Numerical Solution of L-C-R Oscillatory Electrical Circuit Problems Using Adomian Decomposition Method and Haar Wavelet Method

Irfan Ul Haq[1*], S. K. Tiwari[2], Naveed Ul Haq[3], Pradeep Porwal[4]
[1,2,3 & 4] *School of Studies in Mathematics, Vikram University, Ujjain (M.P.).*

***Abstract*—For the purpose of evaluating and determining the solution of some L-C-R Oscillatory electric circuit problems, Adomian decomposition and Haar wavelet method are utilized for this purpose. We present two numerical methods Adomian Decomposition method (ADM) and Haar wavelet method (HWM) to solve few electrical circuit problems. A comparative study of two traditional numerical methods is conducted. Our research highlights the importance of the numerical methods Adomian decomposition method and Haar wavelet method as these methods are quick convergence of solutions. Haar wavelet method provides more accurate numerical solutions than Adomian decomposition method.**

***Keywords*—L-C-R Circuit, ADM), HWM , Convergence.**

I. INTRODUCTION

A numerical technique based on Adomian polynomials and decomposition method is developed in this research for solving the approximate solution of delay differential equation. The algorithm is illustrated by using initial value problems. The results obtained show that only few terms are required to obtain an approximate solution [1]. An alternate algorithm for computing Adomian polynomials and illustrate some numerical examples to show the simplicity and efficiency of the new proposed method [2]. The theory and applications of a new computational method based on Adomian polynomials and decomposition method has been applied for solving ordinary differential equations. The obtained solution takes the form $y' = f(x,y)$, $y(x_0) = y_0$ of a convergent series with computable component. It has been found that the method is suitable to obtain the solution of initial value problems either it has oscillatory solution or exponential solutions [3]. Some modifications in Adomian decomposition method have been presented to find the numerical solution of initial value problems arising in ordinary differential equations. The efficiency of the modified method is illustrated by several numerical examples. a new algorithm has been presented for calculating Adomian polynomial for nonlinear operators. This algorithm helps studying various forms of nonlinearity. Nonlinear evolution model has discussed in this research [4,5]. Adomian decomposition method (ADM) has been used to solve the one-dimensional nonlinear Burgers' equation. Convergence analysis of such method is also presented. The probable solution of this equation is computed in the form of a series with simple computable components. The exactness of the presented numerical scheme is analyzed by comparing numerical and analytical results. Numerical data show that the presented numerical technique is more accurate and easy than other methods [6].

Adomian decomposition method is applied to solve the fuzzy heat equations for which it is hard to find the solution by applying classical methods [7]. Nhawu, G. et al For numerical solutions of first order differential equations, Adomian decomposition method has been used in this research. Adomian decomposition method is shown as a powerful numerical technique which considers the numerical solutions of a non-linear equation as an infinite series that normally converges to the analytical solution [8]. Haar wavelet collocation method (HWCM) is applied to nonlinear Volterra-Fredholm Hammerstein integral equations to find the numerical solutions. Properties of Haar wavelets are used to convert an integral equation into a system of algebraic equations. Numerical results are compared with exact solution to illustrate the accuracy of the proposed scheme [9]. The main advantages of the Haar wavelet method are its simplicity and low computation costs, which are caused by the scarcity of transform matrices and the small number of significant wavelet coefficients. In this research paper, we present two classical numerical techniques to find the solutions of oscillatory problems arising in science and engineering and discuss various cases that have arisen during oscillatory processes. This is because solutions of ordinary and partial differential equations that are not sufficiently smooth, when approximated by cubic, quadratic, and linear polynomials, result in poor convergence or no convergence in results [10-13].

II. THE ADOMIAN DECOMPOSITION METHOD

Consider differential equation

$$Ly + Ry + Ny = g(x), \quad (1)$$

where N is a non-linear operator, L is the higest order derivative which is assumed to be invertible and R is a linear differential operator of order less than L. Making Ly subject to formula, we get

$$Ly = g(x) - Ry - Ny. \quad (2)$$

By solving (2) for Ly, since L is invertible, we can write

$$L^{-1}Ly = L^{-1}g(x) - L^{-1}Ry - L^{-1}Ny. \quad (3)$$

For initial value problems, we define L^{-1} for $L = \frac{d^n}{dx^n}$ as the definite integration from 0 to x. If L is second-order operator, L^{-1} is integral and by solving (3) for y, we get

$$y = A + Bx + L^{-1}g(x) - L^{-1}Ry - L^{-1}Ny, \quad (4)$$

where A and B are constants of integration and can be found from the initial or boundary conditions.

The Adomian method consists of approximating the solution of (1) as an infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x). \quad (5)$$

And decomposing the non-linear operator N as

$$N(y) = \sum_{n=0}^{\infty} A_n, \quad (6)$$

where A_n are Adomian polynomial [6,7] of $y_0, y_1, y_2, \dots, y_n$ given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i y_i\right)\right]_{\lambda=0}, \qquad n = 0,1,2,\dots$$

The first few Adomian polynomials are

$$\left.\begin{aligned} A_0 &= N(y_0)\\ A_1 &= N'(y_0)y_1\\ A_2 &= N'(y_0)y_2 + N''(y_0)\frac{{y_1}^2}{2!}\\ A_3 &= N'(y_0)y_3 + N''(y_0)y_1y_2 + N'''(y_0)\frac{{y_1}^3}{3!}\end{aligned}\right\}$$

By substituting (5) and (6) into (4), we get

$$\sum_{n=0}^{\infty} y_n = A + Bx + L^{-1}g(x) - L^{-1}R\left(\sum_{n=0}^{\infty} y_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right).$$

The recursive relationship is found to be

$$\left.\begin{aligned} y_0 &= y(0) + xy'(0) + L^{-1}g\\ y_1 &= -L^{-1}\big(R(y_0)\big) - L^{-1}\big(A_0(y_0)\big)\\ y_2 &= -L^{-1}\big(R(y_1)\big) - L^{-1}\big(A_1(y_0, y_1)\big)\\ &\;\;\vdots\\ y_{n+1} &= -L^{-1}\big(R(y_n)\big) - L^{-1}\Big(A_n\big(y_0, y_1, y_2 \dots y_n\big)\Big)\end{aligned}\right\}$$

Using the above recursive relationship, we can make solution of y as

$$y = \lim_{n\to\infty} \Phi_n(y), \quad (7)$$

where

$$\Phi_n(y) = \sum_{i=0}^{n} y_i. \quad (8)$$

2.1. CONVERGENCE ANALYSIS OF ADM

The Adomian Decomposition Method is same to the sequence defined as follows [14] [15]

$$S_n = y_1 + y_2 + y_3 + \cdots + y_n$$

through the iterative scheme

$$S_0 = 0$$

$$S_{n+1} = N(y_0 + S_n) \quad (9)$$

is associated with the functional equation

$$S = N(y_0 + S)$$

Cherruault [14] [15] used the fixed-point theorem to solve Equation (16) numerically.

III. NUMERICAL OBSERVATIONS

This section compares the accuracy and effectiveness of the two methods. We use the Adomain decomposition method and the Haar wavelet method to solve some oscillator circuit problems Firstly, we discuss the problems of oscillating circuits arising in physics and engineering.

3.1. L-C-R CIRCUIT:

Now consider the discharge of a condenser C through an inductance L and the resistance R

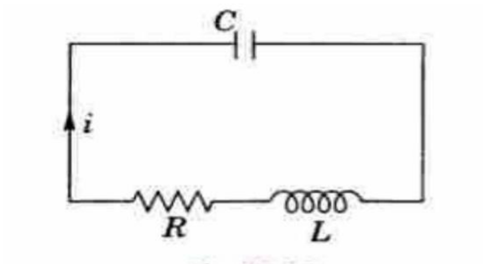

Figure 1: L-C-R Circuit.

Since the voltage drops across L, C and R are respectively

$$L\frac{d^2q}{dt^2}, \frac{q}{C}, \text{and } R\frac{dq}{dt}.$$

Therefore, by Kirchhoff's law, we have $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0.$

Or writing $\frac{R}{L} = 2\lambda$ and $\frac{1}{LC} = \mu^2$, we have

$$\frac{d^2q}{dt^2} + 2\lambda\frac{dq}{dt} + \mu^2 q = 0. \quad (16)$$

Therefore, its auxiliary equation is

$$D^2 + 2\lambda D + \mu^2 = 0, \text{ where } D = -\lambda \pm \sqrt{\lambda^2 - \mu^2}.$$

To study the physical significance of the solution of the above equation, Three cases arises.

Case I. The auxiliary equation's roots are real and distinct, say (β_1, β_2). When $\lambda > \mu$, as a result, the solution of equation (16) is of the form

$$q = c_0 e^{\beta_1 t} + c_1 e^{\beta_2 t} \quad (17)$$

To determine constants, c_0, c_1, let the spring be stretched to a length $q = l$ and then released so that

$$q = l \text{ and } dq/dt = 0 \text{ at } t = 0.$$

From equation (17), we obtain

$$l = c_0 + c_1 \quad (18)$$

Also, from equation (17), by differentiation, we obtain

$$\frac{dq}{dt} = c_0\beta_1 e^{\beta_1 t} + c_2\beta_2 e^{\beta_2 t}$$

After using the conditions, we obtain

$$c_1\beta_1 + c_2\beta_2 = 0 \quad (19)$$

Solving equation (18) and equation (19), we obtain

$$c_0 = \frac{-l\beta_2}{\beta_1 - \beta_2}, \quad c_1 = \frac{l\beta_1}{\beta_1 - \beta_2}$$

Therefore, the solution of (16) is

$$q = \frac{l}{\beta_1 - \beta_2}(\beta_1 e^{\beta_2 t} - \beta_2 e^{\beta_1 t})$$

As $t \to \infty$, which shows that q is always positive and decreases to zero.

Numerical Example 1: Let $\mu = \sqrt{3}$, $\lambda = 2$ From (16), we have

$$q'' + 4q' + 3q = 0 \quad (20)$$

with initial conditions $q(0) = 2$, $q'(0) = 0$. The exact solution is

$$q(t) = 3e^{-t} - e^{-3t}$$

Apply Adomian decomposition method (ADM):

Define linear operator $L = \frac{d^2}{dt^2}$

The inverse operator is $L^{-1} = \iint_0^t (.)dtdt$

$$q_0 = 2,$$
$$q_1 = -3t^2,$$
$$q_2 = -4t^3 + \frac{3}{4}t^4,$$
$$q_3 = -4t^4 - \frac{6}{5}t^5 - \frac{3}{40}t^6,$$
$$q_4 = \frac{16}{5}t^5 + \frac{6}{5}t^6 + \frac{9}{70}t^7 + \frac{9}{2240}t^8 + \cdots \ldots$$

Therefore,

$$q(t) = q_0 + q_1 + q_2 + q_3 + q_4 + \cdots$$
$$q(t) = 2 - 3t^2 + 4t^3 - \frac{13}{4}t^4 + 2t^5 - \cdots$$

Apply Haar Wavelet Method:

Now consider the approximation

$$q''(t) = \sum_{i=1}^{n} a_i h_i(t)$$

In terms of t, integrating, we have

$$q'(t) = q'(0) + \sum_{i=1}^{n} a_i P_{1,i}(t)$$

Again, integrating with respect to t, we obtain

$$q(t) = q(0) + q'(0)t + \sum_{i=1}^{n} a_i P_{2,i}(t) \quad (21)$$

From equation (20), we obtain

$$\sum_{i=1}^{n} a_i[h_i(t) + 4P_{1,i}(t) + 3P_{2,i}(t)] = -4q'(0) - 3q(0) - 3tq'(0)$$

The numerical solution is obtained by substituting the value of wavelet coefficients into equation (21). From here, wavelet coefficients are achieved.

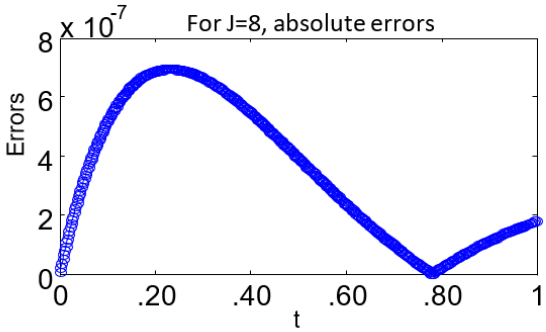

Figure 2: Haar wavelet method absolute errors for Example1

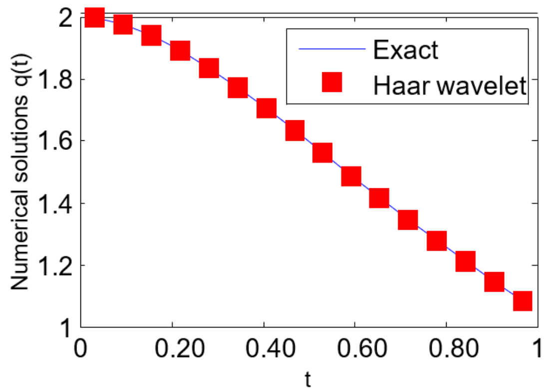

Figure 3: Exact and Haar wavelet solutions for Example 1 are compared.

Case II. The roots of auxiliary equation are real and equal,(each being= $-\lambda$). When $\lambda = \mu$.The general solution of equation (16) is

$$q = (c_0 + c_1 t)e^{-\lambda t}$$

when $q = l$ and $dq/dt = 0$ at $t = 0$, we obtain

$$c_0 = l, \quad c_1 = \lambda l$$

Therefore, the solution of (16) is

$$q = l\,(1 + \lambda t)e^{-\lambda t}$$

As $t \to \infty$, which shows that q is positive and decreases to zero.

Numerical example 2: Letting $\lambda = \mu = 1$

Now consider the second order linear differential equation

$$q'' + 2q' + q = 0 \qquad (22)$$

with initial conditions $q(0) = 2,\ q'(0) = 0$

The exact solution is

$$q(t) = 2(1 + t)e^{-t}$$

Apply Adomian Decomposition Method :

Define linear operator

$$L = \frac{d^2}{dq^2}$$

The inverse operator is then

$$L^{-1} = \iint_0^t (.)\, dtdt$$

$$q_0 = t,$$

$$q_1 = -t^2 - \frac{t^3}{6}$$

$$q_2 = \frac{2}{3}t^3 + \frac{t^4}{6} + \frac{t^5}{120},$$

$$q_3 = -\frac{t^4}{3} - \frac{t^5}{10} - \frac{t^6}{120} - \frac{t^7}{5040}$$

$$q_4 = \frac{2}{15}t^5 + \frac{2}{45}t^6 + \frac{t^7}{210} + \frac{t^9}{362800}, \ldots$$

Therefore,

$$q(t) = q_0 + q_1 + q_2 + q_3 + \cdots$$

$$q(t) = t - t^2 + \frac{t^3}{2} - \frac{t^4}{6} + \frac{t^5}{24} - \frac{t^6}{120} + \cdots$$

Using Haar Wavelet Method:

From equation (22), we have

$$\sum_{i=1}^{n} a_i\left[h_i(t) + 2P_{1,i}(t) + P_{2,i}(t)\right] = -2q'(0) - q(0) - q'(0).t$$

The numerical solution is obtained by substituting the wavelet coefficients into equation (21). From here, wavelet coefficient are obtained.

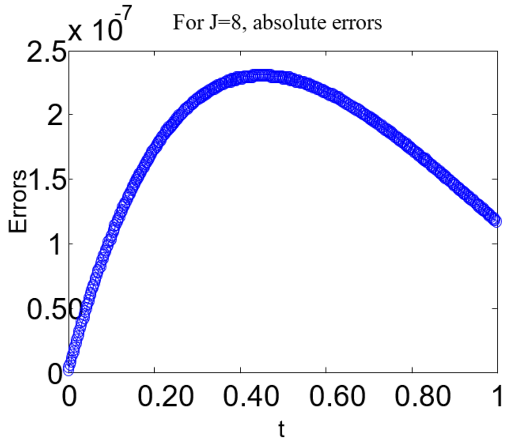

Figure 4: Haar wavelet and exact solutions are compared for example 2.

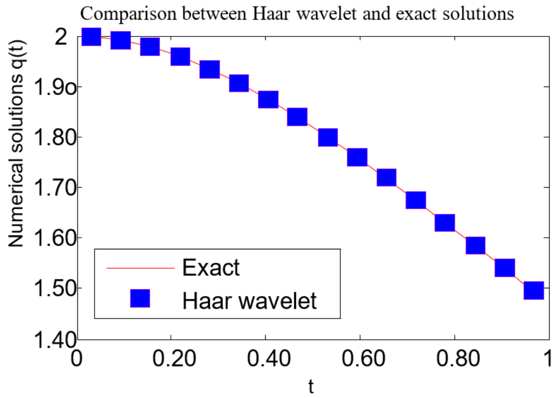

Figure 5: Comparison of exact and Haar wavelet solutions for Example 2

Case III. When $\lambda < \mu$, the roots of the auxiliary equation are imaginary, that is $D = -\lambda \pm i\delta$, where $\delta^2 = \mu^2 - \lambda^2$

So the solution of equation (16) is

$$q = e^{-\lambda t}(c_0 cos\delta t + c_1 sin\delta t)$$

when $q = l$ and $dq/dt = 0$ at $t = 0$, we obtain

$$c_0 = l, c_1 = \lambda l/\delta$$

Hence the solution of equation (16) is

$$q = le^{-\lambda t}\left(cos\alpha t + \frac{\lambda}{\delta} sin\delta t\right).$$

Numerical example 3: Let , $\mu = 1\ \lambda = \frac{1}{2}$

Consider the oscillatory equation

$$q'' + q' + q = 0 \quad (23)$$

with initial conditions, $q(0) = 2,\ q'(0) = 0$

The exact solution is

$$q(t) = 2.e^{-t/2}\left(cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}sin\frac{\sqrt{3}}{2}t\right)$$

Apply Adomian Decomposition Method:

Define linear operator $L = \frac{d^2}{dt^2}$

The inverse operator is $L^{-1} = \iint_0^t (.)dtdt$

$$q_0 = 2\,,$$

$$q_1 = -t^2$$

$$q_2 = \frac{t^3}{3} + \frac{t^4}{12}\,,$$

$$q_3 = -\frac{t^4}{12} - \frac{t^5}{30} - \frac{t^6}{360},$$

$$q_4 = \frac{t^5}{60} + \frac{t^6}{120} + \frac{t^7}{840} + \frac{t^8}{20160}, + \cdots$$

Therefore,

$$q(t) = q_0 + q_1 + q_2 + q_3 + q_4 + \cdots$$

$$q(t) = 2 - t^2 + \frac{t^3}{3} - \frac{t^5}{60} + \frac{t^6}{180} - \cdots$$

Apply Haar Wavelet Method (HWM):

From equation (23), we obtain

$$\sum_{i=1}^{n} a_i\left[h_i(t) + P_{1,i}(t) + P_{2,i}(t)\right] = -q(0) - q'(0) - q'(0).t$$

The numerical solution is obtained by substituting the wavelet coefficients into equation (21). From here, wavelet coefficients are achieved.

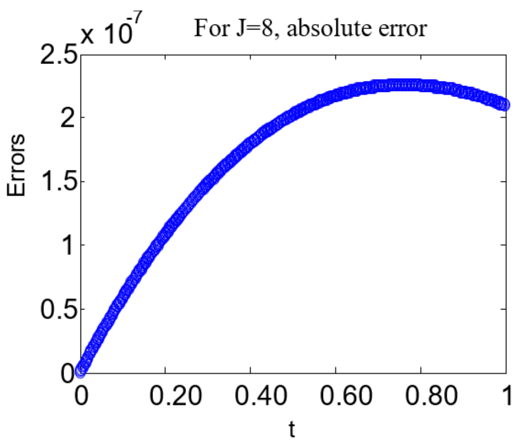

Figure 6: The Haar wavelet method's absolute errors for Example 3.

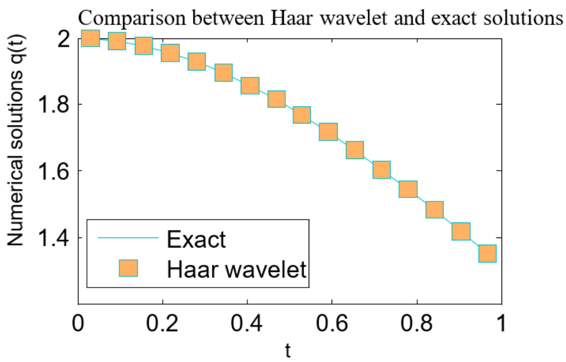

Figure 7: Comparing the Haar wavelet and exact solutions for Example 3.

IV. CONCLUSION

From above numerical data, it is concluded that Adomian decomposition method and Haar wavelet method are powerful mathematical technique for solving electric circuit problems arising in many applications of science and engineering. The numerical findings show that the Haar wavelet method yields more accurate numerical solutions than the Adomian decomposition method. We believe that when compared to the classical methods, the Haar wavelet method is completely competitive in comparison with classical method. For more accuracy the number terms may be increased.

REFERENCES

[1] Evans, D. J. and Raslan, K. R., (2004) "The Adomian Decomposition Method for Solving Delay Differential Equation" International Journal of Computer Mathematics, 1, pp. 1–6.

[2] Biazar, J., Babolian, E., Kember, G., Nouri, A. and Islam, R. (2003) "An Alternate Algorithm for Computing Adomian Polynomials in Special Cases" Applied Mathematics and Computation,138, pp.523–529.

[3] Ibijola.E.A, Adegboyegun.B.J and Halid.O.Y (2008) "On Adomian Decomposition Method (ADM) for Numerical Solution of Ordinary Differential Equations". Advances in Natural and Applied Sciences, 2(3): 165-169.

[4] Hendi.F.A, Bakodah.H.O, Almazmumy.M & Alzumi.H (2012) "A Simple Program for Solving Nonlinear Initial Value Problem using Adomian Decomposition Method". *International Journal of Research and Reviews in Applied Sciences,* 12 (3), pp. 397-406.

[5] Egbetade. S. A, Salawu. I. A (2016) "Solving Differential-Algebraic Equations by Adomian

Decomposition Method”. IJSRSET, 2, pp.139-142 .
[6] Wazwaz, A.M., (2000) “A New Algorithm for Calculating Adomian Polynomial for Nonlinear Operator” Appl. Math. Comput., 111, pp.53-69.
[7] Pirzada. U. M, Vakaskar. D. C. (2015) “Solution of Fuzzy Heat Equations Using Adomian Decomposition Method”. International Journal of Applied Mathematics and Mechanice, 3(1), pp. 87–91.
[8] Nhawu. G, Mafuta.P, Mushanyu. J (2016) ”The Adomain Decompostion Method for Numerical Solutions of First Order Differential Equations”. J.Math. Comput. Sci., 6(3), pp-307-314.
[9] Shiralashetti. S. C, Mundewadi. R. A, Naregal. S. S., Veeresh. B (2017) “Haar Wavelet Collocation Method for the Numerical Solution of Nonlinear Volterra-Fredholm-Hammerstein Integral Equations”. Global Journal of Pure and Applied Mathematics, 13(2), pp. 463-474.
[10] Singh, I., Kumar, S. (2018) “Numerical Solution of Two Dimension Time Dependent Schrodinger Equation Using Haar Wavelet”. International Journal of Scientific Research in Mathematical and Statistical Sciences,5(4), pp.278-282.
[11] Hariharan.G, Padma.S, Sathiyaseelan.D and Kannan.K (2016) “Wavelet Based Approximation Method for Solving Wave and Fractional Wave Equation Arising in Ship Dynamics”. International Journal of Oceans and Oceanography, 10(2), pp. 109-122.
[12] Khaireddine, F., Fateh, E.(2016) “Numerical Solution of Linear Integro-Differential Equation by Modified Haar wavelet”. Hacettepe Journal of Mathematics and Statistics, 45(5),pp-1393-1410.
[13] Inderdeep Singh, Sangeeta Arora, Sheo Kumar (2015), “Numerical Solution of Wave Equation Using Haar Wavelet”. International Journal of Pure and Applied Mathematics, 98(4), 457-469.
[14] Cherruault, Y. (1989) Convergence of Adomian’s Method. Kybernetes, 18, 31-38.
[15] Hosseini, M.M. and Nasabzadeh, H. (2006) On the Convergence of Adomian Decomposition Method. Applied Mathematics and Computation, 182, 536-543.
[16] Zhou, J.K. (1986) Differential Transform and Its Application for Electrical Circuits. Huazhong University Press, Wuhan