

# Beal's Conjecture

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**Abstract**—This idea gives a path to make new proofs using this method. Namely to prove Fermat's last theorem in simple and general way and it links the number theory and linear algebra.

## I. INTRODUCTION

Beal's Conjecture is a conjecture in number theory formulated in 1993 while investigating generalizations of Fermat's Last theorem set forth in 1997 as a Price problem by the United States of America's Dalls, Texas number theory enthusiast and billionaire banker, Mr. Daniel Andrew Beal.

Beal's conjecture states that: The equation  $A^x + B^y = C^z$  (1)

if  $A, B, C, x, y, z$  are integers with all exponents  $(x, y, z)$  greater than 2 then the bases  $A, B, C$  must share a common prime factor. In other words if  $A, B, C$  are relatively prime then the equation (1) has no solution.

## II. THEOREM

Statement: The equation  $x^l + y^m = z^n$  has no solution when  $x, y, z$  are relatively prime and  $l, m, n$  are integers greater than 2.

PROOF: - Assume,  $ax - by = x, cx - ay = z, ex - fz = y$  then we going to write

$$\begin{matrix} 1 & b & -a & x \\ -c & d & 1 & y \\ -e & 1 & f & z \end{matrix} \quad \begin{matrix} X \\ Y \\ Z \end{matrix} = \begin{matrix} A \\ B \\ C \end{matrix}$$

these equation in the form of matrix as  $A \begin{bmatrix} 1 & b & -a \\ -c & d & 1 \\ -e & 1 & f \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$

then  $AX = B$  (2).

now, we get determinant of A we get zero. with simplification we obtain

$$df + bcf + ac = be + aed + 1 \quad (3)$$

Consider

$$(az^2 - by^2)x^2 + (bx^2 - cz^2)y^2 = (ax^2 - cy^2)z^2$$

Here, take  $az^2 - by^2 = x, bx^2 - cz^2 = y$  and  $ax^2 - cy^2 = z$  then we get

$$x^3 + y^3 = z^3 \quad (4)$$

it has no solution in Euler's proof.

by our assumption, replace  $a, b, e, f, c, d$  as  $az, by, bx, cz, ax, cy$  then by equation (3) we get  $c^2yz +$

$a^2zx = b^2yx + 1$  (5) by comparing with equation (4) we have no solution where  $x, y, z$  are relatively prime. If we replace  $y$  by  $y^r, z$  by  $z^s$  and  $x$  by  $x^t$  (in assumption replacement) where  $r, s, t$  are greater than 1 then by equation (5) we get no solution because it still relatively prime with each other. so we conclude that  $x^l + y^m = z^n$  has no solution where  $x, y, z$  are relatively prime and for all integers  $l, m, n$  greater than 2. Hence the proof.

## III. CONCLUSION

This proof concludes that every Beal's conjecture pattern conjectures can be solvable using matrix methods.

## REFERENCES

- [1] Zhang Yue's Fermat's last theorem for  $n=3$ . <https://www.mathematicaljournal.com>.
- [2] linear differential equation wikipedia. <https://en.wikipedia.org>