frac{T^2}{3}\right)+\frac{a_2}{3}\left(t_1^3-\frac{T^3}{4}\right)+\frac{a_3}{4}\left(t_1^4-\frac{T^4}{5}\right)+\frac{a_4}{5}\left(t_1^5-\frac{T^5}{6}\right)\right]+\delta T\left[\frac{a_0}{2}\left(t_1^2-\frac{T^2}{3}\right)+\frac{a_1}{3}\left(t_1^3-\frac{T^3}{4}\right)+\frac{a_2}{4}\left(t_1^4-\frac{T^4}{5}\right)+\frac{a_3}{5}\left(t_1^5-\frac{T^5}{6}\right)+\frac{a_4}{6}\left(t_1^6-\frac{T^6}{7}\right)\right]-t_1^2(1-\delta T)\left[\frac{a_0}{2}+\frac{a_1t_1}{3}+\frac{a_2t_1^2}{4}+\frac{a_3t_1^3}{5}+\frac{a_4t_1^4}{6}\right]-\delta t_1^3\left[\frac{a_0}{3}+\frac{a_1t_1}{4}+\frac{a_2t_1^2}{5}+\frac{a_3t_1^3}{6}+\frac{a_4t_1^4}{7}\right]\right\}+C_L\delta\left(\frac{a_0T^2}{2}+\frac{a_1T^3}{6}+\frac{a_2T^4}{12}+\frac{a_3T^5}{20}+\frac{a_4T^6}{30}-a_0Tt_1-\frac{t_1^2}{2}(a_1T-a_0)-\frac{t_1^3}{3}(a_2T-a_1)-\frac{t_1^4}{4}(a_3T-a_2)-\frac{t_1^5}{5}(a_4T-a_3)+\frac{et_1^6}{6}\right) \quad (13)$$

Our objective is to minimize the total cost.

The necessary condition is $\frac{\partial TC}{\partial t_1}=0$ and $\frac{\partial^2 TC}{\partial {t_1}^2}>0$ for all $t_1>0$

We get

$$\frac{\partial TC}{\partial t_1} = C_p\left\{a_0\alpha t_1^{\beta}+a_1\alpha t_1^{\beta+1}+a_2\alpha t_1^{\beta+2}+a_3\alpha t_1^{\beta+3}+a_4\alpha t_1^{\beta+4}+a_0\delta T+a_1\delta Tt_1+a_2\delta Tt_1^2+a_3\delta Tt_1^3+a_4\delta Tt_1^4-a_0\delta t_1-a_1\delta t_1^2-a_2\delta t_1^3-a_3\delta t_1^4-a_4\delta t_1^5\right\}+C_h\left\{a_0t_1+a_1t_1^2+a_2t_1^3+a_3t_1^4+a_4t_1^5+a_0\alpha t_1^{\beta+1}+a_1\alpha t_1^{\beta+2}+a_2\alpha t_1^{\beta+3}+a_3\alpha t_1^{\beta+4}+\right.$$

$a_4\alpha t_1^{\beta+5} - \frac{a_0\alpha t_1^{\beta+1}}{\beta+1} - \frac{a_1\alpha t_1^{\beta+2}}{\beta+1} - \frac{a_2\alpha t_1^{\beta+3}}{\beta+1} - \frac{a_3\alpha t_1^{\beta+4}}{\beta+1} - \frac{a_4\alpha t_1^{\beta+5}}{\beta+1} - \frac{a_0\alpha^2 t_1^{2\beta+1}}{\beta+1} - \frac{a_1\alpha^2 t_1^{2\beta+2}}{\beta+1} - \frac{a_2\alpha^2 t_1^{2\beta+3}}{\beta+1} - \frac{a_3\alpha^2 t_1^{2\beta+4}}{\beta+1} - \frac{a_4\alpha^2 t_1^{2\beta+5}}{\beta+1}\Big\} - C_s\{T(1-\delta T)[a_0 + a_1t_1 + a_2t_1^2 + a_3t_1^3 + a_4t_1^4] + \delta T[a_0t_1 + a_1t_1^2 + a_2t_1^3 + a_3t_1^4 + a_4t_1^5] - (1-\delta T)[a_0t_1 + a_1t_1^2 + a_2t_1^3 + a_3t_1^4 + a_4t_1^5] - \delta[a_0t_1^2 + a_1t_1^3 + a_2t_1^4 + a_3t_1^5 + a_4t_1^6]\} + C_d\Big\{a_0\alpha t_1^{\beta} + a_1\alpha t_1^{\beta+1} + a_2\alpha t_1^{\beta+2} + a_3\alpha t_1^{\beta+3} + a_4\alpha t_1^{\beta+4}\Big\} + C_L\delta\{-a_0T - t_1(a_1T - a_0) - t_1^2(a_2T - a_1) - t_1^3(a_3T - a_2) - t_1^4(a_4T - a_3) + et_1^5\}$ (14)

and

$\frac{\partial^2 TC}{\partial {t_1}^2} = C_p\Big\{a_0\alpha\beta t_1^{\beta-1} + a_1\alpha(\beta+1)t_1^{\beta} + a_2\alpha(\beta+2)t_1^{\beta+1} + a_3\alpha(\beta+3)t_1^{\beta+2} + a_4\alpha(\beta+4)t_1^{\beta+3} + a_1\delta T + 2a_2\delta Tt_1 + 3a_3\delta Tt_1^2 + 4a_4\delta Tt_1^3 - a_0\delta - 2a_1\delta t_1 - 3a_2\delta t_1^2 - 4a_3\delta t_1^3 - 5a_4\delta t_1^4\Big\} + C_h\Big\{a_0 + 2a_1t_1 + 3a_2t_1^2 + 4a_3t_1^3 + 5a_4t_1^4 + a_0\alpha(\beta+1)t_1^{\beta} + a_1\alpha(\beta+2)t_1^{\beta+1} + a_2\alpha(\beta+3)t_1^{\beta+2} + a_3\alpha(\beta+4)t_1^{\beta+3} + a_4\alpha(\beta+5)t_1^{\beta+4} - a_0\alpha t_1^{\beta} - \frac{a_1\alpha(\beta+2)t_1^{\beta+1}}{\beta+1} - \frac{a_2\alpha(\beta+3)t_1^{\beta+2}}{\beta+1} - \frac{a_3\alpha(\beta+4)t_1^{\beta+3}}{\beta+1} - \frac{a_4\alpha(\beta+5)t_1^{\beta+4}}{\beta+1} - \frac{a_0\alpha^2(2\beta+1)t_1^{2\beta}}{\beta+1} - 2a_1\alpha^2t_1^{2\beta+1} - \frac{a_2\alpha^2(2\beta+3)t_1^{2\beta+2}}{\beta+1} - \frac{a_3\alpha^2(2\beta+4)t_1^{2\beta+3}}{\beta+1} - \frac{a_4\alpha^2(2\beta+5)t_1^{2\beta+4}}{\beta+1}\Big\} - C_s\{T(1-\delta T)[a_1 + 2a_2t_1 + 3a_3t_1^2 + 4a_4t_1^3] + \delta T[a_0 + 2a_1t_1 + 3a_2t_1^2 + 4a_3t_1^3 + 5a_4t_1^4] - (1-\delta T)[a_0 + 2a_1t_1 + 3a_2t_1^2 + 4a_3t_1^3 + 5a_4t_1^4] - \delta[2a_0t_1 + 3a_1t_1^2 + 4a_2t_1^3 + 5a_3t_1^4 + 6a_4t_1^5]\} + C_d\Big\{a_0\alpha\beta t_1^{\beta-1} + a_1\alpha(\beta+1)t_1^{\beta} + a_2\alpha(\beta+2)t_1^{\beta+1} + a_3\alpha(\beta+3)t_1^{\beta+2} + a_4\alpha(\beta+4)t_1^{\beta+3}\Big\} + C_L\delta\{-(a_1T - a_0) - 2t_1(a_2T - a_1) - 3t_1^2(a_3T - a_2) - 4t_1^3(a_4T - a_3) + 5et_1^4\}$ (15)

V. NUMERICAL EXAMPLE

Let us consider an inventory system with the following parameter values:

$$a_0 = 15, a_1 = 6, a_2 = 8, a_3 = 4, a_4 = 2, \alpha = 0.1, \beta = 2,$$
$$\delta = 0.1, T = 2, C_h = 3, C_p = 8, C_d = 6, C_s = 16, C_L = 2, C_o = 90.$$

Using these inputs in the proposed EOQ model with quartic demand, Weibull deterioration, and time-dependent partial backlogging, the optimal results are obtained as

t_1 =1.4872, Q=95.7892 and the total cost TC=1189.7

This numerical example demonstrates the applicability of the developed model and provides insight into its behaviour under the given parameter set. The obtained values indicate an optimal balance between ordering cost, holding cost, and deterioration effects. The results also show that appropriate selection of replenishment time t_1 and order quantity Q can significantly minimize the total inventory cost while accounting for deterioration and partial backlogging.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis is conducted to study how variations in key parameters influence the optimal values of the decision variables and total cost of the system. By changing one parameter at a time while keeping others constant, the responsiveness of the model can be observed.

Table – 1 : Variation in (α)

α	t_1	Q	TC
0.08	1.5231	95.3922	1175.3
0.09	1.5047	95.6051	1182.7
0.1	1.4872	95.7892	1189.7
0.2	1.3466	96.6811	1244.4
0.3	1.2460	96.8011	1281.8

Table 1 shows the impact of changes in the deterioration rate parameter (α) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). As observed, when the value of α increases from 0.08 to 0.3, the time period t_1 gradually decreases, while both Q and TC increase.

This behaviour indicates that as the deterioration rate becomes higher, items perish more quickly, causing the inventory to deplete faster. Consequently, the firm must reorder more frequently, leading to a shorter cycle time. To compensate for the higher rate of deterioration, the optimal order quantity slightly increases to maintain service levels. However, the increase in deterioration also results in greater wastage and higher holding and replenishment costs, which together raise the total cost of the system.

Thus, the analysis confirms that the deterioration parameter (α) plays a crucial role in determining the optimal inventory policy, and managing deterioration effectively can significantly reduce overall cost.

Table – 2 : Variation in (β)

β	t_1	Q	TC
1	1.5617	96.3121	1178.9
2	1.4872	95.7892	1189.7
3	1.4162	95.1517	1200.3
4	1.3557	94.4862	1209.4
5	1.3068	93.8806	1216.9

Table 2 illustrates the effect of changes in the Weibull shape parameter (β) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). It is observed that as β increases from 1 to 5, the time period t_1 and order quantity Q gradually decrease, while the total cost TC increases.

This trend can be explained by the fact that a higher β value in the Weibull distribution implies a faster rate of deterioration over time. As a result, inventory items tend to perish more quickly, reducing the duration for which the stock remains usable. Consequently, the optimal replenishment cycle becomes shorter (t_1 decreases), and firms place smaller orders (Q decreases) to avoid excessive deterioration losses. However, frequent replenishment and increased deterioration elevate the overall operational cost, leading to an increase in the total cost (TC).

Thus, the analysis highlights that as the shape parameter (β) rises, the system experiences higher deterioration sensitivity, which negatively impacts both inventory duration and cost efficiency. Proper control of deterioration and improvement in storage conditions can help minimize these cost escalations.

Table – 3 : Variation in (δ)

δ	t_1	Q	TC
0.08	1.4962	96.1573	1192.4
0.09	1.4918	95.9751	1191.1
0.1	1.4872	95.7892	1189.7
0.2	1.4358	93.6903	1174.9
0.25	1.4051	92.4376	1166.7

Table 3 presents the influence of the backlogging parameter (δ) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). From the table, it is observed that as δ increases from 0.08 to 0.25, all three values t_1, Q and TC gradually decrease.

This outcome indicates that when δ increases, the rate of partial backlogging becomes more time-dependent, meaning that customers are more willing to wait for backordered items as time progresses. As a result, the shortage portion of the cycle can be managed more effectively, allowing the firm to operate with a shorter cycle time (t_1) and a smaller replenishment quantity (Q). Since fewer items are ordered and held in stock, the holding and deterioration costs reduce, which in turn lowers the total cost (TC).

Thus, the analysis demonstrates that increasing the backlogging rate parameter (δ) improves inventory efficiency by reducing both order quantity and total cost. This highlights the importance of considering customer backlogging behaviour when determining optimal inventory policies for deteriorating items.

Table – 4 : Variation in (a_0)

a_0	t_1	Q	TC
13	1.4882	91.6135	1143.6
14	1.4877	93.7013	1166.7
15	1.4872	95.7892	1189.7
16	1.4867	97.8771	1212.7
17	1.4862	99.9649	1235.7

Table 4 shows the effect of varying the demand parameter (a_0) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). As observed, when the value of a_0 increases from 13 to 17, the time period t_1 slightly decreases, while both the order quantity (Q) and total cost (TC) increase steadily.

This behaviour is expected because a_0 represents the base level of demand in the quartic demand function. When a_0 increases, the overall demand for the product rises, requiring larger replenishment quantities to meet the higher consumption rate. As a result, the inventory depletes faster, slightly reducing the non-shortage period (t_1). The increase in demand also leads to higher purchasing, holding, and deterioration costs, which collectively raise the total cost of the system.

Thus, the analysis confirms that as the demand rate parameter (a_0) increases, the firm must order more frequently and in larger quantities, which consequently increases the total cost. Efficient demand forecasting and proper inventory control are therefore crucial to maintaining cost-effectiveness in such situations.

Table – 5 : Variation in (a_1)

a_1	t_1	Q	TC
4	1.4887	91.6139	1140.7
5	1.4879	93.7015	1165.2
6	1.4872	95.7892	1189.7
7	1.4865	97.8769	1214.1
8	1.4858	99.9646	1238.6

Table 5 depicts the effect of changing the demand coefficient (a_1) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). It can be observed that as a_1 increases from 4 to 8, the time period t_1 slightly decreases, whereas both the order quantity (Q) and total cost (TC) consistently increase.

This trend arises because the parameter a_1 determines the linear growth rate of demand with time in the quartic demand function. When a_1 increases, demand rises more rapidly, causing inventory to deplete faster. Consequently, the duration of the non-shortage period (t_1) shortens, as the stock is exhausted sooner. To satisfy the higher demand, the system requires a larger replenishment quantity (Q), which in turn increases the total cost due to higher purchasing, holding and deterioration expenses.

Thus, the results indicate that an increase in the linear demand coefficient (a_1) leads to greater consumption rates and higher total costs, emphasizing the need for efficient replenishment planning to control inventory expenses under growing demand conditions.

Table – 6 : Variation in (a_2)

a_2	t_1	Q	TC
6	1.4773	90.0768	1123.8
7	1.4824	92.9321	1156.7
8	1.4872	95.7892	1189.7
9	1.4917	98.6480	1222.7
10	1.4960	101.5084	1255.7

Table 6 presents the effect of the quadratic demand parameter (a_2) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). From the table, it is observed that as a_2 increases from 6 to 10, all three values t_1, Q, and TC increase gradually.

This behaviour can be explained by the fact that the coefficient a_2 controls the quadratic component of the quartic demand function, influencing how demand accelerates over time. A higher value of a_2 signifies a faster growth in demand during the replenishment cycle. As a result, the inventory must be replenished with larger quantities (Q) to meet the rising demand, and the system slightly extends the non-shortage period (t_1) to balance the increased consumption rate. The rise in demand and replenishment quantity subsequently raises the total inventory cost (TC) due to higher purchasing and holding expenses.

Thus, the analysis concludes that an increase in the quadratic demand parameter (a_2) leads to higher order quantities and costs, highlighting the importance of carefully monitoring demand patterns to optimize inventory decisions and maintain cost efficiency.

Table – 7 : Variation in (a_3)

a_3	t_1	Q	TC
2	1.4046	86.3436	1093.9
3	1.4457	91.0045	1140.7
4	1.4872	95.7892	1189.7
5	1.5287	100.7084	1241.3
6	1.5696	105.7709	1296.4

Table 7 shows the effect of the cubic demand parameter (a_3) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). It is observed that as a_3 increases from 2 to 6, all three values— t_1, Q, and TC—increase consistently.

This trend occurs because the parameter a_3 represents the cubic component in the quartic demand function, which significantly influences the rate of change in demand over time. When a_3 increases, demand rises more sharply in later periods of the cycle, requiring a larger replenishment quantity (Q) to meet the higher demand. To accommodate this increased consumption, the inventory system extends the non-shortage period (t_1) slightly, ensuring sufficient stock availability. However, the higher demand and larger order quantities lead to increased holding, deterioration, and purchasing costs, thereby raising the total cost (TC).

Thus, the analysis indicates that an increase in the cubic demand parameter (a_3) amplifies both demand intensity and cost. Proper demand forecasting and effective control strategies are essential to maintain inventory balance and minimize the resulting cost escalation.

Table – 8 : Variation in (a_4)

a_4	t_1	Q	TC
1	1.6087	91.4821	1130.0
2	1.4872	95.7892	1189.7
3	1.4066	100.9134	1266.4
4	1.3470	106.3962	1348.6
5	1.3000	112.0730	1432.9

Table 8 illustrates the effect of changes in the quartic demand parameter (a_4) on the optimal cycle time (t_1), order quantity (Q), and total cost (TC). As shown, when a_4 increases from 1 to 5, the time period t_1 decreases, while both the order quantity (Q) and total cost (TC) increase significantly.

This trend occurs because the coefficient a_4 represents the highest-order term in the quartic demand function, which governs the steepness or acceleration of demand growth over time. When a_4 increases, demand rises sharply toward the end of the replenishment cycle, causing inventory to deplete faster and shortening the duration of the non-shortage period (t_1). To meet the rapidly increasing demand, the system must order larger quantities (Q), which leads to higher purchasing, holding, and deterioration costs. Consequently, the total cost (TC) also increases.

Thus, the analysis reveals that an increase in the quartic demand parameter (a_4) intensifies demand fluctuations, resulting in higher order quantities and total costs while reducing the effective inventory cycle time. This emphasizes the need for efficient demand monitoring and timely replenishment decisions to minimize cost escalation in highly dynamic demand environments.

VII. CONCLUSION

In this paper, an Economic Order Quantity (EOQ) model has been developed for deteriorating items considering a quartic time-dependent demand pattern, Weibull distributed deterioration rate, and time-dependent partial backlogging. The proposed model extends traditional EOQ frameworks by incorporating more realistic assumptions that reflect complex market and product behaviours. The quartic demand function effectively captures fluctuating demand trends over time, while the Weibull deterioration rate provides flexibility to represent different types of decay processes. Furthermore, the inclusion of time-dependent partial backlogging recognizes that customers' willingness to wait decreases as the waiting time increases.

The total average cost of the inventory system was formulated by combining ordering, holding, deterioration, and shortage costs. Analytical methods were applied to determine the optimal replenishment time and order quantity that minimize the total cost. A numerical example was presented to illustrate the applicability of the model, and a sensitivity analysis revealed the influence of various system parameters on the optimal solution. The findings indicate that both demand and deterioration parameters have a significant impact on the total cost and inventory decisions.

This study provides a more generalized framework for inventory management involving deteriorating items with variable demand and shortage characteristics. The model can assist inventory managers in optimizing order policies for perishable or time-sensitive goods in dynamic market environments. Future research may extend this work by incorporating factors such as price- or advertisement-dependent demand, inflation, trade credit, or multi-item systems to enhance the model's applicability to broader real-world scenarios.

VIII. MANAGERIAL IMPLICATIONS

The proposed model offers valuable insights for practitioners and decision-makers in industries dealing with deteriorating or seasonal items, such as food, pharmaceuticals, and consumer electronics. By understanding how quartic demand and Weibull deterioration jointly affect inventory dynamics, managers can better plan replenishment schedules and prevent excessive holding or shortage costs. The time-dependent partial backlogging feature helps decision-makers design more realistic shortage management policies, aligning with customer tolerance levels. Implementing such a model allows firms to respond effectively to changing demand patterns, reduce waste from deterioration, and enhance overall profitability through optimized inventory control.

REFERENCES

[1] Suman, Vinod Kumar, "To develop a Deterministic Inventory Model for Deteriorating Items with Biquadratic Demand Rate and Constant Deterioration Rate", International Journal of Mathematics Trends and Technology, 2021, Volume 67 Issue 9, 96-104. https://doi.org/10.14445/22315373/IJMTT-V6719P511

[2] M. Pradeepa, S. Sobia, M. Valliathal, "A Deterministic Inventory Model for Deteriorating Items with Biquadratic Demand, Constant Deterioration Rate and Salvage Value",

International Journal of Mathematics Trends and Technology, 2025, Volume 71 Issue 9, 17-31. https://doi.org/10.14445/22315373/IJMTT-V71I9P103.

[3] M. Pradeepa, S. Sobia, M. Valliathal, "A Deterministic Inventory Model for Deteriorating Items with Biquadratic Demand, Linear Deterioration Rate, and Constant Holding Cost", EDU Journal of International Affairs and Research (EJIAR), 2025, Volume 4, Issue 1, 7-16. https://edupublications.com/index.php/ejiar

[4] Pooja Soni, and Rajender Kumar, "A Deterministic Inventory Model with Biquadratic Demand Variable Deterioration Rate and Carrying Cost," Turkish Online Journal of Qualitative Inquiry, vol. 13, no. 1, pp. 1530-1538, 2022.

[5] Suman, and Vinod Kumar, "Analytical Study of Deterministic Inventory Model with Consideration of Biquadratic Demand Rate and Weibull Distribution as Deterioration Rate," International Journal of Innovative Research in Technology, vol. 9, no. 4, pp. 642-651, 2022

[6] Atiqur Rahman, and Mohammed Forhad Uddin, "Analysis of Inventory Model with Time Dependent Quadratic Demand Function Including Time Variable Deterioration Rate without Shortage," Asian Research Journal of Mathematics, vol. 16, no. 12, pp. 97-109, 2020.

[7] R.P. Tripathi, and Shweta Singh Tomar, "Establishment of EOQ Model with Quadratic Time-sensitive Demand and Parabolic-time Linked Holding Cost with Salvage Value," International Journal of Operations Research, vol. 15, no. 3, pp. 135-144, 2018.

[8] Biswaranjan Mandal, "An EOQ Inventory Model for Time-varying Deteriorating Items with Cubic Demand under Salvage Value and Shortages", International Journal of Systems Science and Applied Mathematics, 2020; 5(4): 36-42.

[9] Chandan Kumar Sahoo, Kailash Chandra Paul & Saumya Suchi Sahoo, "An EOQ Form amid Cubic Demand Carry Forwarded with Three Parameter Weibull Distribution Deteriorating Item, Exclusive of Scarcity and Salvage Value", International Journal of Applied Engineering Research, 2021, Volume 16, Number 9, pp. 765-771.

[10] Anthony Limi, K. Rangarajan, P. Rajadurai, A. Akilbasha, K. Parameswari, " Three warehouse inventory model for non-instantaneous deteriorating items with quadratic demand, time-varying holding costs and backlogging over finite time horizon", Ain Shams Engineering Journal, 2024,

[11] R. Amutha, "An Inventory Model for Deteriorating Items with Quadratic Demand, Time Dependent Holding Cost and Salvage Value", International Journal of Innovative Research in Science, Engineering and Technology, 2017, Vol. 6, Issue 8, 16674 - 16678.

[12] Anshu Sharma, Sumeet Gill, Anil Kumar Taneja, "Inventory Optimization Model with Quadratic Demand Pattern and Non-instantaneous Deterioration Rate with Mixed Cash and Advance Payment Scheme", Indian Journal of Science and Technology, 2024, 17(26): 2763-2777. https://doi.org/10.17485/IJST/v17i26.688.